Previous Years Problems on System of Particles and Rotional Motion for NEET

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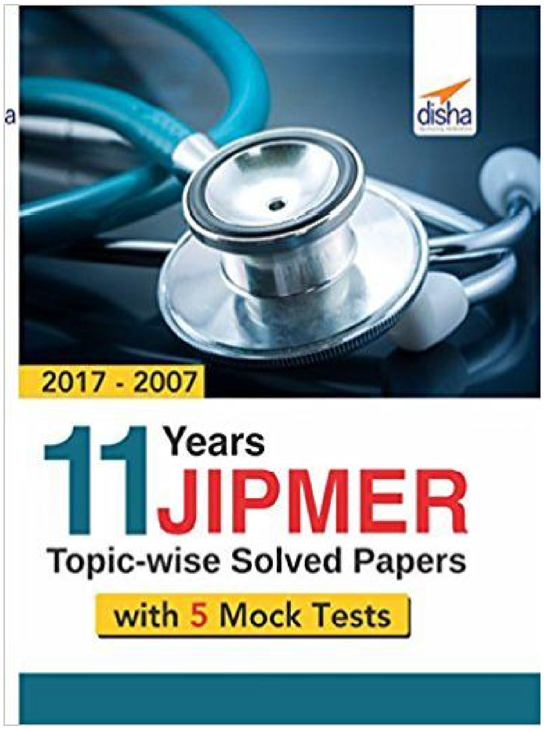

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Product Name : 11 year JIPMER Topic-wise Solved Papers (2017-2007) with 5 Mock Tests

Product Description : 11 years JIPMER Topic-wise Solved Papers with 5 Mock Tests consists of past years (memory based) solved papers from 2008 onwards till date, distributed in 29, 31, 38, 1 and 1 topics in Physics, Chemistry, Biology, English Language and Comprehension and Logical and Quantitative Reasoning respectively.

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Chapter 7 System of Particles and Rotational Motion

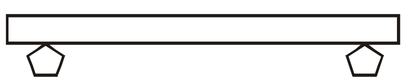
1. The ratio of the radii of gyration of a circular disc about a tangential axis in the plane of the disc and of a circular ring of the same radius about a tangential axis in the plane of the ring is **[2017]**

(a) $1 : \sqrt{2}$ (b) $1 : 3$
(c) $2 : 1$ (d) $\sqrt{5} : \sqrt{6}$

2. Two bodies have their moments of inertia I and 2I respectively about their axis of rotation. If their kinetic energies of rotation are equal, their angular momenta will be in the ratio **[2017]**

(a) 2 : 1 (b) 1 : 2
(c) $\sqrt{2} : 1$ (d) $1 : \sqrt{2}$

3. A uniform thin rod of weight W is supported horizontal by two vertical props at its ends. At $t = 0$, one of these supports is kicked out. The force on the other support immediately there after is **[2016]**



(a) 4 W (b) $\frac{W}{2}$ (c) 2 W (d) $\frac{W}{4}$

4. Two objects A and B of weight 10 kg and 20 kg respectively are kept at two points on the x-axis. B is moved 9 cm along the x-axis. By what distance A should be moved to keep the centre of mass the same? **[2016]**

(a) 21 cm (b) 18 cm
(c) 15 cm (d) 12 cm

5. A sphere of radius r is rolling without sliding. What is the ratio of rotational K.E. and total K.E. associated with the sphere? **[2016]**

(a) $\frac{1}{2}$ (b) $\frac{2}{7}$ (c) $\frac{2}{5}$ (d) 1

6. A tube of length L is filled completely with an incompressible liquid of mass M and closed at both the ends. The tube is then rotated in a horizontal plane about one of its ends with a uniform angular velocity w. The force exerted by the liquid at the other end is **[2016]**

(a) $M\omega^2 L/2$ (b) $M\omega^2 L$
(c) $M\omega^2 L/4$ (d) $M\omega^2 L^2/2$

7. The cylinders P and Q are of equal mass and length but made of metals with densities ρ_P and ρ_Q ($\rho_P > \rho_Q$). If their moment of inertia about an axis passing through centre and normal to the circular face be I_P and I_Q, then **[2016]**

