

Test *RIDER*

JEE Advanced

A Single Door Entry to the Various IITs...

Paper 1

Duration : 3 Hours

Max. Marks : 180

Please read the instructions carefully. You are allotted 5 minutes specially for this purpose.

- ▶ This booklet is your question paper. Attempt all the questions.
- ▶ Blank papers, clipboards, log tables, slide rules, calculators, cameras, cellular phones, pagers and electronic gadgets are not allowed.
- ▶ Write your name and roll number in the space provided on the bottom of this page.

Question Paper Format

- ▶ The question paper consists of three parts (Physics, Chemistry and Mathematics). Each part consists of three sections.
- ▶ **Section 1** contains 10 multiple choice questions. Each question has four choices (a), (b), (c) and (d) out of which only one is correct.
- ▶ **Section 2** contains 5 multiple choice questions. Each question has four choices (a), (b), (c) and (d) out of which one or more than one is/are correct.
- ▶ **Section 3** contains 5 questions. The answer to each question is a single-digit integer, ranging from 0 to 9 (both inclusive)

Marking Scheme

- ▶ For each question in Section 1, you will be awarded **2 marks** for correct answer and zero mark for unattempted questions. No negative marks will be awarded for incorrect answers in this section.
- ▶ For each question in Section 2, you will be awarded **4 marks** for correct answer(s) and zero mark for unattempted questions. In all other cases, minus one (–1) mark will be awarded.
- ▶ For each question in Section 3, you will be awarded **4 marks** for the correct answer and zero mark for unattempted questions. In all other cases, minus one (–1) mark will be awarded.

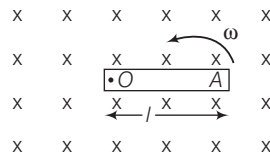
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Part I

Section 1 Single Correct Option Type

This section contains **10 multiple choice questions**. Each question has four choices, (a), (b), (c) and (d) out of which **only one** is correct.

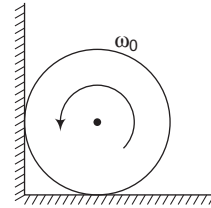
1. A conducting rod is rotating in a uniform magnetic perpendicular to it as shown in the figure. If the rotation is now become clockwise with the same angular speed then what will be the change in the potential difference between O and AP ?



- a. $-B\omega l^2$ b. $B\omega^2 l^2$ c. $2B^2\omega l^2$ d. $3B\omega l^2$

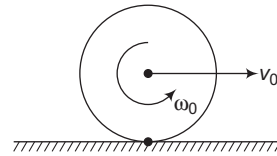
2. A uniform sphere of mass m and radius r is placed in a corner with initial counter clockwise angular velocity ω_0 as given. If $\mu = 1$ for both surfaces, then time required for sphere to come to rest is

- a. $\frac{\omega_0 r}{g}$ b. $\frac{\omega_0 r}{5g}$ c. $\frac{2\omega_0 r}{g}$ d. $\frac{2\omega_0 r}{5g}$



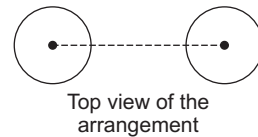
3. A uniform circular disc placed on a rough horizontal surface has initially a velocity v_0 and angular velocity ω_0 as shown. Disc comes to rest after moving certain distance, then $\frac{v_0}{r\omega_0}$ will be

- a. $\frac{1}{2}$ b. 1 c. $\frac{3}{2}$ d. 2



4. Two long straight wires with the same cross-section are arranged in air parallel to one another. The distance between the axis of the wire is η times larger than the radius of wires cross-section. The capacitance of the wires per unit length would be [Take, $\eta \gg 1$]

- a. $\frac{2\pi\epsilon_0}{\ln \eta}$ b. $\frac{\pi\epsilon_0}{2 \ln \eta}$
 c. $\frac{\pi\epsilon_0}{\ln \eta}$ d. Information is insufficient

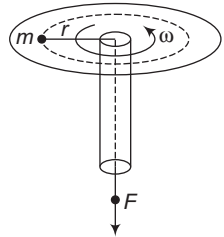


5. A boy with mass m is standing on the edge of the disc with moment of inertia I , radius R and initial angular velocity ω . Boy jumps off the disc with tangential speed v with respect to ground. The new angular velocity of disc will be

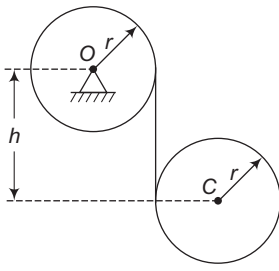
- a. $\sqrt{\frac{I\omega^2 - mv^2}{I}}$ b. $\sqrt{\frac{(I + mR^2)\omega^2 - mv^2}{I}}$
 c. $\frac{I\omega - mvR}{I}$ d. $\frac{(I + mR^2)\omega - mvR}{I}$

6. When a thin transparent sheet of refractive index $\mu = \frac{3}{2}$ is placed near one of the slit in YDSE, the intensity at centre of screen reduces to half of maximum intensity. Then, minimum thickness of sheet should be
- a. $\frac{\lambda}{4}$ b. $\frac{\lambda}{8}$ c. $\frac{\lambda}{2}$ d. $\frac{\lambda}{3}$

7. A small particle of mass m and its retaining cord are spinning with angular velocity ω on the horizontal surface of a smooth disc. As force F is slightly relaxed, r increases and ω changes. Determine the rate of change of ω with respect to r
- a. $+\frac{\omega}{r}$ b. $-\frac{\omega}{r}$
 c. $-\frac{2\omega}{r}$ d. $+\frac{2\omega}{r}$

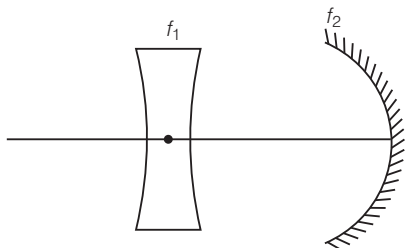


8. Consider the combination of two discs, then what will be the velocity of falling disc centre as a function of h . Both discs are identical and string does not slip relative to disc



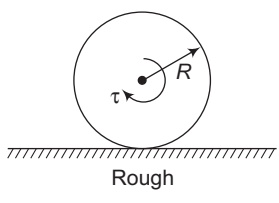
- a. $\sqrt{\frac{8gh}{5}}$ b. $\sqrt{\frac{6gh}{5}}$ c. $\sqrt{\frac{gh}{5}}$ d. \sqrt{gh}

9. A combination of concave lens and concave mirror is placed with the common principal axis. The rays coming from the infinity parallel to principal axis incident on the concave lens and final image is formed at the infinity. Choose the correct options.



- a. $|f_1| = |f_2|$ b. $|f_1| < 2|f_2|$ c. $|f_1| > 2|f_2|$ d. Information is sufficient

10. A homogeneous cylinder of mass m and radius r is at rest on horizontal plane when a couple is applied as shown. Determine magnitude of coefficient of friction between the wheel and plane so that rolling occur.

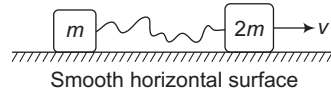


- a. $\mu > \frac{\tau}{mgR}$ b. $\mu > \frac{3\tau}{2mgR}$
 c. $\mu > \frac{3\tau}{mgR}$ d. $\mu > \frac{2\tau}{3mgR}$

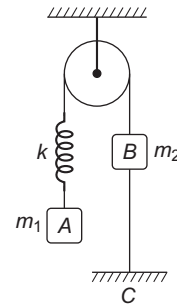
Section 2 More Than One Correct Option

This section contains 5 multiple choice questions. Each question has four choices, (a), (b), (c) and (d) out of which one or more than one is/are correct.

11. Two blocks are connected by a ideal string which is initially slacked and one block is projected along surface. Identify correct statements about subsequent motion

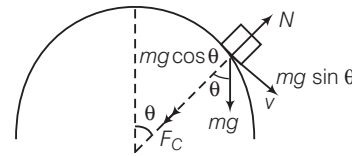


- a. initial speed of CM is $2v/3$
 b. speed of CM will remain same after string get taught
 c. impulse due to tension on block of mass m will increase its momentum
 d. impulse due to tension on block will decrease momentum of block of mass $2m$
12. In a given system $m_1 > m_2$, system is held at rest by thread BC . Just after the thread BC is burnt



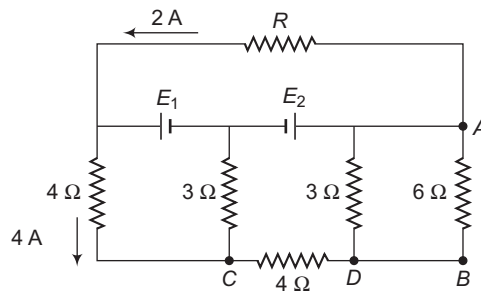
- a. initial acceleration of m_2 will be upwards
 b. magnitude of acceleration of blocks is $\left(\frac{m_1 - m_2}{m_1 + m_2}\right)g$
 c. initial acceleration of m_1 will be zero
 d. magnitude of acceleration of block is non-zero and unequal

13. A box is placed on the top most point of circular path as shown in the figure. It is given a push and it starts sliding downwards. Choose the correct options



- a. It will leave the circular path at a point where $N = 0$
 b. It will leave the circular path just after the instant when $N = 0$
 c. When it leave the circular path the contact force at that instant on the circular path is $\frac{mv^2}{R}$, where v is the speed of block at that instant
 d. When $N = 0$, there contact force on the circular path is $\frac{mv^2}{R}$, where v is the speed at that instant.

14. In the circuit shown in figure, E_1 and E_2 are two ideal sources of unknown emfs. Some currents are shown. Potential difference appearing across 6Ω resistance is $V_A - V_B = 10\text{ V}$,



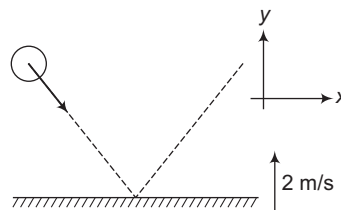
- a. the current in the 4Ω resistor in between C and D is 5 A
 b. the unknown emf E_1 is 36 V
 c. the unknown emf E_2 is 54 V
 d. the resistance R is equal to 9Ω

15. A photon moves vertically up in a region with gravitational field g downwards. The frequency of photon at an instant is ν . After it has moved up by height h ,
- its speed decreases
 - its energy decreases
 - its frequency is $\nu \left(1 - \frac{gh}{c^2}\right)$
 - its frequency is $\nu e^{\frac{-gh}{c^2}}$

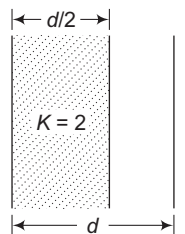
Section 3 Integer Answer Type

This section contains 5 multiple choice questions. The answer to each question is a single-digit integer, ranging from 0 to 9 (both inclusive).

16. A hydrogen atom is in its ground state and stationary, when a neutron of kinetic energy 25.5 eV collides with it. Take mass of hydrogen atom = mass of neutron = 1.6×10^{-27} kg and $1 \text{ eV} = 1.6 \times 10^{-19}$ J, if the collision is head-on, find the maximum possible number of photons of different wavelength that can be emitted after the collision from the hydrogen atom.
17. Let us consider that the value of gravitational potential on the surface of earth to be zero. A particle was initially at point at distance R_e from the earth's surface. The work done to bring this particle to earth's surface is $\left[\frac{-XGM_e m}{2R_e} \right]$, so the value of X is? (m = mass of particle)
18. Consider a horizontal surface moving vertically upward with velocity 2 m/s. A small ball of mass 2 kg is moving with velocity $2\hat{i} - 2\hat{j}$ (m/s). If coefficient of restitution is $1/2$ and friction coefficient is $1/3$. Find horizontal component of velocity of ball after collision



19. A certain series R - C circuit is formed using a resistance R , a capacitor without dielectric having a capacitance $C = 2$ F and a battery of emf $E = 3$ V. The circuit is completed and it is allowed to attain the steady state. After this, at $t = 0$, half the thickness of the capacitor is filled with a dielectric constant $K = 2$ as shown in the figure. The system is again allowed to attain a steady state. What will be the heat generated (in joule) in the capacitor between $t = 0$ and $t = \infty$?



20. A uniform rope of length L and mass m is held at one end and whirled in a horizontal circle with angular velocity ω . You can ignore the force of gravity on rope. The time required for a transverse wave to travel from one end of rope to other is $\frac{\pi}{\sqrt{n} \omega}$, then the value of n is

Part II

Section 1 Single Correct Option Type

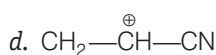
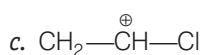
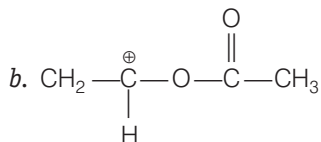
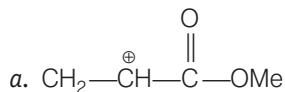
This section contains **10 multiple choice questions**. Each question has four choices, (a), (b), (c) and (d) out of which **only one** is correct.

- 21.** ${}_{92}\text{U}^{238}$ by successive decay changes to ${}_{82}\text{Pb}^{206}$, when a sample of uranium ore was analysed it was found that it contains 1g of U^{238} and 0.1g of Pb^{206} , considering that all the Pb^{206} had accumulated due to decay of U^{238} . Calculate the age of the ore. (Half-life of $\text{U}^{238} = 4.5 \times 10^9$ yrs).
- a. 0.1155×10^8 yrs b. 7.099×10^8 yrs
c. 0.154×10^{-9} yrs d. 7.099×10^{10} yrs

- 22.** Given that, λ_0 and λ are threshold wavelength and wavelength of incident light respectively then what will be the velocity of ejected electron from the metal surface?

a. $\sqrt{\frac{2h}{m}(\lambda_0 - \lambda)}$ b. $\sqrt{\frac{2hc}{m}(\lambda_0 - \lambda)}$ c. $\sqrt{\frac{2hc}{m}\left(\frac{\lambda_0 - \lambda}{\lambda\lambda_0}\right)}$ d. $\sqrt{\frac{2h}{m}\left(\frac{1}{\lambda_0} - \frac{1}{\lambda}\right)}$

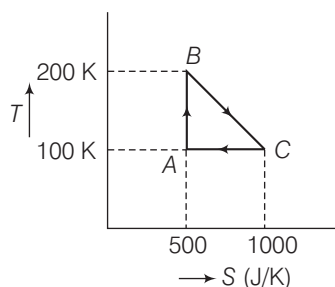
- 23.** Which of the following is the suitable monomer for cationic polymerisation?



