Question Booklet Code 3

Test RIDER

JEE Advanced

A Single Door Entry to the Various IITs...

Paper 1

Duration : 3 Hours

Max. Marks : 180

Please read the instructions carefully. You are allotted 5 minutes specially for this purpose.

- This booklet is your question paper. Attempt all the questions.
- Blank papers, clipboards, log tables, slide rules, calculators, cameras, cellular phones, pagers and electronic gadgets are not allowed.
- Write your name and roll number in the space provided on the bottom of this page.

Question Paper Format

- The question paper consists of three parts (Physics, Chemistry and Mathematics). Each part consists of three sections.
- **Section 1** contains 10 multiple choice questions. Each question has four choices (a), (b), (c) and (d) out of which only one is correct.
- **Section 2** contains 5 multiple choice questions. Each question has four choices (a), (b), (c) and (d) out of which one or more than one is/are correct.
- **Section 3** contains 5 questions. The answer to each question is a single–digit integer, ranging from 0 to 9 (both inclusive)

Marking Scheme

- For each question in Section 1, you will be awarded **2 marks** for correct answer and zero mark for unattempted questions. No negative marks will be awarded for incorrect answers in this section.
- For each question in Section 2, you will be awarded **4 marks** for correct answer(s) and zero mark for unattempted questions. In all other cases, minus one (–1) mark will be awarded.
- For each question in Section 3, you will be awarded **4 marks** for the correct answer and zero mark for unattempted questions. In all other cases, minus one (–1) mark will be awarded.

Practice Set 4

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Part I

Section 1 Single Correct Option Type

This section contains **10 *multiple choice questions***. *Each question has four choices, (a), (b), (c) and (d) out of which* ***only one*** *is correct.*

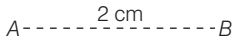
1. Imagine an atom made up of a proton and a hypothetical particle of double the mass of the electron but having the same charge as the electron. The wavelength of the radiation that will be emitted when this particles jumps from first excited state to ground state (in terms of the Rydberg constant R for the hydrogen atom) is equal to

a. $\frac{2}{3R}$ *b.* $\frac{1}{3R}$ *c.* $\frac{1}{2R}$ *d.* None of these

2. Let ω be the angular velocity of earth's rotation about its axis. Assume that the acceleration due to gravity on earth's surface has value at equator and the poles. An object weighed at the equator gives same reading as taken at depth d below surface at pole ($d << R$). The value of d is

a. $\frac{\omega^2R^2}{g}$ *b.* $\frac{\omega^2R^2}{2g}$ *c.* $\frac{2\omega^2R^2}{g}$ *d.* $\frac{\sqrt{R}}{\sqrt{g}}$

3. The potential difference between the points A and B is + 40 V. If the electric field is uniform then the value of electric field at B

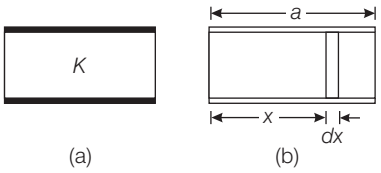


a. may be equal to 20 V/cm *b.* may be less than 20 V/cm
c. Both (a) and (b) are correct *d.* Both (a) and (b) are not correct.

4. Figure (a), shows a parallel-plate capacitor having square plates of edge a and plate-separation d. The gap between the plates is filled with a dielectric of dielectric constant K which varies parallel to an edge a, where K and α are constant and x is the distance from the left end.

$$K = K_o + \alpha x$$

Calculate the capacitance?

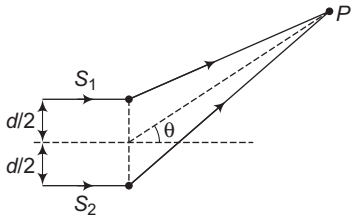


a. $\frac{\varepsilon_o a^2}{d}\left[K_o + \frac{a\alpha}{2}\right]$ *b.* $\frac{\varepsilon_o 2a^2}{d}\left[3K_o + \frac{a\alpha}{2}\right]$
c. $\frac{3\varepsilon_o a^2}{4d}\left[K_o + \frac{a\alpha}{4}\right]$ *d.* $\frac{\varepsilon_o a^2}{6d}\left[K_o + \frac{a\alpha}{2}\right]$

5. A rigid body oscillates about point O. Distance between O and center of gravity is L and radius of gyration is K, then what will be equivalent length?