(a) $I_P = I_Q$ (b) $I_P > I_Q$
(c) $I_P < I_Q$ (d) $I_P \leq I_Q$

8. A wheel whose moment of inertia is 2 kg m^2 has an initial angular velocity of 50 rad s^{-1}. A constant torque of 10 N m acts on the wheel. The time in which the wheel is accelerated to 80 rad s^{-1} is **[2015]**

(a) 12 s (b) 3 s (c) 6 s (d) 9 s

9. Particles of masses m, $2m$, $3m$, ... , nm grams are placed on the same line at distances l, $2l$, $3l$, ..., nl cm from a fixed point. The distance of centre of mass of the particles from the fixed point in centimetres is **[2015]**

(a) $\frac{(2n+1)l}{3}$ (b) $\frac{l}{n+1}$

(c) $\frac{n(n^2+1)l}{2}$ (d) $\frac{2l}{n(n^2+1)}$

10. From a circular disc of radius R and mass $9M$, a small disc of mass M and radius $\frac{R}{3}$ is removed concentrically. The moment of inertia of the remaining disc about an axis perpendicular to the plane of the disc and passing through its centre is **[2015]**

(a) $\frac{40}{9}MR^2$ (b) MR^2

(c) $4\ MR^2$ (d) $\frac{4}{9}MR^2$

11. The diameter of a flywheel is increased by 1% Increase in its moment of inertia about the central axis is **[2015]**

(a) 1% (b) 0.51% (c) 2% (d) 4%

12. A child is standing with folded hands at the centre of a platform rotating about its central axis. The kinetic energy of the system is K. The child now stretches his arms so that the moment of inertia of the system doubled. The kinetic energy of the system now is **[2014]**

(a) $2K$ (b) $\frac{K}{2}$ (c) $\frac{K}{4}$ (d) $4K$

13. A solid sphere of mass M and radius R rotates about an axis passing through its centre making 600 rpm. Its kinetic energy of rotation is **[2013]**

(a) $\frac{2}{5}\pi^2 MR^2$ (b) $\frac{2}{5}\pi MR^2$
(c) $80\,\pi^2 MR^2$ (d) $80\,\pi MR^2$

14. Moment is inertia of a unifrom rod of lenght L and mass M, about an axis passing through $L/4$ form one end and perpendicular to its length is

(a) $\frac{7}{36}ML^2$ (b) $\frac{7}{48}ML^2$ **[2013]**
(c) $\frac{11}{48}ML^2$ (d) $\frac{ML^2}{12}$

15. A flywheel rotates with a uniform angular acceleration. Its angular velocity increases from 20π rad s^{-1} to 40π rad s^{-1} in 10 seconds. How many rotations did it make in this period? **[2013]**

(a) 80 (b) 100 (c) 120 (d) 150

16. A 2 kg mass is rotating on a circular path of radius 0.8 m with angular velocity of 44 rad s^{-1}. If radius of path becomes 1 m, then what will be the value of angular velocity? **[2012]**

(a) 19.28 rad s^{-1} (b) 28.16 rad s^{-1}
(c) 8.12 rad s^{-1} (d) 35.26 rad s^{-1}

17. A uniform thin bar of mass $6m$ and length $12L$ is bent to make a regular hexagon. Its moment of inertia about an axis passing through the centre of mass and perpendicular to the plane of hexagon is **[2012]**

(a) $20mL^2$ (b) $30mL^2$
(c) $\left(\frac{12}{5}\right)mL^2$ (d) $6mL^2$

18. Two rings of radii R and nR made up of same material have the ratio of moment of inertia about an axis passing through centre is 1 : 8. The value of n is **[2012]**

(a) 2 (b) $2\sqrt{2}$ (c) 4 (d) $\frac{1}{2}$

19. A solid sphere and a hollow sphere of the same material and of same size can be distinguished without weighing **[2011]**

(a) by determining their moments of inertia about their coaxial axes
(b) by rolling them simultaneously on an inclined plane
(c) by rotating them about a common axis of rotation
(d) by applying equal torque on them

20. Point masses 1, 2, 3 and 4 kg are lying at the points (0, 0, 0), (2, 0, 0), (0, 3, 0) and (–2, –2, 0) respectively. The moment of inertia of this system about X-axis will be **[2011]**