- 24.** Which of the following pairs of complexes are isomeric with each other but their aqueous solutions exhibit different molar conductivities?

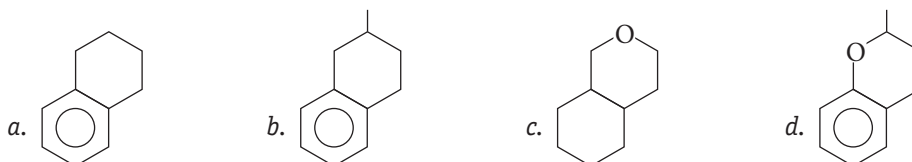
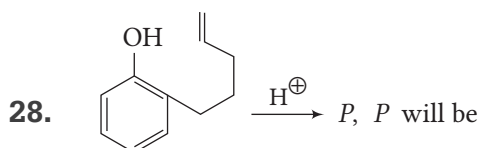
- a. $[\text{PtCl}_2(\text{NH}_3)_4]\text{Br}_2$ and $[\text{PtBr}_2(\text{NH}_3)_4]\text{Cl}_2$
b. $[\text{CoCl}_2(\text{NH}_3)_4]\text{NO}_2$ and $[\text{CoCl}(\text{NO}_2)(\text{NH}_3)_4]\text{Cl}$
c. $[\text{Co}(\text{NO}_2)(\text{NH}_3)_5]\text{Cl}_2$ and $[\text{Co}(\text{ONO})(\text{NH}_3)_5]\text{Cl}_2$
d. $[\text{CoBr}_2(\text{NH}_3)_4]\text{SO}_4$ and $[\text{Co}(\text{SO}_4)(\text{NH}_3)_4]\text{Br}_2$

- 25.** The efficiency of a reversible cycle is shown as in figure is



- a. 33.33% b. 52.3% c. 10% d. 29.2%

26. Which of following is a method for preparation of Engel's sulphur?
 a. $S_2Cl_2 + H_2S_4$ b. $2HNO_3 + H_2S$ c. $2H_2S + SO_2$ d. All of these
27. Phosphorus on reaction with hot solution of NaOH produces, A, which on reaction with nitric acid produces B. B can also be obtained by direct oxidation of phosphorus in air, which of the following is correct statement regarding A and B?
 a. A is, hydrate of phosphorus, B is oxide of phosphorus
 b. A is, hydride of phosphorus, B is oxide of phosphorus
 c. A is, oxide of phosphorus, B is hydride of phosphorus
 d. A is, hydroxide of phosphorus, B is oxide of phosphorus



29. The standard emf of $S^{2-}/CuS/Cu$ is, if $K_{sp}(CuS) = K$ and $E^0_{Cu^{2+}/Cu} = V$ volt at $25^\circ C$.
 a. $V + \frac{0.059}{2} \log K$ b. $V - \frac{0.059}{2} \log K$
 c. $2V + \log K$ d. $V^2 - \frac{0.059}{2} \log K$
30. Which of the following is an oxidation reaction?
 a. $CH_3-CH=CH_2 \xrightarrow[CCl_4]{Br_2}$ b. $CH_3-CH=CH_2 \xrightarrow[KMnO_4]{OH^\ominus}$
 c. $CH_3-CH=CH_2 \xrightarrow{H^\oplus/H_2O}$ d. Both a and b

Section 2 More Than One Correct Option

This section contains 5 multiple choice questions. Each question has four choices, (a), (b), (c) and (d) out of which one or more than one is/are correct.

31. Arrange the following inorganic compounds ions in their increasing orders of % s character.
 I. CO_3^{2-} II. XeF^4 III. I_3^- IV. NCl_3 V. $BeCl_2$
 a. II < III < IV < I < V b. II < IV < III < V < I
 c. III < II < I < V < IV d. II < IV < III < I < V
32. Nitrogen forms more than one type of oxide which may or may not contain H—N bond. The wide range of these types of compound is due to possibility of N to extend its oxidation upto 5. Which of the following oxides of nitrogen will contain atleast one N—N bond?
 a. N_2O_4 b. N_2O_5 c. N_2O_3 d. NO_2

33. Choose correctly matched columns.

A	B
a. $\text{MeOH} > \text{EtOH} > \text{Me}_2\text{CHOH} > \text{Me}_3\text{COH}$	Rate of esterification
b. $\text{Me}_3\text{COH} > \text{Me}_2\text{CHOH} > \text{MeCH}_2\text{OH}$	Rate of dehydration
c. $\text{HCOOH} > \text{MeCO}_2\text{H} > \text{EtCO}_2\text{H} > \text{Me}_3\text{C}-\text{CO}_2\text{H}$	Rate of esterification
d. $\text{MeCOCl} > (\text{MeCO})_2\text{O} > \text{MeCO}_2\text{Me} > \text{MeCONH}_2$	Rate of reactivity towards CH_3MgX

34. A compound X on treatment with sodium hexanitro cobaltate (III) produces yellow coloured precipitate and on reaction with hexachloroplatinum hydride also produces yellow coloured precipitate. Here X is

- a. NH_4^+ b. Na^+ c. Mg^{2+} d. K^+

35. Amino acid contains an acidic group $-\text{COOH}$ as well as a basic group $-\text{NH}_3$ the structure of

alanine is $\text{NH}_3^+-\text{CH}-\text{COO}^-$, which of the following statements, is/are incorrect statement

|
 CH_3

about it?

- a. It is an acidic amino acid
 b. It is an essential amino acid
 c. It will move to cathode in electrolysis at $\text{pH} = 5$
 d. It is an optically active amino acid

Section 3 Integer Answer Type

This section contains 5 multiple choice questions. The answer to each question is a single-digit integer, ranging from 0 to 9 (both inclusive).

36. Considering Z as internuclear axis. Find the number of delta bonding molecular orbital among following set?

$$d_{x^2-y^2} \text{ and } d_{xy}, d_{x^2-y^2} \text{ or } d_{xz}$$

37. If the K_{sp} of $\text{Mg}(\text{OH})_2$ is 1×10^{-12} , then 0.01 M MgCl_2 will precipitate at a limiting pH of.

38. Find the number of X-M-X angles in MX_6 , which has an octahedral geometry. If it is x then $\frac{x}{2}$ equals

39. Two vessels divided by a partition contain 1 mol of N_2 and 2 mol of O_2 gas, If the partition is removed and gases are mixed isothermally, then find the change in entropy due to mixing assuming initial and final pressure are same. The answer after dividing by a factor of 2 comes as.

40. In the Lindemann theory of unimolecular reactions, it is shown that apparent rate constant for such a reaction is $k_{\text{app}} = \frac{k_1 c}{1 + \alpha c}$, where c is the concentration of the reactant, k_1 and α are constants. Calculate the value of c for which k_{app} has 90% of its limiting value at c tending to infinitely large values, given $\alpha = 9 \times 10^5$. If answer is x then $|\log x|$ equals to

49. The equation of the straight line in the plane $\mathbf{r} \cdot \mathbf{n} = d$, which is parallel to $\mathbf{r} = \mathbf{a} + \lambda \mathbf{b}$ and passes through the foot of perpendicular drawn from the point $P(\mathbf{a})$ to the plane $\mathbf{r} \cdot \mathbf{n} = d$ is (where, $\mathbf{n} \cdot \mathbf{b} = 0$)
- a. $r = a + \left(\frac{d - a \cdot n}{n}\right) n + \lambda b$ b. $r = a + \left(\frac{d - a \cdot n}{|n|^2}\right) n + \lambda b$
 c. $r = a + \left(\frac{a \cdot n - d}{n^2}\right) n + \lambda b$ d. $r = a + \left(\frac{a \cdot n - d}{n}\right) n + \lambda b$
50. $\lim_{x \rightarrow 0} \left[\min(y^2 - 4y + 11) \cdot \frac{\sin x}{x} \right]$, (where $[\cdot]$ denotes the greatest integer function) is
- a. 5 b. 6 c. 7 d. does not exist

Section 2 More Than One Correct Option

This section contains 5 multiple choice questions. Each question has four choices, (a), (b), (c) and (d) out of which one or more than one is/are correct.

51. Given, $\lim_{x \rightarrow 0} \frac{f(x)}{x^2} = 2$, where $[\cdot]$ denotes greatest integer function, then
- a. $\lim_{x \rightarrow 0} [f(x)] = 0$ b. $\lim_{x \rightarrow 0} [f(x)] = 1$
 c. $\lim_{x \rightarrow 0} \left[\frac{f(x)}{x} \right]$ does not exist d. $\lim_{x \rightarrow 0} \left[\frac{f(x)}{x} \right]$ exists
52. Let E_1 and E_2 be two ellipses $\frac{x^2}{a^2} + y^2 = 1$ and $x^2 + \frac{y^2}{a^2} = 1$ (where a is a parameter). Then, the locus of the points of intersection of the ellipses E_1 and E_2 is a set of curves comprising
- a. two straight lines b. one straight lines
 c. one circle d. one parabola
53. The sum to n terms of the series $\tan^{-1}\left(\frac{1}{2}\right) + \tan^{-1}\left(\frac{2}{9}\right) + \tan^{-1}\left(\frac{1}{8}\right) + \tan^{-1}\left(\frac{2}{25}\right) + \tan^{-1}\left(\frac{1}{8}\right) + \dots + n$ terms
- a. $\tan^{-1}\left(\frac{1}{3}\right)$ b. $\tan^{-1}(3)$ c. $\cot^{-1}\left(\frac{1}{3}\right)$ d. None of these
54. p and q are non-zero constants. The equation $x^2 + px + q = 0$ has roots u and v , then
- a. $qx^2 + px + 1 = 0$ has roots $\frac{1}{u}, \frac{1}{v}$ b. $(x - p)(x - q) = 0$ has roots $u + v$ and $u \cdot v$
 c. $x^2 - (p^2 - 2q)x + q^2 = 0$ has roots u^2 and v^2 d. $x^2 + qx + p = 0$ has roots $\frac{u}{v}$ and $\frac{v}{u}$
55. Let $\alpha = a\hat{i} + b\hat{j} + c\hat{k}$, $\beta = b\hat{i} + c\hat{j} + a\hat{k}$ and $\gamma = c\hat{i} + a\hat{j} + b\hat{k}$ be three coplanar vectors with $a \neq b$ and $v = \hat{i} + \hat{j} + \hat{k}$, then v is perpendicular to
- a. α b. β c. γ d. None of these

Section 3 Integer Answer Type

This section contains 5 multiple choice questions. The answer to each question is a single-digit integer, ranging from 0 to 9 (both inclusive).

56. The minimum value of the expression $p = |z|^2 + |z - 3|^2 + |z - 6i|^2$ is q , then the value of $\frac{q}{6}$ is
57. If $x, y \in R$, satisfying the equation $\frac{(x-4)^2}{4} + \frac{y^2}{9} = 1$, then the difference between the largest and smallest value of the expression $\frac{x^2}{4} + \frac{y^2}{9}$ is
58. If A, B, C and D are four points in spaces, then $|\mathbf{AB} \times \mathbf{CD} + \mathbf{BC} \times \mathbf{AD} + \mathbf{CA} \times \mathbf{BD}| = k$ (area of ΔABC), where k is equal to
59. If the only integral solution of the equation $(x-1)^3 + (x-2)^3 + (x-3)^3 + \dots + (x-2007)^3 = 0$ is K then $\left(\frac{K}{502}\right)$ is
60. The equation to the side BC of ΔABC is $x = 2$. If the altitude through A meets the circumcircle of ΔABC at $P(7, -2)$, then the absolute value of the sum of the x and y -coordinates of the orthocentre of ΔABC is equal to

Analytical Explanations

1. (a) **Idea** The problem is based on rotating rod in a magnetic field (\mathbf{B}) perpendicular to it, then it's different parts will be in circular motion with linear speed $v = r\omega$. So, due to motional emf we can find out the emf generated between the two ends O and A . Then by change in rotation, the polarity of emf will also change.

When the rod is rotating anti-clockwise

$$\begin{aligned} e &= \int_0^l B\omega x dx \\ &= B\omega \left[\frac{x^2}{2} \right]_0^l \\ &= \frac{B\omega}{2} [l^2] \\ e_1 &= (V_0 - V_A) = \frac{B\omega l^2}{2} \end{aligned}$$

When the rod is rotating clockwise, then

$$\begin{aligned} e_2 &= V_0 - V_A = -\frac{B\omega l^2}{2} \\ \Rightarrow \text{So, } e_2 - e_1 &= -\frac{B\omega l^2}{2} - \left(\frac{B\omega l^2}{2} \right) = -B\omega l^2 \end{aligned}$$

TEST Edge In JEE Advanced/IIT JEE, different types of questions were asked on induced emf. There are different ways by which emf could be induced. Consider one such way as

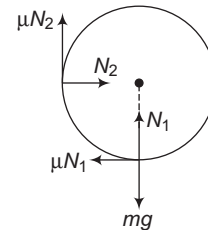
- If the conductor is not in motion but magnetic field is varying then low the electrons will feel the force to induce an emf between the ends of loop.
- We can learn a new thing that a varying magnetic field generates an electric field and due to this electric field, the electrons will feel some force and an emf will be induced between the ends.

2. (d) **Idea** The question is based on circular motion. When sphere is rotating against the wall and floor, so there will be frictional force on it. The normal reaction will adjust itself and due to it the $f_k = \mu_k N$ can be adjusted. The sphere will be in translational equilibrium but it will have angular retardation.

FBD of arranged object, it does not move it just rotates. So, resultant force will be zero.

$$\begin{aligned} N_2 &= \mu N_1 \\ \mu N_2 + N_1 &= mg \\ N_1 &= \frac{mg}{1 + \mu^2} \end{aligned}$$

and
$$N_2 = \frac{\mu mg}{1 + \mu^2}$$



Braking torque

$$\tau = \mu N_1 r + \mu N_2 r = \frac{\mu(1 + \mu)}{1 + \mu^2} mgr = mgr \quad (\because \mu = 1)$$

$$\tau = I\alpha$$

$$\frac{2}{5} mr^2 \alpha = mgr$$

$$\alpha = \frac{5g}{2r} \text{ (constant)}$$

So, it will come to rest

$$\omega = \omega_0 + \alpha t$$

$$0 = \omega_0 - \frac{5g}{2r} t$$

$$t = \frac{2\omega_0 r}{5g}$$

TEST Edge In JEE Advanced, rolling motion questions may come as it is an important part of mechanics.