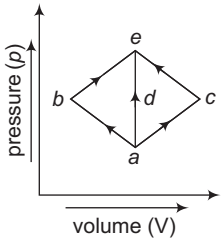
a. $\frac{K}{L}$ *b.* $\frac{K^2}{L}$ *c.* $L + \frac{L^2}{K}$ *d.* $L + \frac{K^2}{L}$

6. Two coherent light source radiate in phase. The distance between sources is d, where $d/4$ is the wavelength of light produced. The angles are measured from the line connecting the two sources. The distance from source to point of observation is significantly larger than wavelength, then



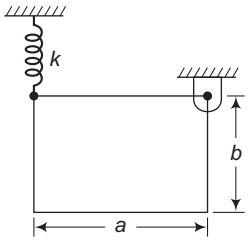
a. at θ = 60°, maxima at *P*
b. at θ = 30°, minima at *P*
c. at θ = 60°, minima at *P*
d. at θ = 30°, maxima at *P*

7. The initial state of an ideal gas is represented by the point *a* on the *p-V* diagram and its final state by the point *e*. The gas goes from the state *a* to the state *e* by three quasi stationary processes represented by (i) *abe* (ii) *ace* (iii) *ade*. The heat absorbed by the gas is



a. the same in all the processes
b. the same in processes (i) and (ii)
c. greater in process (i) than in (iii)
d. less in process (ii) than in (iii)

8. Time period of small oscillation of plate of mass *m* and given dimension is



a. $2\pi\sqrt{\frac{m}{k}}$
b. $2\pi\sqrt{\frac{m(a^2+b^2)}{3ka^2}}$
c. $2\pi\sqrt{\frac{ma}{kb}}$
d. None of these

9. A particle moves with simple harmonic motion in straight line.
In 1^{st} second, it covers d_1 and in 2^{nd} second, it covers d_2. What will be the amplitude of SHM (particle start from rest)?

a. d_1d_2
b. $\frac{d_1}{d_2}$
c. $\frac{d_1^2}{2d_1-d_2}$
d. $\frac{2d_1^2}{3d_1-d_2}$

10. A projectile is fired with velocity *u* at an angle θ, so to strike a point on the inclined plane at an angle α with the horizontal. The point of projection is at a distance *d* from the inclined plane on the ground. The angle α is adjusted in such a way, so that the projectile can strike the inclined plane in minimum time, find that minimum time.

a. $t_{min}=\left(\frac{u-\sqrt{u^2-gd\tan 2\alpha}}{g\tan\alpha}\right)$
b. $t_{min}=\left(\frac{u-\sqrt{u^2-gd\sin 2\alpha}}{g\cos\alpha}\right)$
c. $t_{min}=\left(\frac{u-\sqrt{u^2-gd\sin 2\alpha}}{g\tan\alpha}\right)$
d. $t_{min}=\left(\frac{u-\sqrt{u^2-gd\cos 2\alpha}}{g\cos\alpha}\right)$

Section 2 More Than One Correct Option

This section contains **5 multiple choice questions**. *Each question has four choices, (a), (b), (c) and (d) out of which* **one** *or* **more than one** *is/are correct.*

11. First overtone of open organ pipe (of length L_o) beats with first overtone of a closed organ pipe (of length L_c) and beat frequency is 10 beats/s. Fundamental frequency of close organ pipe is 110 Hz and speed of sound is 330 m/s. Find out the correct choices for length L_o and L_c.

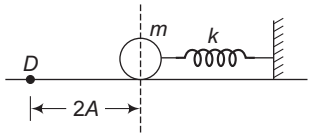
a. $L_c = 0.75$ m

b. $L_o = \frac{33}{34}$ m or $\frac{33}{35}$ m

c. $L_o = \frac{33}{32}$ m or 1 m

d. $L_o = \frac{33}{32}$ m or $\frac{33}{34}$ m

12. In a given diagram, detector D is stationary and source is oscillating with amplitude A and given situation is mean position. Mass of block m and spring constant k. Assuming no damping, pick correct options $\left(T = 2\pi\sqrt{\frac{m}{k}}\right)$



a. detector will detect maxima at intervals of T

b. detector will detect minima at intervals less than T

c. detector will detect frequency less than actual frequency for longer time interval

d. detector will detect the frequency greater than actual frequency for time interval $\left(\frac{T}{2} - \frac{2A}{v}\right)$, where v speed of sound.