(a) 43 kg –m^2 (b) 34 kg–m^2
(c) 27 kg – m^2 (d) 72 kg – m^2

21. The radius of gyration of a body about an axis at a distance 6 cm from its centre of mass is 10 cm. Then, its radius of gyration about a parallel axis through its centre of mass will be **[2011]**

(a) 80 cm (b) 8 cm (c) 0.8 cm (d) 80 m

22. A small disc of radius 2 cm is cut from a disc of radius 6 cm. If the distance between their centres is 3.2 cm, what is the shift in the centre of mass of the disc? **[2010]**

(a) 0.4 cm (b) 2.4 cm
(c) 1.8 cm (d) 1.2 cm

23. The moment of inertia of a circular disc of mass M and radius R about an axis passing through the centre of mass is I_0. The moment of inertia of another circular disc of same mass and thickness but half the density about the same axis is **[2009]**

(a) $\frac{I_0}{8}$ (b) $\frac{I_0}{4}$ (c) $8I_0$ (d) $2I_0$

24. Whan a ceiling fan is switched on, it makes 10 revolutions in the first 3 seconds. Assuming a uniform angular acceleration, how many rotations it will make in the next 3 seconds? **[2009]**

(a) 10 (b) 20 (c) 30 (d) 40

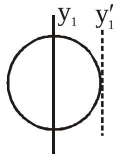
25. A ball of radius 11 cm and mass 8 kg rolls from rest down a ramp of length 2 m. The ramp in inclined at 35° to the horizontal. When the ball reaches the bottom, its velocity is (Take, sin 35° = 0.57 and cos 35° = 0.81) **[2008]**

(a) 2 m s^{-1} (b) 5 m s^{-1}
(c) 4 m s^{-1} (d) 6 m s^{-1}

26. A drum of radius R and mass M, rolls down without slipping along an inclined plane of angle θ. The frictional force. **[2007]**

(a) decreases the rotational and translational motion.
(b) dissipates energy as heat.
(c) decreases the rotational motion.
(d) converts translational energy to rotational energy.

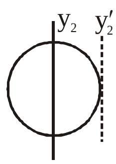
Solutions

1. **(d)** 

Circular disc ...(1)

$$I_{y_1} = \frac{MR^2}{4}$$

$$\therefore I'_{y_1} = \frac{MR^2}{4} + MR^2 = \frac{5}{4}MR^2$$



Circular ring ...(2)

$$I_{y_2} = \frac{MR^2}{2}$$

$$\therefore I'_{y_2} = \frac{MR^2}{2} + MR^2 = \frac{3}{2}MR^2$$

$$I'_{y_1} = MK_1^2,\ I'_{y_2} = MK_2^2$$

$$\therefore \frac{K_1^2}{K_2^2} = \frac{I'_{y_1}}{I'_{y_2}} \Rightarrow K_1 : K_2 = \sqrt{5} : \sqrt{6}$$

2. **(d)** $K = \frac{L^2}{2I} \Rightarrow L^2 = 2KI \Rightarrow L = \sqrt{2KI}$

$$\frac{L_1}{L_2} = \sqrt{\frac{K_1}{K_2} \cdot \frac{I_1}{I_2}} = \sqrt{\frac{K}{K} \cdot \frac{I}{2I}} = \frac{1}{\sqrt{2}}$$

$L_1 : L_2 = 1 : \sqrt{2}$

3. **(d)**

4. **(b)** To keep centre of mass at same point
r_{CM}(before) = r_{CM} (Final)

$$\frac{m_1r_1 + m_2r_2}{m_1 + m_2} = \frac{m_1(r_1 + x) + m_2(r_2 + 9)}{m_1 + m_2}$$

$m_1r_1 + m_2r_2 = m_1r_1 + m_1x_1 + m_2r_2 + 9m_2$
$m_1x = -9m_2$

$x = \frac{9m_2}{m_1}$ [neglecting negative sing]

$$x = 9 \times \frac{20}{10} = 18 \text{ cm}$$

5. **(b)** We know, $KE_{rotational} = \frac{1}{2}I\omega^2$

$$= \frac{1}{2}\frac{2}{5}MR^2\frac{\upsilon^2_{CM}}{R^2}$$

and $KE_{total} = \frac{1}{2}M\upsilon^2_{CM}\left[1 + \frac{K^2}{R^2}\right]$

$$\frac{KE_{rotational}}{KE_{total}} = \frac{\frac{1}{2} \times \frac{2}{5}MR^2\,\frac{\upsilon^2 CM}{R^2}}{\frac{1}{2}Mv^2_{CM}\left[1 + \frac{\frac{2}{5}R^2}{R^2}\right]}$$

$$\frac{KE_{rotational}}{KE_{total}} = \frac{2}{7}$$

6. **(a)** The centre of mass is at $\frac{L}{2}$ distance from the axis.