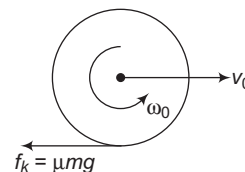
- Rolling can be divided into two parts, pure rolling and rolling with slipping. Even, we can have pure accelerated rolling where with $v_{CM} = R\omega$, we have $a_{CM} = R\alpha$
- In pure rolling, there is no need of any external force. But in pure accelerated rolling, external forces such as friction may also needed.

3. (a) **Idea** It is based on the concept of moment of inertia of uniform circular disc and angular acceleration of a disc on a horizontal surface.

In FBD of system,

Retarding angular acceleration $f_k = \mu mg$

$$\mu mgr = \frac{mr^2 \alpha}{2}, \quad \alpha = \frac{2\mu g}{r}$$



For final rest, $v_f = 0$ and $\omega_f = 0$

$$v_f = v_0 + at = v_0 - \mu g t, t = \frac{v_0}{\mu g}$$

$$\omega_f = \omega_0 + \alpha t = \omega_0 - \frac{2\mu g}{r} t, t = \frac{\omega_0 r}{2\mu g}$$

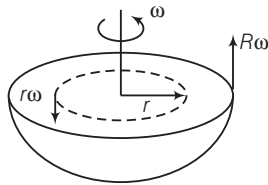
So,

$$\frac{v_0}{\mu g} = \frac{\omega_0 r}{2\mu g}$$

$$\frac{v_0}{\omega_0 r} = \frac{1}{2}$$

TEST Edge This concept is important according to JEE Advanced. Every year atleast two questions are asked and students should concentrate on moment of inertia and its rotational mechanism.

Remember if the angular velocity of rotation varies with the magnitude of tangential velocity as $v = r\omega$. The rate of change of tangential velocity is known as tangential acceleration (a_t). Given as



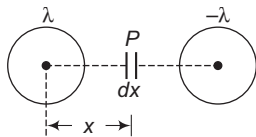
$$a_t = \frac{\Delta v}{\Delta t} = r \cdot \frac{\Delta \omega}{\Delta t}$$

$$a_t = r\alpha \quad \left[\because \alpha = \frac{\Delta \omega}{\Delta t} \right]$$

Note ω and α are same for all particles in the body while v and a_t are different for different points.

4. (c) **Idea** The problem is based on the combination of two long charged wires will act as a capacitor and you have to find the capacitance of the combination. $C = \frac{Q}{V}$, where V is potential difference between two wires. By using formula it's capacitance could be found.

Let us give equal and opposite charges to two wires so that they would have linear charge densities as $+\lambda$ and $-\lambda$.



Electric field at point P

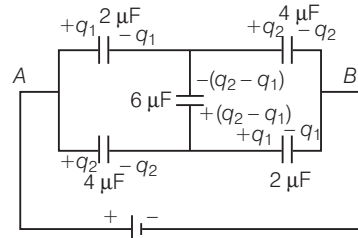
$$E = \frac{\lambda}{2\pi\epsilon_0 x} + \frac{\lambda}{2\pi\epsilon_0 (\eta a - x)}$$

$$\int dV = -\int E dx = -\int_a^{\eta a - x} E dx$$

where, a is radius of wire

$$\Rightarrow C = \frac{Q}{|V|} = \frac{\pi \epsilon_0}{\ln \eta}$$

TEST Edge Capacitor is important part of electrostatics and question may come from capacitors in JEE Advanced. The questions in the combination of capacitors may also come so let us consider an example



Here, concentrate on charge distribution and then apply Kirchoff's loop rule to solve the question.

5. (d) **Idea** We can apply conservation of angular momentum to solve this problem. We consider the angular momentum of the system with respect to centre (or any other point) and then apply conservation of angular momentum.

See the motion of system (boy + platform) from the axis of rotation with respect to axis, jump of body will not develop any external force or torque, so angular momentum of system will be conserved.

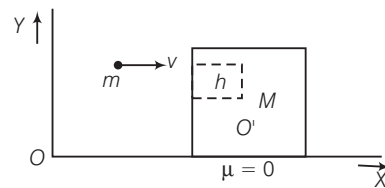
$$L_i = L_f$$

$$I_i \omega_i = I_f \omega_f$$

$$(I + mR^2)\omega = I\omega_f + m v R$$

$$\omega_f = \frac{(I + mR^2)\omega - m v R}{I}$$

TEST Edge Conservation of angular momentum is important for rotational motion and for JEE Advanced. So, let us consider a situation where we have to apply conservation of angular momentum.



If a particle hits a box at height h and strikes to it from its centre then the angular momentum of system after collision about the point O' .

Note Apply linear momentum conservation first and then angular momentum conservation.

6. (c) **Idea** It is based on refraction of light through a glass slab and intensity variation in Young's double slit experiment

$$i.e., \quad I = 4I' \cos^2 \frac{\delta}{2}$$

where, I is the resultant intensity, I' is the intensity due to single slit and δ is the angle of deviation.

$$I_R = 4I_0 \cos^2\left(\frac{\Delta\theta}{2}\right)$$

As given I_R at centre is $2I_0$, then

$$2I_0 = 4I_0 \cos^2\left(\frac{\Delta\theta}{2}\right)$$

$$\Rightarrow \left(\frac{\Delta\theta}{2}\right) = \frac{\pi}{4} \quad \text{[for minimum result]}$$

$$\Delta\theta = \frac{\pi}{2}$$

$$\Delta x = \Delta\theta \times \frac{\lambda}{2\pi} = \frac{\lambda}{4}$$

For slab of thickness $\Delta x = (\mu - 1)t$

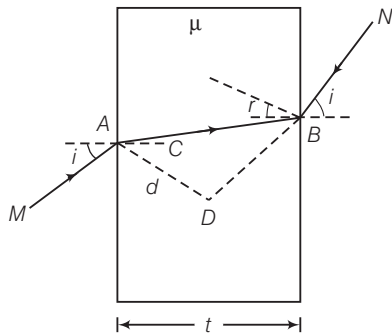
$$\left(\frac{3}{2} - 1\right)t = \frac{\lambda}{4}$$

$$t = \frac{\lambda}{2}$$

TEST Edge The problem is based on refraction of light, it's intensity and relate to lateral shift interference pattern are important according to JEE Advanced.

Remember In this case, ray MA is parallel to ray BN . But the emergent ray is displayed laterally by a distance d which depends upon μ , t and i and its value is given by

$$d = t \left[1 - \frac{\cos i}{\sqrt{\mu^2 - \sin^2 i}} \right] \sin i$$



where d is lateral shift.

7. (c) **Idea** It is based on the concept of angular velocity and conservation of angular momentum of a body.

i.e., $\mathbf{L} = \mathbf{r} \times \mathbf{p}$

where, \mathbf{L} is angular momentum.

During the motion of object, there is only one force tension which is acting towards a fixed point (centre of table/disc). So that motion is under the effect of central force and thus, angular momentum will be conserved.

At any radial distance r is

$$L = m\omega r^2$$

$$\frac{dL}{dr} = mr^2 \frac{d\omega}{dr} + m\omega (2r) = 0$$

So, $\frac{d\omega}{dr} = -\frac{2\omega}{r}$

TEST Edge This concept is more important according to JEE Advanced. Every year one or two questions are asked and students should relate angular momentum with torque such as

Angular momentum $\mathbf{L} = \mathbf{r} \times \mathbf{p}$

$$\frac{d\mathbf{L}}{dt} = \frac{d\mathbf{r}}{dt} \times \mathbf{p} + \mathbf{r} \times \frac{d\mathbf{p}}{dt}$$

$$= \mathbf{p} \times m\mathbf{v} = \mathbf{r} \times \mathbf{F}$$

$$= 0 + \mathbf{r} \times \mathbf{F} = \boldsymbol{\tau}_{\text{ext}}$$

$$\Rightarrow \frac{d\mathbf{L}}{dt} = \boldsymbol{\tau}_{\text{ext}}$$

Note Thus, rate of change of angular momentum is equal to the torque due to an external force.

8. (d) **Idea** This problem is based on the moment of inertia of a disc and conservation of mechanical energy

Mechanical energy = Potential energy + All the types of energy (i.e., rotational + translational)

String does not slip relative to disc so only conservative force acting in system, so mechanical energy is conserved.

loss in PE = gain in KE

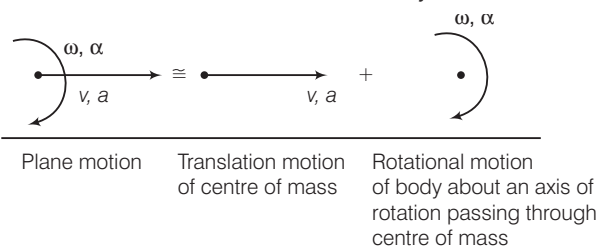
$$mgh = \frac{1}{2}mv^2 + \frac{1}{2}\left(\frac{mr^2}{2}\right)\omega^2 + \frac{1}{2}\left(\frac{mr^2}{2}\right)\omega^2$$

$$= \frac{1}{2}mv^2 + \frac{mr^2\omega^2}{2} = \frac{2mv^2}{2} = mv^2$$

$$v = \sqrt{gh}$$

TEST Edge These type of problems are important according to the JEE Advanced point of view. Every year two or three questions are asked. Students should focus on kinetic energy of a body in combined rotational and translation effect.

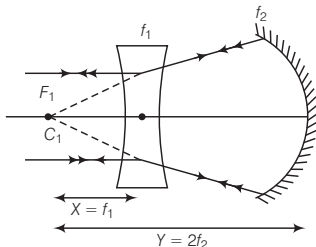
Remember, a plan motion can be considered as combination of translational motion of the centre of mass and rotational motion of body.



Net KE = K_T (Translating KE) + K_R (Rotating KE)

$$\therefore K_N = \frac{1}{2}mv^2 \left(1 + \frac{(KE)^2}{R^2} \right)$$

9. (b) **Idea** By drawing the ray diagram one can easily solve this question. Here, the focus of lens and centre of curvature of mirror will coincide. So, the relation between the $|f_2|$ and $|f_1|$ could be easily found.



The rays incident on the lens will refract as shown and if the focus of lens and centre of curvature of mirror coincide then the rays will retrace it's path and find image will be at infinity.

So, from the given situation $2|f_2| > |f_1|$.

TEST Edge In JEE Advanced/IIT-JEE, the questions from ray optics are very probable. On the combination of lenses and mirrors different questions could be asked.

Let us consider an example If we consider two lenses one is convex and the other is concave the combination may act as converging or diverging lens depending on their focal lengths. We can find the effective focal length by $\frac{1}{F} = \frac{1}{f_1} + \frac{1}{f_2}$.

10. (d) **Idea** This problem is based on rolling friction, torque and angular acceleration.

i.e., Torque (τ) = force \times perpendicular distance between force and axis of rotation

$$\tau = \mathbf{r} \times \mathbf{F} = rF\sin\theta$$

(where θ is the angle between \mathbf{r} and \mathbf{F})

FBD for disc

Equation of force $f = ma$

$$\tau - fR = \frac{mR^2}{2}\alpha \quad [a = R\alpha]$$

$$\frac{\tau}{R} - f = \frac{m}{2}(R\alpha) = \frac{ma}{2} = \frac{f}{2}$$

$$\frac{\tau}{R} = \frac{3f}{2}$$

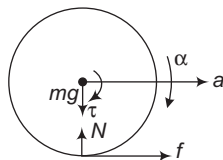
$$f = \frac{2\tau}{3R}$$

$$f \leq \mu N$$

(where μ , is coefficient of friction)

$$\frac{2\tau}{3R} \leq \mu(mg)$$

$$\mu \geq \frac{2\tau}{3mgR}$$



TEST Edge Every year, two or three questions are based on this concept in JEE Advanced. Students should concentrate on work done by torque, power and angular impulse in rotational motion.

(i) In rotational motion, total work done by a torque is given by $\omega \cdot D \int \tau d\theta$

(ii) Instantaneous power in rotational motion, $p = \tau \cdot \omega$

(iii) Angular impulse = $\int \tau dt = L_f - L_i$

where L_f and L_i are angular momentum of initial and final position.

In other word, angular impulse of torque is equal to total change in angular momentum of the body in given time.

11. (a,b,c,d) **Idea** It is based on the conservation of momentum we can find the speed of CM.

As there is no external force on the system so, $a_{CM} = 0$. Finally both the blocks will move with constant velocity.

$$v_{CM} = \frac{2mv + m(0)}{3m} = \frac{2v}{3}$$

No external force on system so $a_{CM} = 0$, v_{CM} will remain constant.

Impulse on m will be in direction of its motion, so it will increase its momentum.

Impulse on $2m$ is opposite to the direction of motion, so momentum will be decreased.

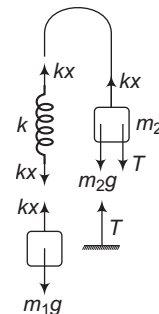
TEST Edge CM concept is used in many other topics such as rotation, gravitation, so it is important from JEE Advanced point of view.

In the above case, if we replace string by spring the spring will stretch and some KE of the system will convert into PE of the system. Even in the case of string the string has to be elastic otherwise the above case will not be possible.

12. (a,c) **Idea** Just after burning the thread the spring force does not change instantaneously. So just after burning the thread we can consider the force and we could easily solve the question.

Spring force does not change its direction and magnitude instantaneously.

FBD just before the cutting,



$$\therefore kx = m_1g$$

Just before and after cutting acceleration of a will be zero.

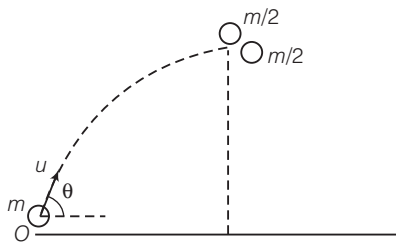
But T becomes zero just after cutting string so motion of m_2

$$kx - m_2 g = m_2 a$$

$$m_1 g - m_2 g = m_2 a$$

$$\therefore a = \frac{(m_1 - m_2)g}{m_2}$$

TEST Edge This problem is the combination of spring force and constraint motion. The multiple concept questions are more likely to asked in JEE Advanced. Let us consider one such example.

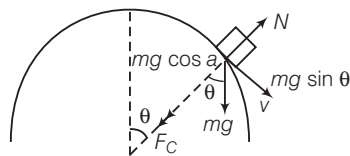


A ball is projected at speed u at angle θ . At the highest point it explodes into two equal parts. If one part reached back to its initial projection point O then the position of second ball where it hits the ground can be find out.

This question is based on the combination of CM concept and projectile motion.

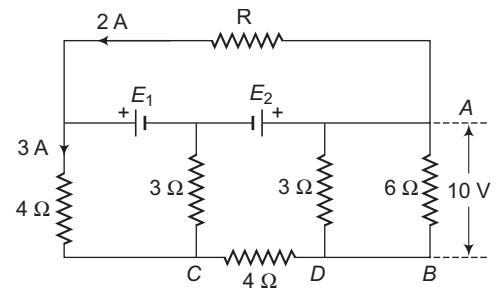
13. (b,c) **Idea** It is based on the concept of circular motion. As the box move downwards, its speed as well as required centripetal force $\left(\frac{mv^2}{R}\right)$ will also increase. Force F_C will be provided by the component of weight $mg \cos \theta$. Note carefully that as box is coming downwards, then F_C is increasing and $mg \cos \theta$ is decreasing.

The box will leave the circular surface just after the $N = 0$ condition, not at $N = 0$ instant because at $N = 0$, the component $mg \cos \theta$ will provide the required centripetal force $\frac{mv^2}{R} \cdot mg \cos \theta = \frac{mv^2}{R}$ at $N = 0$.

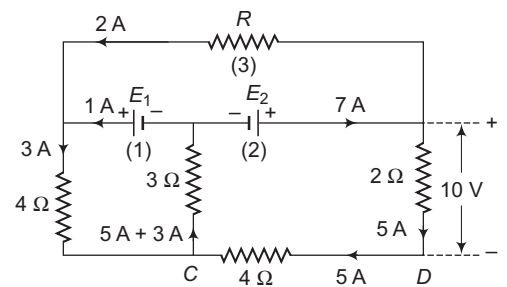


TEST Edge Some of the questions were asked on circular motion in IIT-JEE and may asked in JEE Advanced. In most of the circular motion questions, we just have to equate the required **centripetal force** and the **real force**. For example the required centripetal force for moon to revolve is provided by **real gravitational force**, so $F_C = F_G$.

14. (a,b,c,d) **Idea** In this problem, first see distribution of current in the circuit. Use Kirchhoff's first rule (junction rule) to see the distribution of current and then use Kirchhoff's second rule by considering loops to find the value of currents and emf of batteries.



After redrawing the circuit



a. $I_{CD} = 5 \text{ A}$

b. c. From loop (1),

$$-8(3) + E_1 - 4(3) = 0$$

$$E_1 = 36 \text{ V}$$

From loop (2) $4(5) + 5(2) - E_2 + 8(3) = 0$

$$E_2 = 54 \text{ V}$$

d. From loop (3), $-2R - E_1 + E_2 = 0$

$$R = \frac{E_2 - E_1}{2} = \frac{54 - 36}{2} = 9 \Omega$$

TEST Edge The **Combination of resistances** is an important topic of current electricity and it may come in JEE Advanced from this topic.

There are two to three methods that we use to solve this type of problem.

1. By Wheatstone bridge method
2. By Kirchhoff's rule
3. By removing the parts of circuit if the potential difference between the ends of that part is zero. In this case, there will be no current in that part of circuit.

15. (b,d) **Idea** The problem is based on work-energy theorem.

Work done by all the force acting on a body is equal to change in kinetic energy of the same body.

i.e., $\Delta KE = W_{\text{net}}$

Loss in the energy of photon = $h\nu - h(\nu - d\nu)$
 $= h(d\nu)$
 $= \text{work done by gravity}$

Let say $m = \text{mass of photon} = \frac{h\nu}{c^2}$

$$hd\nu = -du \begin{cases} u = \frac{-GMm}{x} \\ \frac{du}{dx} = + \frac{GMm}{x^2} \end{cases}$$

$$hd\nu = \frac{-GMm}{x^2} dx = - \frac{GM \left(\frac{h\nu}{c^2} \right)}{x^2} dx$$

$$hd\nu = - \frac{GMh\nu}{c^2 x^2} dx$$

$$\int_{\nu_0}^{\nu} \frac{d\nu}{\nu} = - \frac{GM}{c^2} \int_R^{R+H} \frac{dx}{x^2}$$

$$h \left(\frac{\nu}{\nu_0} \right) = + \frac{GM}{c^2} \left[\frac{1}{R+H} - \frac{1}{R} \right] = - \frac{GMh}{R^2 c^2}$$

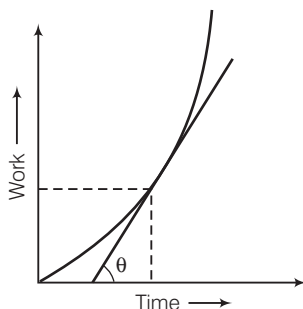
$$\nu = \nu_0 e^{\frac{-GMh}{R^2 c^2}}$$

$$\nu = \nu_0 e^{\frac{-gh}{c^2}} \quad \left[\because g = \frac{GM}{R^2} \right]$$

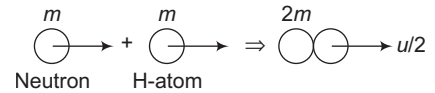
TEST Edge It is important according to JEE Advanced. Every year one question is asked and student should focus on work energy theorem and relate to power and potential energy of a spring.

Remember This theorem can be applied to non-inertial frame also in an non-inertial frame it can be written as work done by all the force (including the pseudo forces) change in kinetic energy in non-inertial frame.

The slope of work-time curve gives the instantaneous power as $p = \frac{dW}{dt} = \tan\theta$ as shown in figure.



16. (3) **Idea** The electron of hydrogen atom will gain some part of KE of the system and it will jump to the higher state. While returning to the ground state it will emit photons. By observing the excited state we can find the possible number of photons that could be emitted.



Maximum possible loss of energy is $\frac{KE_{in}}{2}$, when collision is perfectly inelastic as shown.

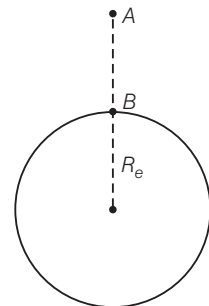
$$\therefore (KE_{loss})_{max} = \frac{25.5}{2} = 12.75 \text{ eV}$$

This is sufficient to raise the electron upto 3rd shell. While returning to first shell three photons could be possibly emitted.

TEST Edge Atom is the part of modern physics and the questions from modern physics are relatively easier than mechanics or electricity and magnetism. So, it is easier to score marks from this part. So, concentrate on this part as well and remember its formulae and theory to solve its questions quickly.

17. (3) **Idea** Here don't be confused with the choice of zero potential. As $W_{ext} = \Delta V$ does not depend on the choice of zero potential. Just apply the formula for W_{ext} with considering zero potential at infinity and you will get the answer.

When we consider the value of potential to be zero at infinity then formula of potential $\frac{-GM_e}{r}$

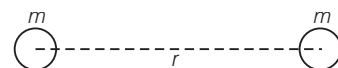


$$\begin{aligned} \Rightarrow W_{ext} (A \rightarrow B) &= [V_B - V_A] m \\ &= \left[\frac{-GM_e}{R_e} - \frac{GM_e}{2R_e} \right] m \\ &= \left[\frac{-3}{2} \frac{GM_e}{R_e} \right] m \end{aligned}$$


by comparing it $\left[\frac{-x GM_e m}{2R_e} \right]$

Don't be confused by the choice of zero potential because we want to find W_{ext} which depends on ΔV and ΔV is not dependent in the choice of zero potential.

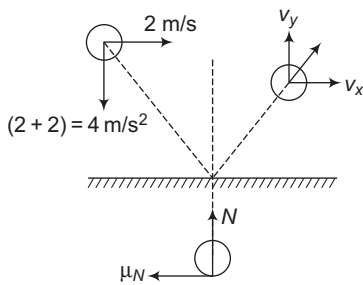
TEST Edge From gravitation, the questions based on conservation of mechanical energy may be asked in JEE Advanced. Let us consider one example.



- (i) Two masses are at a distance r as shown in the figure. Initially they were at rest. Now if they attract each other with gravitational force then their speeds when they are at a distance $\frac{r}{2}$ with each other can be find out.
- (ii) Just apply conservation of mechanical energy to solve it.

18. (0)  **Idea** The problem is based on impulse and conservation of energy in elastic collision.

Ball with respect to surface



Upward impulse on ball

$$\int N dt = p_f - p_i$$

$$\int N dt = 2v_y - (-2 \times 4)$$

$$\int N dt = 2v_y + 8 \quad \dots(i)$$

Equation of e along normal direction

$$e = \frac{v_y}{4} \quad \dots(ii)$$

$$v_y = 4e = 2 \text{ m/s}$$

So, $\int N dt = 4 + 8 = 12$

Horizontal impulse

$$-\int \mu N dt = p_f - p_i$$

$$-\mu \int N dt = 2v_x - 2 \times 2$$

$$-\frac{1}{3} \times 12 = 2v_x - 4$$

$$v_x = 0$$

TEST Edge It is important according to JEE Advanced. Every year one question is asked and student should concentrate in elastic, inelastic collision and impulsive force.

Remember In a perfectly elastic collision,

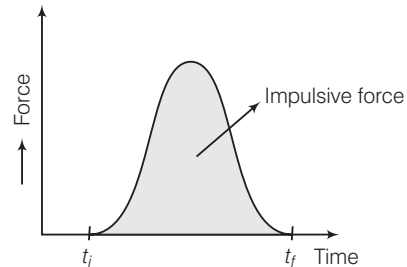
velocity of separation = velocity of approach and for a perfectly inelastic collision,


velocity of separation = 0

- (i) If coefficient of restitution (e) is equal to 1, then collision is perfectly elastic and if $e = 0$, then collision is perfectly inelastic.

- (ii) The change in momentum produced by such an impulsive force is

$$\mathbf{p}_f - \mathbf{p}_i = \int_{p_i}^{p_f} d\mathbf{p} = \int_{t_i}^{t_f} \frac{d\mathbf{p}}{dt} dt = \int_{t_i}^{t_f} \mathbf{F} \cdot dt$$



19. (3)  **Idea** The problem is based on energy stored in a capacitor and to calculate amount of heat energy stored when capacitor is filled with a dielectric constant.

Initial charge on capacitor = CE

Initial potential energy of capacitor = $CE^2/2$

Now, $C = \epsilon_0 A/d$

and new capacitance $C' = \frac{\epsilon_0 A}{d/2}$, in series with $\frac{\epsilon_0 KA}{d/2}$

$$\Rightarrow C' = \frac{2\epsilon_0 A}{d} \text{ series } \frac{4\epsilon_0 A}{d}$$

$$= \frac{\epsilon_0 A}{d} \left(\frac{2 \times 4}{2 + 4} \right) = \frac{4\epsilon_0 A}{3d} = \frac{4}{3} C$$

$$\Rightarrow \text{New charge} = \frac{4}{3} CE$$

$$\text{New energy} = \frac{1}{2} \times \frac{4}{3} CE^2$$

$$= \frac{2}{3} CE^2$$

Now, $W_{\text{battery}} = \Delta H + \Delta U$

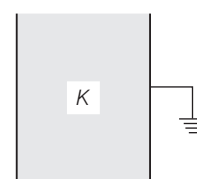
$$\Rightarrow E \left(\frac{4}{3} CE - CE \right) = \Delta H + \frac{2}{3} CE^2 - \frac{1}{2} CE^2$$

$$\Rightarrow \Delta H = \frac{1}{6} CE^2 = \frac{1}{6} \times 2 \times (3)^2 = 3 \text{ J}$$

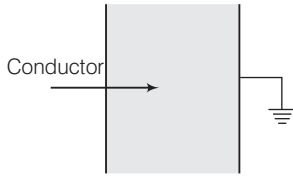
TEST Edge This concept is generally asked in JEE Advanced. Student should relate to energy density in a electric field and parallel plate capacitor with a dielectric.

Case 1. If a conducting slab ($k = \infty$) is placed between the plates, then

$$C = \frac{\epsilon_0 A}{d - t + \frac{t}{\infty}} = \frac{\epsilon_0 A}{d - t}$$



Case 2. If the space between plates is completely filled with a conductor, then $t = d$ and $k = \infty$



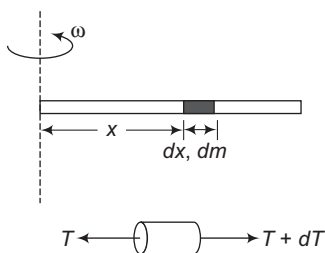
then, $C = \frac{\epsilon_0 A}{d - d + \frac{d}{\infty}} = \infty$

where C is capacitance.

20. (2) **Idea** It is based on the concept of velocity of transverse wave and centrifugal force on a string.

Velocity of transverse wave on rope is $= \sqrt{\frac{T}{\mu}}$

where, T = tension in string.
Variation in tension in a horizontal rotating string.
For small element of rope



$$T - (T + dT) = dm\omega^2 r$$

$$-dT = dm\omega^2 x$$

$$-\int_{T_0}^T dT = \mu\omega^2 \int_0^x x dx$$

$$T - T_0 = -\frac{\mu x^2 \omega^2}{2}$$

$$T = T_0 - \frac{\mu x^2 \omega^2}{2}$$

At $x = L, T = 0,$

$$0 = T_0 - \frac{\mu L^2 \omega^2}{2}$$

$$T_0 = \frac{\mu \omega^2 L^2}{2}$$

$$T = \frac{\mu \omega^2 (L^2 - x^2)}{2}$$

So, velocity of wave on string

$$v = \sqrt{\frac{T}{\mu}} = \sqrt{\frac{\mu \omega^2 (L^2 - x^2)}{2\mu}} = \omega \sqrt{\frac{L^2 - x^2}{2}}$$

$$\frac{dx}{dt} = \frac{\omega}{\sqrt{2}} \sqrt{L^2 - x^2}$$

$$\int_0^L \frac{dx}{\sqrt{L^2 - x^2}} = \int_0^t \frac{\omega}{\sqrt{2}} dt$$

$$\sin^{-1}\left(\frac{x}{L}\right)_0^L = \frac{\omega}{\sqrt{2}} t$$

$$\sin^{-1}(1) = \frac{\pi}{2} = \frac{\omega}{\sqrt{2}} t$$

$$\therefore t = \frac{\pi}{\sqrt{2} \omega}$$

As given in question $\frac{\pi}{\sqrt{2} \omega} = \frac{\pi}{\sqrt{n} \omega} \Rightarrow$ So, $n = 2$

TEST Edge It is important according to JEE Advanced, every year one question is asked from this concept students should focus in transverse of waves.