Hence, centripetal force $F_C = M\left(\frac{L}{2}\right)\omega^2$

So, the reaction at the other end will be equal to F_C.

7. **(c)** Mass of cylinder $M = \pi R^2 \ell\rho$

i.e $R^2 \propto \frac{1}{\rho}$ ...(i)

$$\frac{I_P}{I_Q} = \frac{\frac{1}{2}MR_P^2}{\frac{1}{2}MR_Q^2} \quad \text{(from eqn (i))}$$

$$\frac{I_P}{I_Q} = \frac{P_Q}{P_P} \Rightarrow I_P < I_Q$$

$(\because \rho_P > \rho_Q)$

8. (c) Initial angular velocity = 50 rad s^{-1}
Final angular velocity = 80 rad s^{-1},
Torque = 10 N m
Moment of inertia = 2 kg m^2
Angular acceleration α is given by $\tau = I\alpha$

$$\alpha = \frac{\tau}{I} = \frac{10}{2} = 5 \text{ rad s}^{-2}$$

Hence if t is the time,
$5t = 80 - 50 = 30 \Rightarrow t = 6$ s

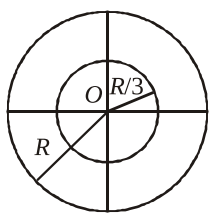
9. (a) $$X_{CM} = \frac{m_1x_1 + m_2x_2 + ...}{m_1 + m_2 + ...}$$

$$= \frac{ml + 2m\cdot 2l + 3m.3l + ...}{m + 2m + 3m + ...}$$

$$= \frac{ml(1+4+9+...)}{m(1+2+3+...)}$$

$$= \frac{l\frac{n(n+1)(2n+1)}{6}}{\frac{n(n+1)}{2}} = \frac{l(2n+1)}{3}$$

10. (a) Mass of the disc = 9M
Mass of removed portion of disc = M



The moment of inertia of the complete disc about an axis passing through its centre 0 and perpendicular to its plane is $I_1 = \frac{9}{2}MR^2$

Now, the moment of inertia of the disc with removed portion

$$I_2 = \frac{1}{2}M\left(\frac{R}{3}\right)^2 = \frac{1}{18}MR^2$$

Therefore, moment of inertia of the remaining portion of disc about O is

$$I = I_1 - I_2 = \frac{9MR^2}{2} - \frac{MR^2}{18} = \frac{40MR^2}{9}$$

11. (c) As we know,
Taking log on both sides
$\therefore$ $\log I = \log M + 2 \log R$

Differentiating, we get $\frac{dI}{I} = 0 + 2\frac{dR}{R}$

$$\therefore \quad \frac{dI}{I} \times 100 = 2\left(\frac{dR}{R}\right) \times 100$$

$= 2 \times 1\% = 2\%$

12. (b) From the law of conservation of angular momentum, $I\omega$ = constant
As I is doubled, ω becomes half.

K.E. of rotation, $K = \frac{1}{2}I\omega^2$

$\because$ I is doubled and ω is halved, therefore K.E. will become half i.e. $\frac{K}{2}$.

13. (c) Kinetic energy of rotation = $\frac{1}{2}I\omega^2$

$$= \frac{1}{2} \times \frac{2}{5}MR^2 \times (2\pi\upsilon)^2$$

$$= \frac{1}{5} \times 4\pi^2\upsilon^2MR^2 = 0.8\pi^2\left(\frac{600}{60}\right)^2 MR^2$$

$= 80\pi^2MR^2$

14. (b) Moment of inertia of a uniform rod of length L and mass M about an axis passing through the centre and perpendicular to its length is given by

$$I_0 = \frac{ML^2}{12} \qquad ...(i)$$

Applying the theorem of parallel axes, moment of inertia of a uniform rod of length L and mass M about an axis passing through $L/4$ from one end and perpendicular to its length is given by

$$I = I_0 + M\left(\frac{L}{4}\right)^2 = \frac{ML^2}{12} + \frac{ML^2}{16} = \frac{7ML^2}{48}$$

(Using (i))

15. (d) As we know,
$\omega_2 = \omega_1 + \alpha t \therefore 40\pi = 20\pi + \alpha \times 10$
or $\alpha = 2\pi$ rad s^{-2}

From, $\omega_2^2 - \omega_1^2 = 2\alpha\theta$
$(40\pi)^2 - (20\pi)^2 = 2 \times 2\pi\theta$

or $\theta = \frac{1200\pi^2}{4\pi} = 300\pi$

No. of rotations completed = $\frac{\theta}{2\pi} = \frac{300\pi}{2\pi}$
= 150

16. (b) Given: Mass (m) = 2kg
Initial radius of the path (r_1) = 0.8 m
Initial angular velocity (ω_1) = 44 rad s^{-1}
Final radius of the path (r_2) = 1 m
Initial moment of inertia,

$I_1 = mr_1^2 = 2 \times (0.8)^2 = 1.28$ kg m^2

Final moment of inertia

$I_2 = mr_1^2 = 2 \times (1)^2$

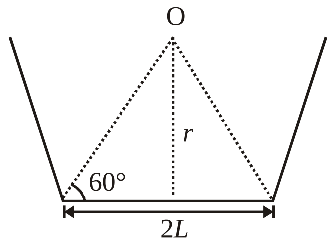
$= 2$ kg m^2

From the law of conservation of angular momentum, we get

$I'\omega_1 = I_2\omega_2$

$$\omega_2 = \frac{I_1 \times \omega_1}{I_2} = \frac{1.28 \times 44}{2} = 28.16 \text{ rad s}^{-1}$$

17. (a) Length of each side of hexagon = $2L$ and mass of each side = m.



Let O be centre of mass of hexagon.

Therfore perpendicular distance of O from each side, $r = L\tan 60° = L\sqrt{3}$.

The desired moment of inertia of hexagon about O is

$$I = 6[I_{\text{one side}}] = 6\left[\frac{m(2L)^2}{12} + mr^2\right]$$

$$= 6\left[\frac{mL^2}{3} + m(L\sqrt{3})^2\right] = 20mL^2$$

18. (a) The moment of inertia of circular ring whose axis of rotation is passing throught its centre is $I = mR^2$

$\therefore \quad I_1 = m_1R^2$ and $I_2 = m_2(nR)^2$

Since, both have same density

$$\therefore \quad \frac{m_2}{2\pi(nR) \times A} = \frac{m_1}{2\pi R \times A}$$

where A is cross-section area of ring.

$\therefore \quad m_2 = nm_1$

$$\because \quad \frac{I_1}{I_2} = \frac{m_1R^2}{m_2(nR)^2} = \frac{m_1R^2}{m_1 n(nR)^2} = \frac{1}{n^3}$$

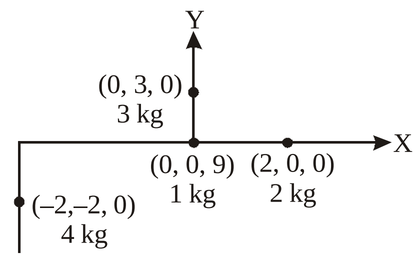
$$\because \quad \frac{I_1}{I_2} = \frac{1}{8} \text{ (Given)}$$

$$\therefore \quad \frac{1}{8} = \frac{1}{n^3} \text{ or } n = 2$$

19. (b) Acceleration of solid sphere is more than that of hollow sphere, it rolls faster, and reaches the bottom of the inclined plane earlier.

Hence, solid sphere and hollow sphere can be distinguished by rolling them simultaneously on an inclined plane.

20. (a) Moment of inertia of the whole system about the axis of rotation will be equal to the sum of the moments of inertia of all the particles.



$\therefore \quad I = I_1 + I_2 + I_3 + I_4$

$= 0 + 0 + 27 + 16 = 43$ kg m^2

21. (b) From the theorem of parallel axes, the moment of inertia

$I = I_{CM} + Ma^2$

where I_{CM} is moment of inertia about centre of mass and a is the distance of axis from centre.