Remember While wave travelling though the string, then tension (T) is developed in a string is given by

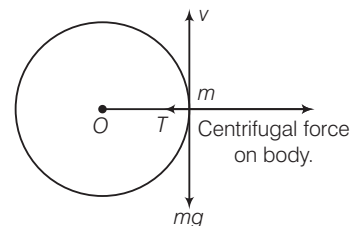
$$T = v^2 \mu^2$$

$$\Rightarrow v = \sqrt{\frac{T}{\mu}}, \text{ where } v \text{ is speed of wave}$$

when string is whirled, there is a centrifugal force on the string as whole, which is given by

$$\therefore F_{\text{centrifugal}} = m\omega^2 r$$

where, r = radius of a circle.



ω = angular speed at each and every section of the body of the string.

21. (b) **Idea** Problem is based on concept of determination of age of ore. While solving the problem students are advised to go through following steps.

Calculate the total amount of U^{238} decay.

Then calculate the total amount of U^{238} present and then after age of ore using following equation

$$t = \frac{2.303}{\lambda} \log \frac{N_0}{N}$$

Considering that the whole of Pb^{206} comes from U^{238} ,
Amount of U^{238} decayed = $0.1g Pb^{206}$

$$= \frac{0.1 \times 238}{206} g U^{238}$$

$$= 0.1155 g U^{238}$$

Hence, initial amount of $U^{238} = (1 + 0.1155) g U^{238}$
Value of disintegration constant can be calculated as

$$\text{Now, } \lambda = \frac{0.693}{T_{1/2}} = \frac{0.693}{0.5 \times 10^9 \text{ yrs}}$$

$$= 0.154 \times 10^{-9} \text{ yrs}^{-1}$$


$$\begin{aligned}
 t &= \frac{2.303}{\lambda} \log \frac{N_0}{N} \\
 &= \frac{2.303}{0.154 \times 10^{-9}} \log \left(\frac{1.1155}{1} \right) \\
 &= 7.099 \times 10^8 \text{ yrs}
 \end{aligned}$$

TEST Edge Problem based on concept of half-life time and isotopic composition can also be asked in JEE Advanced while solving this type of problem with conceptual mixing of more than two terms, students must calculate important parameters first followed by other problems such as

A mixture containing ^{239}Pu and ^{240}Pu has specific gravity of $6.0 \times 10^9 \text{ dis/s}$. The half-lives of the isotopes are 2.44×10^4 and 6.58×10^3 years respectively then what will be the isotopic composition of these samples.

Answer = ^{239}Pu , ^{240}Pu = 39:61

This problem can be solved by calculating concentration of both isotopes separately and then their ratio by using concept of mole fraction.


22. (c)  **Idea** Problem is based on concept of Einstein equation of photoelectric effect while solving the problem students are advised to use the Einstein equation.

According to the Einstein equation of photoelectric effect

$$\begin{aligned}
 h\nu &= h\nu_0 + KE \\
 \therefore \frac{1}{2}mv^2 &= h\nu - h\nu_0 \\
 &= \frac{hc}{\lambda} - \frac{hc}{\lambda_0} \\
 v^2 &= \frac{2hc}{m} \left(\frac{1}{\lambda} - \frac{1}{\lambda_0} \right) \\
 v &= \sqrt{\frac{2hc}{m} \left(\frac{\lambda_0 - \lambda}{\lambda\lambda_0} \right)}
 \end{aligned}$$

Hence, (c) is the correct choice.

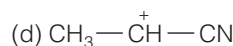
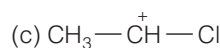
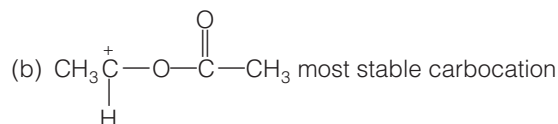
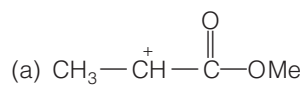
TEST Edge Similar type of problem based on conceptual mixing of photoelectric effect, de-Broglie equation and Hisenberg uncertainty principle can also be asked in JEE Advanced.

23. (b)  **Idea** This problem includes concept of polymerisation. While solving the problem students are advised to undergo knowledge of carbocation as well.

More will be the stability of carbocation produced by monomer, more will be the extent of cationic polymerisation.

Since, cationic polymerisation proceeds through formation of carbocation hence most stable monomer for cationic polymerisation is that which produces more stable carbocation.


Intermediate of cationic polymerisation will be carbocation.



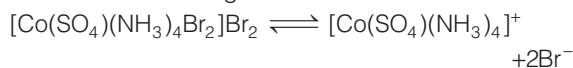
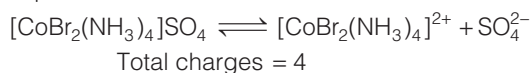
Hence, correct choice is (b).

In option (b) $-\overset{\text{O}}{\parallel}{\text{C}}-\text{CH}_3$ group stabilise carbocation.

TEST Edge Problems based on properties and uses of important polymers such as nylon, Teflon, PVC can be asked in JEE Advanced.

24. (d)  **Idea** This problem is based on conceptual mixing of isomerism and molar conduction of coordination compound. While solving the problems it is advisable to go through the understanding of isomerism and vant Hoff factor of coordination compound.

Out of given options, choices (d) contain $[\text{CoBr}_2(\text{NH}_3)_4]\text{SO}_4$ and $[\text{Co}(\text{SO}_4)(\text{NH}_3)_4]\text{Br}_2$ both are ionization isomers and when they dissolve in aqueous solution produces different number of ions in aqueous solution as



Total charges = 3

Since, charges are different in a aqueous solution hence, they have different molar conductivities.


TEST Edge Similar problems based on conceptual mixing of isomerism and qualitative analysis of inorganic compounds can also be asked in JEE Advanced the such as.

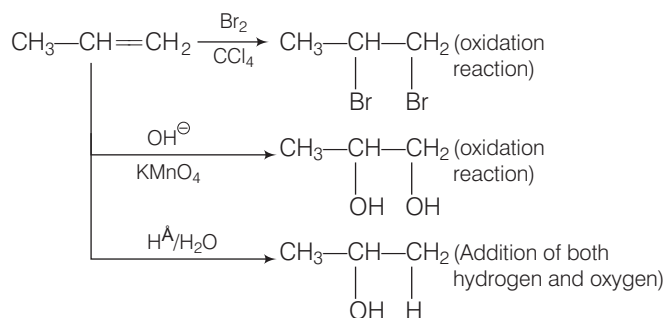
What is the magnetic moment of complex used in qualitative analysis of nitrate ion?

After determining magnetic moment using the formula

$$\mu = \sqrt{n(n+2)} \text{ BM.}$$

One can get answer as 1.73 BM.

25. (a)  **Idea** This problem is based on conceptual efficiency of Carnot engine and diagram representation of Carnot engine, while solving this problem students must have a good emphasis on analysis of diagram which represents Carnot engine.



TEST Edge Similar problem based on conceptual mixing type of reaction, intermediate involved in the reaction and rearrangement of intermediate can also be asked so students are advised to go through clear understanding of these topics and related problems such as

What will be the intermediate involved in Favorski rearrangement after undergoing through proper mechanism?

One can get carbanion as answer.

Remember It also includes formation of cyclopropane ring inside as intermediate during product formation.

31. (a,b) **Idea** This problem includes conceptual mixing of hybridisation and percentage *s* character while solving the problem, students are advised to calculate the hybridisation first followed by percentage *s* character in molecule, % *s* character and hybridisation are inter related as

Hybridisation	% <i>s</i> Character
sp	50%
sp^2	33%
sp^3	25%
sp^3d	20%
sp^3d^2	17%

Molecules	Hybridisation	<i>s</i> character
I. CO_3^{2-}	sp^2	33%
II. XeF_4	sp^3d^2	17%
III. I_3^-	sp^3d	20%
IV. NCl_3	sp^3	25%
V. BeCl_2	sp	50%

Hence, correct order is II < III < IV < I < V

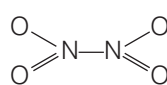
TEST Edge Similar problems including conceptual mixing of hybridisation, percentage *s* character, percentage *p* character, bond angle of molecules can also be asked in JEE Advanced, such as

What will be the hybridisation of non-bonding pair of electrons in NH_3 if bond angle is 107° .

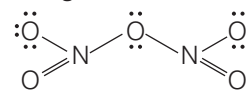
After solving, you will get answer as $sp^{2.13}$.

32. (a,c) **Idea** This problem is based on structure of oxides of nitrogen. Students are suggested to draw the structure of molecules before solving such questions be careful during drawing the structure the oxidation state and number of electrons distributed in compound and check that these must be in proper ratios.

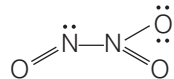
Structure of oxides of nitrogen



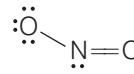
Structure of N_2O_4
1 N—N bond



Structure of N_2O_5
No N—N bond



Structure of N_2O_3
1 N—N bond



Structure of NO_2
No N—N bond

TEST Edge Similar problems based on structure of oxo acids of sulphur can also be asked in JEE Advanced, such as

Which of the following doesn't have *s-s* linkage?

- (a) $\text{S}_2\text{O}_8^{2-}$ (b) $\text{S}_2\text{O}_6^{2-}$ (c) $\text{S}_2\text{O}_5^{2-}$ (d) $\text{S}_2\text{O}_3^{2-}$

After drawing structure, one can get *a* as answer.

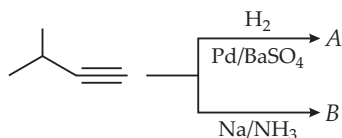
33. (a,b,c,d) **Idea** While solving this problem, students must have knowledge about concept involved in rate of esterification, rate of dehydration, so the reactivity of derivatives of carboxylic acid, students are suggested to go through the study of inductive effect, mesomeric effect, resonance effect, hyperconjugation etc., which will help them in the solving of these types of problems.

- (i) Rate of esterification with respect to alcohol \propto nucleophilicity of alcohol.
 (ii) Rate of dehydration \propto stability of carbocation,
 (iii) Rate of esterification with respect to acid \propto electrophilicity of $\text{R}-\text{C}(=\text{O})$ group.

- (iv) Reactivity of acid derivatives \propto electrophilicity of $\text{R}-\text{C}(=\text{O})$ group.

TEST Edge Problems based on chemical reactivity of organic compounds such as carboxylic acid and their derivatives towards various reducing agents can also be asked in JEE Advanced. So, students are suggested to go through in depth study of these topics, such as

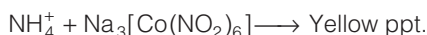
What will be the relation between products of given chemical reactions?



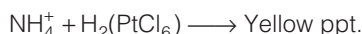
Knowing the mechanism of Lindlars catalyst Birch reduction, one can get answer as geometrical isomers.

34. (a, d) **Idea** This problem is based on the concept of analysis of NH_4^+ and K^+ . To solve this problem, students must have the knowledge of group reagents used in qualitative analysis of ammonium ion.

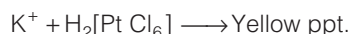
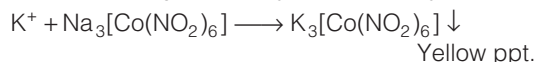
Analysis of NH_4^+ NH_4^+ may be analysed by reaction of solution containing NH_4^+ with sodium hexanitrocobaltate (III), which results as yellow precipitate.



Solution containing NH_4^+ on reaction with $\text{H}_2[\text{PtCl}_6]$ produces yellow precipitate also.



Similarly, K^+ also produces yellow precipitate on reaction with $\text{Na}_3[\text{Co}(\text{NO}_2)_6]$ and $\text{H}_2[\text{PtCl}_6]$



While Na^+ and Mg^{2+} does not give this test.

TEST Edge Similar problems based on concept of group reagents and their qualitative analysis can also be asked.

35. (a,b) **Idea** This problem includes the concept of characteristics and nature of amino acid under different conditions. While solving this problem students are advised to study the statements very carefully for choosing the incorrect statements. Hence, tick the true and false statements first according to information provided, then answer the question as shown below

As alanine is a neutral non-essential amino acid. Hence, answer of this question can be done as mark The statement as

For true statement (✓)

For false statement (✗)

According to the questions

- (a) It is acidic amino acid (✗)
- (b) It is an essential amino acid (✗)
- (c) It will move to cathode in electrolysis at pH = 5 (✓)
- (d) It is an optically active amino acid (✓)

Hence, incorrect statements are (a) and (b) then answer is (a) and (b).

TEST Edge Problems based on characteristics of amino acids as well as preparation and uses of various amino acid can also be asked in JEE Advanced, related such as

Which of the following processes can be used during preparation of amino acid?

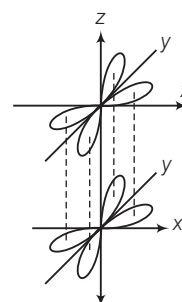
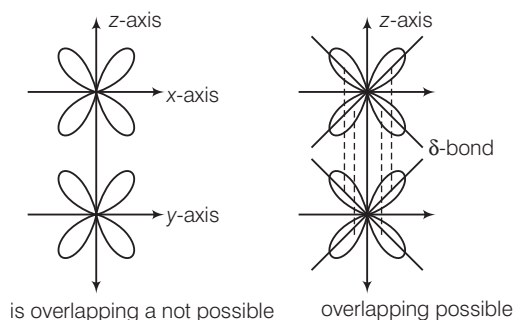
- (a) Gabriel phthalimide process
- (b) Strecker synthesis
- (c) Sandmeyer process
- (d) All of the above

It can also be asked, students can answer such question if he has the knowledge of amino acid precursor and proper mechanisms of reaction.