$mk^2 = m(k^1)^2 + m(6)^2$

($\because$ $I = mk^2$ where k is radius of gyration)

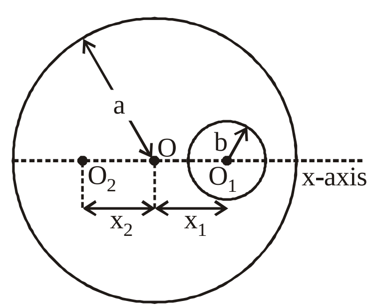
or, $\quad k^2 = (k^1)^2 + 36$

or, $\quad 100 = (k^1)^2 + 36$

$\Rightarrow \quad (k^1)^2 = 64$ cm

$\therefore \quad k^1 = 8$ cm

22. (a) The situation can be shown as : Let radius of complete disc is a and that of small disc is b. Also let centre of mass now shifts to O_2 at a distance x_2 from original centre.



The position of new centre of mass is given by

$$X_{CM} = \frac{-\sigma.\pi b^2.x_1}{\sigma.\pi a^2 - \sigma.\pi b^2}$$

Here, a = 6 cm, b = 2 cm, x_1 = 3.2 cm

Hence, $X_{CM} = \frac{-\sigma \times \pi(2)^2 \times 3.2}{\sigma \times \pi \times (6)^2 - \sigma \times \pi \times (2)^2}$

$= \frac{-12.8\pi}{32\pi} = -0.4$ cm.

23. (d) For circular disc 1
mass = M, radius R_1 = R
moment of inertia $I_1 = I_0$
For circular disc 2, of same thickness *t*,

mass = M, density = $\frac{\rho}{2}$

then $\pi R_2{}^2 t \times \frac{\rho}{2} = \pi R_1{}^2 t \times \rho = M$

$R_2{}^2 = 2R_1{}^2$

$R_2 = \sqrt{2}R_1 = \sqrt{2}R$

As we know, moment of inertia I ∝ $(\text{Radius})^2$

$\therefore \quad \frac{I_1}{I_2} = \left(\frac{R_1}{R_2}\right)^2$

$\frac{I_0}{I_2} = \left(\frac{R}{\sqrt{2}R}\right)^2 \Rightarrow I_2 = 2I_0$

24. (c) In first three seconds, angle rotated
θ = 2π × 10 rad

Using, $\theta = \omega_0 t + \frac{1}{2}\alpha t^2$

$\therefore \quad 2\pi \times 10 = 0 + \frac{1}{2}\alpha \times 3^2 = \frac{9}{2}\alpha$ ...(i)

For the rotation of fan in next three second, the total time of revolutions = 3 + 3 = 6 s

Let total number of revolutions = *N*

Then angle of revolutions, θ' = 2π*N* rad

$\therefore \quad 2\pi N = 0 + \frac{1}{2}\alpha \times 6^2 = 18\alpha$ ...(ii)

Dividing (ii) by (i), we get
N = 40
No. of revolution in last three secons
= 40 – 10 = 30 revolutions

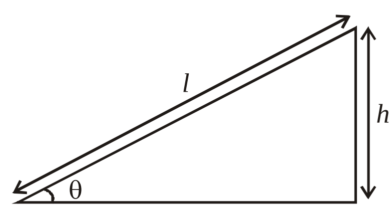
25. (c) Kinetic energy $K = \frac{1}{2}m\upsilon^2 + \frac{1}{2}I\omega^2$

$K = \frac{1}{2}m\upsilon^2 \times \frac{1}{2} \times \left(\frac{2}{5}mr^2\right)\omega^2$

$= \frac{1}{2}m\upsilon^2 \times \frac{1}{5}m\upsilon^2 = \frac{7}{10}m\upsilon^2$ $(\because \upsilon = r\omega)$

According to conservation of energy, we get

$\frac{7}{10}mv^2 = mgh$



$\upsilon = \sqrt{\frac{10}{7}gh} = \sqrt{\frac{10}{7}gl\sin\theta}$

$= \sqrt{\frac{10}{7} \times 9.8 \times 2\sin 35^\circ} = 4 \text{ m s}^{-1}$

26. (d) When a drum rolls without slipping, force of friction provides the torque necessary for rolling. Therefore, the frictional force converts translational energy to rotational energy.