The answers are (a) and (b).

36. (1) **Idea** This problem is based on concept of δ -bond formation which can be solved by using the bonding involved in δ -bond formation.

For formation of δ -bond, the lobes of orbital must be in phase i.e., in overlapping condition to each other as shown below



overlapping possible d_{xy} undergo δ bond formation

37. (9) **Idea** This problem includes the concept of determination of pH using value of K_{sp} . It is advisable to go through the study of equilibrium constant, solubility product and pH determination to solve all these types of problems.



$$1 \times 10^{-12} = [\text{Mg}^{2+}][\text{OH}^-]^2$$

$$[\text{OH}^-] = 10^{-5} \text{ M}$$

$$\text{pOH} = -\log 10^{-5} = 5$$

$$\text{pH} = 14 - 5 = 9$$

$$\text{pH} = 9$$

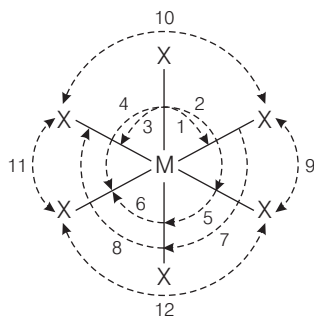
$$[\because \text{pH} + \text{pOH} = 14]$$

TEST Edge Similar problems based on conceptual mixing of pH, K_{sp} electrode potential can also be asked in JEE Advanced, students are advised to go through the basic concepts of these topics and relate all these term.. In order to approach towards the correct answer, students must follow stepwise approach.

38. (6) **Idea** This problem is based on the concept of structure of octahedral compound and number of possible type of MX_6 bond angles, while solving problems, students are advised to consider all the X as different ligands.

All 12 bond angles are shown by numeric below.

$$\therefore \begin{aligned} x &= 12 \\ \frac{x}{2} &= \frac{12}{2} = 6 \end{aligned}$$



TEST Edge Similar structure and hybridisation related problems such as

What is the ratio of σ and π -bonds in $B_3N_3Cl_3$?

It can also be asked students, can answer the question by drawing the structure of inorganic compound and counting number of σ and π -bonds and finally calculating the exact ratio, one can get answer as 3.

39. (8) **Idea** This problem can be solved by using the concept of entropy of mixing. Since, two gases are mixing, so students are advised to use the following formula to solve the problem.

$$\Delta S_{\text{sys}} = -R \sum n_i \ln x_i$$

where, n_i = number of moles of gas

x_i = mole fraction of gas.

$$\begin{aligned} \Delta S_{\text{sys}} &= -R \sum (n_i \ln x_i) \\ &= -R \left[1 \times \ln \frac{1}{3} + 2 \times \ln \frac{2}{3} \right] \\ &= R \left[\ln 3 + 2 \ln \frac{3}{2} \right] \end{aligned}$$

$$= 8.314 \times (1 + 2 \times 0.4) = 15.9 \text{ J/K} \approx \frac{16}{2} = 8$$

TEST Edge Problems based on energy of mixing of two solids two immiscible liquids, one liquid and one gas can also be asked. So, students are advised to go through the concepts of these topics.

40. (5) **Idea** This problem includes the concept of Lindmann theory of unimolecular reactions which is used to calculate apparent rate constant. It is advisable to use the concept of limit and continuity (mathematics) to solve this problem.

$$\begin{aligned} \frac{90}{100} \times \left(\lim_{c \rightarrow \infty} \frac{k_1 c}{1 + \alpha c} \right) &= \frac{k_1 c}{1 + \alpha c} \\ \frac{90}{100} \times \left(\lim_{c \rightarrow \infty} \frac{k_1}{\frac{1}{c} + \alpha} \right) &= \frac{k_1 c}{1 + \alpha c} \\ \Rightarrow \frac{90}{100} \times \frac{k_1}{\alpha} &= \frac{k_1 c}{1 + \alpha c} \Rightarrow \frac{0.9}{\alpha} = \frac{c}{1 + \alpha c} \\ 0.9 &= \frac{\alpha c}{1 + \alpha c} \end{aligned}$$

$$\begin{aligned} \Rightarrow 0.9 + 0.9\alpha c &= \alpha c \Rightarrow 0.1\alpha c = 0.9 \Rightarrow \alpha c = 9 \\ c &= \frac{9}{\alpha} = \frac{9}{9 \times 10^5} \Rightarrow c = 10^{-5} \text{ M.} \therefore |\log c| = 5 \end{aligned}$$

TEST Edge Similar types of problems, with conceptual mixing of mathematical concept and rate constant determination can also be asked in JEE Advanced. Some problems including concept of Clausius theory and transition state theory can also be asked.

41. (a) **Idea** If $ax^2 + bx + c = 0$ is a quadratic equation, then sum of roots $(\alpha + \beta) = -\frac{b}{a}$ and product of roots $(\alpha\beta) = \frac{c}{a}$ and arithmetic mean A of any two numbers a and b is $\frac{a+b}{2}$.

Given equation is $x^2 - x - 1 = 0$

$$\Rightarrow \alpha + \beta = 1, \alpha\beta = -1$$

$$\text{Now, } A_n = \alpha^n + \beta^n$$

$$\therefore A_{n-1} = \alpha^{n-1} + \beta^{n-1}$$

$$\text{Hence, } AM = \frac{A_n + A_{n-1}}{2} = \frac{\alpha^n + \beta^n + \alpha^{n-1} + \beta^{n-1}}{2} \dots (i)$$

$$\text{Now, } A_{n+1} = \alpha^{n+1} + \beta^{n+1}$$

$$= (\alpha + \beta)(\alpha^n + \beta^n) - \alpha\beta(\alpha^{n-1} + \beta^{n-1})$$


$$A_{n+1} = \alpha^n + \beta^n + \alpha^{n-1} + \beta^{n-1} \dots (ii)$$

$$[\because \alpha + \beta = 1, \alpha\beta = -1]$$

From Eqs. (i) and (ii), we get

$$AM = \frac{A_{n+1}}{2}$$

TEST Edge Questions based on quadratic formula to find the roots of the equation and concept related to progression (AP, GP and HP) are asked. So, to solve such questions, students are advised to learn nature of roots, relation between roots and coefficient of quadratic equation and basic facts of arithmetic progression such as n^{th} term of AP i.e., $a_n = a + (n-1)d$, sum of n^{th} term of AP i.e., $S_n = \frac{n}{2} [2a + (n-1)d]$ etc.

42. (c)  **Idea** We have three inverse functions $\sin^{-1}(x-1)$, $\cos^{-1}(x-3)$ and $\tan^{-1}\left(\frac{x}{2-x^2}\right)$ whose domain will be $[-1, 1]$, $[-1, 1]$ and $(-\infty, \infty)$ respectively and from the value of trigonometric function $\cos \frac{\pi}{4} = \frac{1}{2}$. We find the value of k .


For $\sin^{-1}(x-1)$ to exist $x-1 \in [-1, 1]$
 $x \in [0, 2]$... (i)

For $\cos^{-1}(x-3)$ to exist $x-3 \in [-1, 1]$
 $\lambda \in [2, 4]$... (ii)

Eqs. (i) and (ii) $\Rightarrow x = 2$

So, $\sin^{-1}(2-1) + \cos^{-1}(2-3) + \tan^{-1}\left(\frac{2}{2-4}\right)$
 $= \cos^{-1}k + \pi$
 $\sin^{-1}(1) + \cos^{-1}(-1) + \tan^{-1}(-1) = \cos^{-1}k + \pi$
 $\frac{\pi}{2} + \pi - \frac{\pi}{4} = \cos^{-1}k + \pi$
 $\frac{\pi}{4} = \cos^{-1}k \Rightarrow k = \cos \frac{\pi}{4}$
 $k = \frac{1}{\sqrt{2}}$

TEST Edge Domain and range of inverse function and value of trigonometric functions based questions are asked. So, to solve such types of questions students are advised to learn domain and range of various trigonometric functions such as $\tan^{-1}x$, $\operatorname{cosec}^{-1}x$, $\sec^{-1}x$, etc., and its graphical representation and also acquainted yourself with the values of trigonometric functions at the given angle e.g., $\tan^{-1}x = \cot^{-1}\left(\frac{1}{x}\right)$ if $x > 0$ and $\tan^{-1}x = -\pi + \cot^{-1}\left(\frac{1}{x}\right)$ if $x < 0$ because range of these two functions are different.

43. (a)  **Idea** If $\mathbf{a} = a_1\hat{i} + a_2\hat{j} + a_3\hat{k}$ and $\mathbf{b} = b_1\hat{i} + b_2\hat{j} + b_3\hat{k}$ then, $\mathbf{a} \cdot \mathbf{b} = a_1b_1 + a_2b_2 + a_3b_3$, $|\mathbf{a}| = \sqrt{a_1^2 + a_2^2 + a_3^2}$ and $|\mathbf{b}| = \sqrt{b_1^2 + b_2^2 + b_3^2}$ and any point which lie on the curve satisfies the curve equation.

Let $\mathbf{OA} = x_1\hat{i} + y_1\hat{j}$ and $\mathbf{OB} = x_2\hat{i} + y_2\hat{j}$

Since, $\mathbf{OA} \cdot \hat{i} = 2$

$x_1 = 2$

Also, $\mathbf{OB} \cdot \hat{i} = -3$

$\Rightarrow x_2 = -3$

Since, the points (x_1, y_1) and (x_2, y_2) lie on curve.

$\therefore y_1 = x_1^2$ and $y_2 = x_2^2$

When $x_1 = 2$, $y_1 = 4$

When $x_2 = -3$, $y_2 = 9$

$\therefore \mathbf{OA} = 2\hat{i} + 4\hat{j}$


$\mathbf{OB} = -3\hat{i} + 9\hat{j}$

Now, $3(\mathbf{OA}) - 4(\mathbf{OB}) = 3(2\hat{i} + 4\hat{j}) - 4(-3\hat{i} + 9\hat{j})$
 $= 18\hat{i} - 24\hat{j}$

$\therefore |3\mathbf{OA} - 4\mathbf{OB}| = \sqrt{(18)^2 + (24)^2}$
 $= \sqrt{324 + 576}$
 $= \sqrt{900} = 30$ units

Required length is 30 units.

TEST Edge Questions based on basic coordinate geometry, position vector and linearly dependent vectors are asked in conceptual mixing. So, students are advised to understand basics of coordinate geometry and also acquainted yourself with concept of vectors such as dot and cross products of two vectors, types of vectors like position vector, coplanar vector, etc. e.g., if a line passes through the point $A(\mathbf{a})$ and is parallel to the vector \mathbf{b} , then its equation is $\mathbf{r} = \mathbf{a} + t\mathbf{b}$.

44. (a)  **Idea** Modulus function $y = f(x) = |x|$ is defined

as $y = \begin{cases} x, & \text{if } x \geq 0 \\ -x & \text{if } x < 0 \end{cases}$

Range of the trigonometric function $\cos x$ is R and properties of definite interval $\int_a^b f(x) dx = \int_a^c f(x) dx + \int_c^b f(x) dx$. Also, standard result of $\int \cos x dx = \sin x + C$ and

general solution of trigonometrical equation if $\sin \theta = \sin \alpha$, then $\theta = n\pi + (-1)^n \alpha$ where, $\alpha \in \left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$

If $0 < x < \frac{\pi}{2}$, $\int_{-2}^x |\cos x| dx + \int_{-2}^{-\pi/2} |\cos x| dx$
 $+ \int_{-\pi/2}^x |\cos x| dx$

$\Rightarrow \int_{-2}^{-\pi/2} \cos x dx + \int_{-\pi/2}^x \cos x dx + \int_{-2}^x |\cos x| dx = 0$

$\Rightarrow |-\sin x|_{-2}^{-\pi/2} + |-\sin x|_{-\pi/2}^x = 0$

$\Rightarrow 1 - \sin 2 + \sin x + 1 = 0$


$\Rightarrow 2 - \sin 2 + \sin x = 0$

$\Rightarrow \sin x = \sin 2 - 2 < -1$ not possible

\therefore No solution exist in $\left(0, \frac{\pi}{2}\right)$

So, number of solution = 0.

TEST Edge In JEE Advanced, questions based on different types of functions such as trigonometric function, modulus function, greatest integer function and concept related to properties of definite integral and general solution of trigonometrical equations are asked. So, to solve such types of questions learn definition of various functions, different properties of definite integral and methods to find general solutions of trigonometrical equation e.g., general solution of $\tan^2 \theta = \tan^2 \alpha$ is $\theta = n\pi \pm \alpha$, $n \in I$.

45. (a)  **Idea** If $f(-x) = f(x) \forall x$ in the domain of f then f is an even function and if $f(-x) = -f(x)$ then f is an odd function. If $f: A \rightarrow B$, then domain of $f = \{a \mid a \in A, (a, f(a)) \in f\}$ and range of $f = \{f(a) \mid a \in A, f(a) \in B\}$

$$x^2 f(x) - 2f\left(\frac{1}{x}\right) = g(x) \quad \dots(i)$$

Replacing x by $\frac{1}{x}$, we have

$$\frac{1}{x^2} f\left(\frac{1}{x}\right) - 2f(x) = g\left(\frac{1}{x}\right)$$

Multiplying by $2x^2$

$$2f\left(\frac{1}{x}\right) - 4x^2 f(x) = 2x^2 g\left(\frac{1}{x}\right) \quad \dots(ii)$$

Adding Eqs. (i) and (ii), we get

$$-3x^2 f(x) = g(x) + 2x^2 g\left(\frac{1}{x}\right)$$

$$\Rightarrow f(x) = -\left[\frac{g(x) + 2x^2 g\left(\frac{1}{x}\right)}{3x^2}\right]$$

$$\text{Now, } f(-x) = -\left[\frac{g(-x) + 2x^2 g(-1/x)}{3x^2}\right]$$

$$= \left[\frac{g(x) + 2x^2 g(1/x)}{3x^2}\right]$$


$$\therefore f(x) = -f(-x)$$

$f(x)$ is an odd function.

But $f(x)$ is given to be an even function.

$$\therefore f(x) = 0 \quad \forall x \Rightarrow f(5) = 0$$

TEST Edge Questions based on domain and range of different types of functions and their value in any interval are asked in JEE Advanced. So, to solve such types of questions, students are advised to learn definition of different functions with range and domain such as trigonometric functions, even or odd function, etc. e.g., if $f(x) = (1+b^2)x^2 + 2bx + 1$ and $m(b)$ is the minimum value of $f(x)$ then for the range of $m(b)$, we need to find minimum value and the range of given function $f(x)$, so range of $m(b)$ is $(0, 1]$.

46. (d)  **Idea** For any complex number z , we have $\text{Re}(z)$ as $-|z| \leq \text{Re}(z) \leq |z|$ and $\text{Im}(z)$ as $-|z| \leq \text{Im}(z) \leq |z|$ and for any function $f(x)$ if $f'(a) = 0$ and $f''(a) < 0$, then $f(x)$ would have a local maximum at $x = a$, otherwise when $f'(a) = 0$ and $f''(a) > 0$, then $f(x)$ would have a local minimum at $x = a$.

$$\text{Let } z = a + ib, a > 0, b > 0$$

$$\text{Since, } \text{Re}(z) + \text{Im}(z) = 3$$

$$\therefore a + b = 3$$

$$\text{Let } P = (\text{Re } z)^2 \text{Im}(z) = a^2 b$$

$$P = a^2(3-a) = 3a^2 - a^3 \quad (\text{to be maximised})$$

$$\frac{dP}{da} = 6a - 3a^2$$

$$\text{For maxima or minima } \frac{dP}{da} = 0$$

$$\Rightarrow 6a - 3a^2 = 0$$

$$\Rightarrow 3a(2-a) = 0$$

$$\Rightarrow a = 0 \text{ or } a = 2$$


$$\text{Now, } \frac{d^2P}{da^2} = -6a$$

$$\left[\frac{d^2P}{da^2}\right]_{a=2} = -12 < 0$$

$\therefore a = 2$ is point of maxima.

$$\text{Maximum value of } P = (2)^2(3-2) = 4$$

TEST Edge Questions based on the expression of complex number and related to maxima or minima of function, students are advised to learn the basic representation of complex numbers in terms of real and imaginary numbers also acquainted yourself with second order derivative test to find the maximum or minimum value of the function. e.g., the minimum value of function $f(x) = \log_a x + \log_x a$ is to be verified by the second order derivative test so you will find, it is false.

47. (d)  **Idea** Chain rule of differentiation if $y = f(g(x))$ and $u = g(x)$, then $\frac{dy}{dx} = \frac{dy}{du} \cdot \frac{du}{dx}$ and the concept of definite integral $\int_0^b f(x) dx = F(b) - F(a)$, where $\int f(x) dx = F(x) + C$

$$\text{Consider } \frac{df(x)}{dx} = \frac{e^{\cos x}}{x}$$

$$\Rightarrow \frac{df(x^3)}{dx^3} = \frac{e^{\cos x^3}}{x^3}$$

$$\Rightarrow \frac{df(x^3)}{dx^3} \cdot \frac{d(x^3)}{dx} = \frac{e^{\cos x^3}}{x^3} \cdot 3x^2$$

$$\Rightarrow \frac{df(x^3)}{dx} = \frac{3e^{\cos x^3}}{x} \quad \dots(i)$$

Now, consider the given integral

$$\int_1^5 \frac{3e^{\cos x^3}}{x} dx = f(k) - f(1)$$

$$\Rightarrow \int_1^5 \frac{d[f(x^3)]}{dx} dx = f(k) - f(1) \quad [\text{using Eq. (i)}]$$

$$\Rightarrow \int_1^5 d[f(x^3)] = f(k) - f(1)$$

$$\Rightarrow [f(x^3)]_1^5 = f(k) - f(1)$$

$$\Rightarrow f(125) - f(1) = f(k) - f(1)$$

$$\Rightarrow k = 125$$

TEST Edge In JEE Advanced, questions based on differentiation rule and basic concepts related to definite integral are asked. To solve these types of questions learn the basic formulae of differentiation such as quotient rule, product rule, etc., and basic definition of definite integral as area function etc.

eg., if $\int_0^x f(t) dt = x + \int_x^1 t f(t) dt$, then the value of $f(1)$ is obtained by differentiating w.r.t. x , so $f(1) = \frac{1}{2}$.

48. (c) **Idea** If $y = ax^2 + bx + c$ is a quadratic equation, then value of discriminant, $D = \sqrt{b^2 - 4ac}$ and the graph between x and y is always a parabola. If $a > 0$ and $D < 0$, then shape of the parabola is concave upwards $\forall x \in R, y > 0$. Probability of an event is given by $P(E) = \frac{\text{Favourable outcomes}}{\text{Total outcomes}}$

Since, the curve is strictly above the x -axis

$$\therefore x^2 + 2(a+4)x - 5a + 64 \geq 0$$

$$\Rightarrow D < 0$$

$$4(a+4)^2 - 4(-5a+64) < 0$$

$$\Rightarrow a^2 + 13a - 48 < 0$$

$$\Rightarrow (a+16)(a-3) < 0$$

$$\Rightarrow -16 < a < 3$$

Also, $a \in [-5, 30]$

$$\therefore a \in [-5, 2]$$

\therefore Number of favourable cases = 8

Total number of values of a in $[-5, 30]$ i.e., 36

$$\therefore \text{Required probability} = \frac{8}{36} = \frac{2}{9}$$

TEST Edge Questions based on quadratic equation such as roots of the equation, discriminant value, nature of roots and basic definition of probability are asked. So, to solve such types of questions understand the results of quadratic in equalities and its graphical representation in different shapes of the parabola and also understand the application of the probability definition.

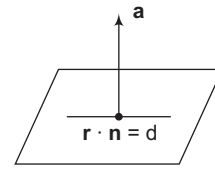
For any quadratic equation $ax^2 + bx + c$, if $D = 0$ then roots are equal i.e., $x_1 = x_2$ in that case $a > 0$

$$\Rightarrow x \in (-\infty, x_1) \cup (x_1, \infty) \text{ and } a < 0$$

$$\Rightarrow x \in \phi$$

49. (b) **Idea** If a line $\mathbf{r} = \mathbf{a} + \lambda \mathbf{b}$ lies in the plane $\mathbf{r} \cdot \mathbf{n} = d$, then $\mathbf{b} \cdot \mathbf{n} = 0$ and $\mathbf{a} \cdot \mathbf{n} = d$, vector equation of a straight line passing through a position vector \mathbf{a} and parallel to a vector \mathbf{b} is $\mathbf{r} = \mathbf{a} + \lambda \mathbf{b}$.

Equation of line passing through \mathbf{a} and \perp to $\mathbf{r} \cdot \mathbf{n} = d$ is $\mathbf{r} = \mathbf{a} + \lambda \mathbf{n}$



For foot of perpendicular,

$$(\mathbf{a} + \lambda \mathbf{n}) \cdot \mathbf{n} = d$$

$$\Rightarrow \mathbf{a} \cdot \mathbf{n} + \lambda |\mathbf{n}|^2 = d$$

$$\Rightarrow \lambda = \frac{d - \mathbf{a} \cdot \mathbf{n}}{|\mathbf{n}|^2}$$

$$\therefore \text{Foot of perpendicular is } \mathbf{a} + \left(\frac{d - \mathbf{a} \cdot \mathbf{n}}{|\mathbf{n}|^2} \right) \mathbf{n}$$

\therefore Equation of line parallel to $\mathbf{r} = \mathbf{a} + \lambda \mathbf{b}$ and passing through foot of perpendicular

$$\mathbf{r} = \mathbf{a} + \left(\frac{d - \mathbf{a} \cdot \mathbf{n}}{|\mathbf{n}|^2} \right) \mathbf{n} + \lambda \mathbf{b}$$

TEST Edge Questions based on planes, lines and also concept related to lines with planes such as angle between them, distance between them, vector form, Cartesian form of lines and planes are asked. To solve such types of questions, students are advised to understand the concept of lines and planes such as length of the perpendicular from a point having position vector \mathbf{a} to the plane $\mathbf{r} \cdot \mathbf{n} = d$ is given by

$$P = \frac{\mathbf{a} \cdot \mathbf{n} - d}{|\mathbf{n}|} \text{ need to use and understand.}$$

50. (b) **Idea** Let $y = f(x) = [x]$ is a greatest integer function. If $n \leq x < n + 1$, then $[x] = n$, i.e., $[2.3] = 2$, $[4.3] = 4$. Now, apply the concept of maxima and minima, then take $\lim x \rightarrow 0$, to solve this problem.

$$y^2 - 4y + 11 = (y - 2)^2 + 7$$

Now, $(y - 2)^2 \geq 0$

$$(y - 2)^2 + 7 \geq 7$$

\therefore Minimum value of $y^2 - 4y + 11 = 7$


$$\Rightarrow \lim_{x \rightarrow 0} \left[\min(y^2 - 4y + 11) \frac{\sin x}{x} \right] = \lim_{x \rightarrow 0} \left[\frac{7 \sin x}{x} \right]$$

Now, $|\sin x| < |x|$, where $x \rightarrow 0$

$$\therefore \lim_{x \rightarrow 0} [\text{a value slightly less than } 7] = 6$$

TEST Edge Evaluation of limits of the logarithmic function, exponential function related questions are asked. To solve such types of questions, students are advised to understand the concept of limit and also acquainted yourself with properties of the limit such as

$$\lim_{n \rightarrow \infty} a^n = \begin{cases} \infty, & \text{if } a > 1 \\ 1, & \text{if } a = 1 \\ 0, & \text{if } -1 < a < 1 \\ \text{doesn't exist,} & \text{if } a \leq -1 \end{cases}$$

51. (a,c)  **Idea** ∴ $f(x) = [x]$ is a greatest integer function. If $n \leq x < n + 1$, then $[x] = n$, such as $[2.3] = 2$, $[9.2] = 9$ and apply the concept of limit to get the required solution.
 $\lim_{x \rightarrow 0} [-x] = \lim_{x \rightarrow 0} [0-x] = \lim_{x \rightarrow 0} (-1) = -1$.

We have $x^2 > 0$ and $\lim_{x \rightarrow 0} \frac{f(x)}{x^2} = 2$

∴ $f(x)$ must be a positive quantity.

Also, $x^2 \rightarrow 0$ as $x \rightarrow 0$ and limit is finite.

∴ $f(x)$ should also approach to zero as $x \rightarrow 0$

$$\lim_{x \rightarrow 0} [f(x)] = 0$$

Now, $\lim_{x \rightarrow 0^+} \left[\frac{f(x)}{x} \right] = \lim_{x \rightarrow 0^+} \left[x \cdot \frac{f(x)}{x^2} \right] = 0$


$$\lim_{x \rightarrow 0^-} \left[\frac{f(x)}{x} \right] = \lim_{x \rightarrow 0^-} \left[x \cdot \frac{f(x)}{x^2} \right] = -1$$

∴ $\lim_{x \rightarrow 0} \left[\frac{f(x)}{x} \right]$ does not exist.

TEST Edge Evaluation of limit of algebraic function, modulus function, or related questions are asked. To solve such types of questions, students are advised to understand the concepts of the limit and also acquainted yourself with the properties of limit such as if

$$\lim_{x \rightarrow a} f(x) = A > 0 \quad \text{and} \quad \lim_{x \rightarrow a} g(x) = B,$$

then $\lim_{x \rightarrow a} [f(x)]^{g(x)} = A^B$.

52. (a,c)  **Idea** ∴ Let the point of intersection be (h, k) , then compare the both equations and solve them in (h, k) to get required solution.

Let the point of intersection of E_1 and E_2 be (h, k)

$$\therefore \frac{h^2}{a^2} + k^2 = 1 \quad \text{and} \quad h^2 + \frac{k^2}{a^2} = 1$$

$$\Rightarrow \frac{h^2}{a^2} = 1 - k^2 \quad \text{and} \quad 1 - h^2 = \frac{k^2}{a^2}$$

$$\Rightarrow \frac{h^2}{1 - k^2} = a^2 \quad \text{and} \quad a^2 = \frac{k^2}{1 - h^2}$$

$$\therefore \frac{h^2}{1 - k^2} = \frac{k^2}{1 - h^2}$$

$$\Rightarrow h^2(1 - h^2) = k^2(1 - k^2)$$

$$\Rightarrow h^2 - h^4 = k^2 - k^4$$

$$\Rightarrow h^2 - k^2 - h^4 + k^4 = 0$$

$$\Rightarrow h^2 - k^2 - (h^4 - k^4) = 0$$

$$\Rightarrow (h^2 - k^2) - (h^2 - k^2)(h^2 + k^2) = 0$$


$$\Rightarrow (h^2 - k^2)[1 - (h^2 + k^2)] = 0$$

$$\Rightarrow h^2 = k^2 \quad \text{or} \quad h^2 + k^2 = 1$$

$$\Rightarrow x^2 = y^2 \quad \text{or} \quad x^2 + y^2 = 1$$

$$\Rightarrow x = \pm y \quad \text{or} \quad x^2 + y^2 = 1$$

TEST Edge Locus of straight line, parabola, and circle related questions are asked. To solve such types of questions, students are advised to understand the concepts of locus and also acquainted yourself with the properties of straight line, parabola, and circle such as, the two lines $a_1x + b_1y + c_1 = 0$ and $a_2x + b_2y + c_2 = 0$ are intersecting, if $\frac{a_1}{a_2} \neq \frac{b_1}{b_2}$ i.e., if they are neither coincident nor parallel.

53. (b,c)  **Idea** To solve this problem first of all find the n^{th} term of the given series, and use method of difference, then apply the concept of summation of series.

$$T_r = \tan^{-1} \left(\frac{\sqrt{2}}{r+1} \right)^2$$

$$S_n = \sum_{r=1}^n \tan^{-1} \left(\frac{2}{r^2 + 2r + 1} \right) = \sum_{r=1}^n \tan^{-1} \left(\frac{r+2-r}{1+r(r+2)} \right)$$

$$= \sum_{r=1}^n \tan^{-1} (r+2) - \tan^{-1} r$$

$$S_n = [\tan^{-1} 3 - \tan^{-1} 1] + [\tan^{-1} 4 - \tan^{-1} 2] + [\tan^{-1} 5 - \tan^{-1} 3] + \dots + [\tan^{-1} (n+2) - \tan^{-1} n]$$

$$= \tan^{-1} (n+2) + \tan^{-1} [n+1] - \tan^{-1} 2 - \tan^{-1} 1$$


$$S_n = \tan^{-1} \left[\frac{3n^2 + 7n}{n^2 + 9n + 10} \right]$$

$$S_n = \lim_{n \rightarrow \infty} \tan^{-1} \left[\frac{3 + \frac{7}{n}}{1 + \frac{9}{n} + \frac{10}{n^2}} \right] = \tan^{-1} (3)$$

$$\text{or } \cot^{-1} \left(\frac{1}{3} \right)$$

TEST Edge Conceptual mixing of trigonometric function, algebraic function and series based questions are asked. To solve such types of questions, students are advised to understand the concepts of series and also acquainted yourself with the properties of the functions, such as if a is the first term, d the common difference and r the common ratio then, sum of first n terms of arithmetic-geometric progression (AGP) then

$$S_n = \frac{d}{(1-r)} + \frac{dr(1-r^{n-1})}{(1-r)^2} - \frac{[a + (n-1)d]r^n}{(1-r)}$$

54. (a,c)  **Idea** ∴ If $ax^2 + bx + c = 0$ has roots α and β then $\alpha + \beta = -b/a$, $\alpha\beta = c/a$ and if sum of roots be S and their product be P then quadratic equation $x^2 - Sx + P = 0$

Given that u and v are roots of equation

$$x^2 + px + q = 0$$

So, $u + v = -p$

$$uv = q$$

a. Now, equation whose roots are $\frac{1}{v}$ and $\frac{1}{u}$

$$\Rightarrow x^2 - \left(\frac{1}{u} + \frac{1}{v}\right)x + \frac{1}{uv} = 0$$

$$x^2 - \frac{u+v}{uv}x + \frac{1}{uv} = 0$$

$$uvx^2 - (u+v)x + 1 = 0$$

$$\Rightarrow qx^2 + px + 1 = 0$$

b. Now equation whose roots are u^2 and v^2

So, equation will be

$$x^2 - (u^2 + v^2)x + u^2v^2 = 0$$

So, equation will be

$$x^2 - (p^2 - 2q)x + q^2 = 0$$

$$\therefore u^2 + v^2 = (u+v)^2 - 2uv = p^2 - 2q$$

$$\text{and } u^2v^2 = q^2$$

TEST Edge Common roots, greatest and least values of a quadratic expression, nature of roots of a quadratic equation with respect to one or two real numbers related questions are asked. To solve such types of questions, students are advised to understand the concept of quadratic equation and expression such as a polynomial equation $f(x) = 0$ has exactly r roots equal to α if $f(\alpha) = f'(\alpha) = f''(\alpha) \dots f^{(r-1)}(\alpha) = 0$ and $f^{(r)}(\alpha) \neq 0$.

55. (a, b, c) **Idea** \therefore If three vectors $\mathbf{a} = a_1\hat{i} + a_2\hat{j} + a_3\hat{k}$, $\mathbf{b} = b_1\hat{i} + b_2\hat{j} + b_3\hat{k}$ and $\mathbf{c} = c_1\hat{i} + c_2\hat{j} + c_3\hat{k}$ are coplanar then $[\mathbf{a} \ \mathbf{b} \ \mathbf{c}] = 0$ or $\begin{vmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{vmatrix} = 0$ and if two vectors \mathbf{p} and \mathbf{q} are perpendicular to each other, then $\mathbf{p} \cdot \mathbf{q} = 0$.

Given vectors are coplanar

$$\Rightarrow \begin{vmatrix} a & b & c \\ b & c & a \\ c & a & b \end{vmatrix} = 0$$

$$\Rightarrow a^3 + b^3 + c^3 - 3abc = 0$$

$$\Rightarrow (a+b+c)(a^2 + b^2 + c^2 - ab - bc - ca) = 0$$

$$= \frac{1}{2}(a+b+c)(2a^2 + 2b^2 + 2c^2 - 2ab - 2bc - 2ca) = 0$$

$$\Rightarrow \frac{1}{2}(a+b+c)[(a-b)^2 + (b-c)^2 + (c-a)^2] = 0$$

$$\Rightarrow a+b+c=0$$

$$\text{or } a=b=c$$

But $a \neq b$

$$\therefore a+b+c=0$$

$$\text{Now, } \alpha \cdot \mathbf{v} = (a\hat{i} + b\hat{j} + c\hat{k}) \cdot (\hat{i} + \hat{j} + \hat{k})$$

$$\Rightarrow a+b+c=0$$

$$\beta \cdot \mathbf{v} = a+b+c=0$$

$$\lambda \cdot \mathbf{v} = a+b+c=0$$

TEST Edge Collinear vectors linearly dependent or independent vector and geometrical application of dot product of two vectors related questions are asked. To solve such types of questions, students are advised to understand the concept of types vector and dot product of two vectors such as if $\mathbf{a} = a_1\hat{i} + a_2\hat{j} + a_3\hat{k}$, $\mathbf{b} = b_1\hat{i} + b_2\hat{j} + b_3\hat{k}$ then by Cauchy-schwarz inequality $(\mathbf{a} \cdot \mathbf{b}) \leq |\mathbf{a}| |\mathbf{b}|$.

56. (5) **Idea** \therefore If $z = a + ib$ is a complex number then $\bar{z} = a - ib$ and $z + \bar{z} = 2a$, $z - \bar{z} = 2ib$ and $|z|^2 = z\bar{z}$, $|z - a|^2 = (\bar{z} - a)(z - a)$. Now, apply the concept of minimum to get the required solution.

$$p = z\bar{z} + (z-3)(\bar{z}-3) + (z-6i)(\bar{z}+6i)$$

$$= 3z\bar{z} - 3(z+\bar{z}) + 9 + 6(z-\bar{z})i + 36$$

$$= 3(x^2 + y^2) - 3(2x) + 9 + 6(2iy)i + 36$$

$$\left. \begin{array}{l} \text{(let } z = x + iy \\ \text{then } \bar{z} = x - iy \\ z + \bar{z} = 2x \\ z - \bar{z} = 2iy \end{array} \right\}$$

$$= 3(x^2 + y^2) - 6x + 9 - 12y + 36$$

$$= 3[x^2 + y^2 - 2x - 4y + 15]$$

$$= 3[(x-1)^2 + (y-2)^2 + 10]$$

For minimum value of p , $x = 1, y = 2$

$$\text{Minimum value of } p = 3(10) = 30 \Rightarrow \frac{q}{6} = \frac{30}{6} = 5$$

TEST Edge Principal value of arg of complex number, properties of modulus and properties of argument related questions are asked. To solve such types of questions, students are advised to understand the concept of complex number such as $||z_1| - |z_2|| \leq |z_1 + z_2| \leq |z_1| + |z_2|$. Thus, $|z_1| + |z_2|$ is the greatest possible value of $|z_1 + z_2|$ and $||z_1| - |z_2||$ is least possible for value of $|z_1 + z_2|$.

57. (8) **Idea** $\therefore x = a \cos \theta, y = b \sin \theta$ are the parametric equations of the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$, where Parameter $0 \leq \theta < 2\pi$. $x = \cos \theta, \theta \in [0, \pi]$ and $x \in [-1, 1]$ also apply the concept of maxima and minima.

$$\text{Let } x - 4 = 2 \cos \theta \Rightarrow x = 2 \cos \theta + 4$$

$$\text{and } y = 3 \sin \theta$$

$$\text{Now, } E = \frac{x^2}{4} + \frac{y^2}{9}$$

$$= \frac{(2 \cos \theta + 4)^2}{4} + \sin^2 \theta$$

$$= \frac{4 \cos^2 \theta + 16 + 16 \cos \theta + 4 \sin^2 \theta}{4}$$

$$= \frac{20 + 16 \cos \theta}{4} = 5 + 4 \cos \theta$$

$$\text{Now, } E_{\max} = 5 + 4 = 9 \quad [\because \cos \theta \in [-1, 1]]$$

$$\Rightarrow E_{\min} = 5 - 4 = 1$$

$$\Rightarrow E_{\max} - E_{\min} = 9 - 1 = 8$$

TEST Edge Properties of an ellipse, eccentricity of the ellipse, auxiliary circle related questions are asked.

To solve such types of questions, students are advised to understand the concepts of ellipse such as the equations of chord joining the points $P(a \cos \theta, b \sin \theta)$ and $Q(a \cos \theta_2, b \sin \theta_2)$ is

$$\frac{x}{a} \cos \left(\frac{\theta_1 + \theta_2}{2} \right) + \frac{y}{b} \sin \left(\frac{\theta_1 + \theta_2}{2} \right) = \cos \left(\frac{\theta_1 - \theta_2}{2} \right)$$

58. (4) **Idea** ∴ Vector area of a ΔABC , when $\mathbf{a}, \mathbf{b}, \mathbf{c}$ are the position vectors of A, B, C respectively is area $(\Delta ABC) = \frac{1}{2}(\mathbf{a} \times \mathbf{b} + \mathbf{b} \times \mathbf{c} + \mathbf{c} \times \mathbf{a})$

Let $\mathbf{OA} = \mathbf{a}, \mathbf{OB} = \mathbf{b}, \mathbf{OC} = \mathbf{c}, \mathbf{OD} = \mathbf{d}$
 $\mathbf{AB} = \mathbf{b} - \mathbf{a}, \mathbf{BC} = \mathbf{c} - \mathbf{b}, \mathbf{CD} = \mathbf{d} - \mathbf{c}$
 $\mathbf{AD} = \mathbf{d} - \mathbf{a}, \mathbf{CA} = \mathbf{a} - \mathbf{c}, \mathbf{BD} = \mathbf{d} - \mathbf{b}$

Then, $\mathbf{AB} \times \mathbf{CD} + \mathbf{BC} \times \mathbf{AD} + \mathbf{CA} \times \mathbf{BD}$
 $= (\mathbf{b} - \mathbf{a}) \times (\mathbf{d} - \mathbf{c}) + (\mathbf{c} - \mathbf{b}) \times (\mathbf{d} - \mathbf{a})$
 $\quad + (\mathbf{a} - \mathbf{c}) \times (\mathbf{d} - \mathbf{b})$
 $= \mathbf{b} \times \mathbf{d} - \mathbf{b} \times \mathbf{c} - \mathbf{a} \times \mathbf{d} + \mathbf{a} \times \mathbf{c}$
 $\quad + \mathbf{c} \times \mathbf{d} - \mathbf{c} \times \mathbf{a} - \mathbf{b} \times \mathbf{d} + \mathbf{b} \times \mathbf{a}$
 $\quad + \mathbf{a} \times \mathbf{d} - \mathbf{a} \times \mathbf{b} - \mathbf{c} \times \mathbf{d} + \mathbf{c} \times \mathbf{b}$
 $= 2(\mathbf{b} \times \mathbf{a}) + 2(\mathbf{c} \times \mathbf{b}) + 2(\mathbf{a} \times \mathbf{c})$

Now, $|\mathbf{AB} \times \mathbf{CD} + \mathbf{BC} \times \mathbf{AD} + \mathbf{CA} \times \mathbf{BD}|$
 $= |2(\mathbf{b} \times \mathbf{a}) + 2(\mathbf{c} \times \mathbf{b}) + 2(\mathbf{a} \times \mathbf{c})|$
 $= 2|\mathbf{a} \times \mathbf{b} + \mathbf{b} \times \mathbf{c} + \mathbf{c} \times \mathbf{a}|$
 $= 4 \cdot \frac{1}{2} |\mathbf{a} \times \mathbf{b} + \mathbf{b} \times \mathbf{c} + \mathbf{c} \times \mathbf{a}|$
 $= 4(\text{area of } \Delta ABC)$

TEST Edge Geometrical application of dot product or cross products of two vectors related questions are asked. To solve such types of questions, students are advised to understand the concept of product of two vectors such as the area of a parallelogram with an adjacent sides of \mathbf{a} and \mathbf{b} is given by $|\mathbf{a} \times \mathbf{b}|$.

59. (2) **Idea** To solve this problem use the concept of theory of equation and apply concept of number of integral solution.

For an integer solution, it must lie between 1 and 2007 (for otherwise all terms will be positive or negative). Now, the terms will cancel in pairs if integer is taken as

$$\frac{2007+1}{2} = 1004$$

$$\Rightarrow \frac{K}{502} = \frac{1004}{502} = 2$$

TEST Edge Location of roots, logarithmic inequalities related questions are asked. To solve such types of questions, students are advised to

understand the concept of theory of equation such as the condition that a quadratic function $f(x, y) = ax^2 + 2hxy + by^2 + 2gx + 2fy + c$ may be resolved into two linear factor is that

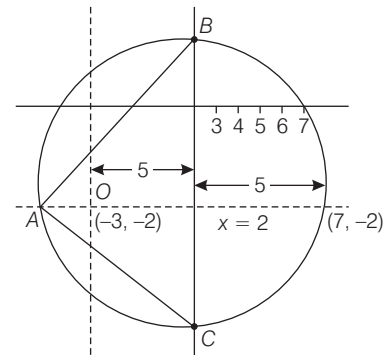
$$\begin{vmatrix} a & h & g \\ h & b & f \\ g & f & c \end{vmatrix} = 0$$

60. (5) **Idea** To solve this problem, use the concepts, image of the orthocentre of a triangle in any of its sides always circumcentre on it lies.

Image of point $P(7, -2)$ in the line $x = 2$ is $(-3, -2)$

Orthocentre of Δ is $(-3, -2)$

To solve this problem, use the concept of the image of point of intersection of altitude with circumcircle is the orthocenter of triangle



∴ Absolute value of sum of coordinates $= |-3 - 2| = 5$

TEST Edge Image or reflection of a point in different cases, and application of image or reflection related questions are asked. To solve such types of questions, students are advised to understand the concept of image or reflection of a points in 2D such as the image of $A(x_1, y_1)$ with respect to line mirror $ax + by + c = 0$ be $B(h, k)$ is given by $\frac{h - x_1}{a} = \frac{k - y_1}{b} = \frac{-2(ax_1 + by_1 + c)}{a^2 + b^2}$.