13. A block is connected to a spring in an elevator. Elongation in spring is $x_1 = 4\sqrt{2}$ mm and $x_2 = 3\sqrt{2}$ mm, when it moves up and down with constant acceleration respectively. Then,

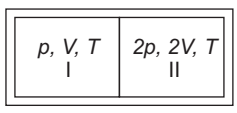
a. acceleration of elevator is $\frac{g}{7}$ m/s^2

b. acceleration of elevator is $\frac{g}{3}$ m/s^2

c. if same elevator moves horizontally with same acceleration then elongation will be 5 mm

d. if same elevator moves horizontally with same acceleration then elongation will be 10 mm

14. A partition divides a container having insulated walls into two compartments I and II. The same gas fills the two compartments whose initial parameters are given. The partition is a conduction wall which can move freely without friction. Which of the following statements is/are correct with reference to the final equilibrium position?



a. The final pressure in the two compartments are equal

b. Volume of compartment I is $\frac{3V}{5}$

c. Volume of compartment II is $\frac{12V}{5}$

d. Final pressure in compartment I is $\frac{5p}{3}$

15. Radiations of monochromatic waves of wavelength 400 nm are made incident on the surface of metals Zn, Fe and Ni of work functions 3.4 eV, 4.8 eV and 5.9 eV respectively. Which of the following is (are) correct?

a. Maximum KE associated with photoelectrons from the surface of any metal is 0.3 eV

b. No photoelectrons are emitted from the surface of Ni

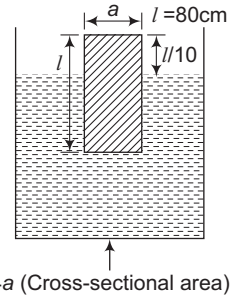
c. If the wavelength of source radiation is doubled then KE of photoelectrons is also doubled

d. Photoelectrons will be emitted from the surface of all the three metals, if the wavelength of incident radiations < 200 nm

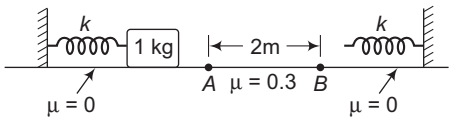
Section 3 Integer Answer Type

This section contains **5 multiple choice questions.** *The answer to each question is a single-digit integer, ranging from* 0 *to* 9 *(both inclusive).*

16. A uniform vertical cylinder of cross-sectional area a floats, 90% submerged in an unknown liquid inside a tank with cross-sectional area four times that of cylinder. When cylinder is pushed down gently and released, it performs SHM. The maximum possible amplitude (in cm) for this SHM is



17. Massless springs, each with $k = 1350\ \text{Nm}^{-1}$ are attached to the left and right walls as shown in the figure. A 1 kg block is initially held against the left-hand spring, compressing the spring by 0.1 m. The block is released to move. The floor is frictionless except for the section AB. Coefficient of friction for AB is 0.3. Find the distance (in cm) from point B, where block finally comes to rest.



18. The prism spectrum is spread out more at violet end than at the red end. Angular dispersion is defined as $\phi = \dfrac{d\delta}{d\lambda}$, if deviation of small prism δ is defined as $\delta = (\mu - 1)\, A_0$ and wavelength is defined as $\mu = A + \dfrac{B}{\lambda^2}$, where A and B are constant and A_0 is prism angle then dispersion $\phi \propto \dfrac{1}{\lambda^N}$. Find value of N.

19. A body cools from 50°C to 40°C in 5 min. The surrounding temperature is 20°C. By how many °C does the temperature decrease in the next 5 min? Round your answer to nearest integer.

20. A hydrogen like atom (atomic number Z) is in a higher excited state of quantum number n. The excited atom can make a transition to the first excited state by successively emitting two photons of energy 10.2 eV and 17.0 eV respectively. Alternatively, the atom from the same excited state can make a transition to the second excited by successively emitting two photons of energies 4.25 eV and 5.95 eV respectively. Determine the value of n. (Ionisation energy of H-atom = 13.6 eV)

Part II

Section 1 Single Correct Option Type

This section contains **10 *multiple choice questions***. *Each question has four choices, (a), (b), (c) and (d) out of which* ***only one*** *is correct.*

21. Which of the following is correct pair of chiral isomers?

a. *Cis* $[CrCl_2(OX)_2]^{3-}$ and *trans* $[CrCl_2(OX)_2]^{3-}$ *b.* *Cis* $[CrCl_2(OX)_2]^{3-}$ and *cis* $[PtCl_2(en)_2]^{2+}$

c. *Trans* $[CrCl_2(OX)_2]^{3-}$ and *trans* $[PtCl_2(en)_2]^{2+}$ *d.* *Trans* $[CrCl_2(OX)_2]^{3-}$ and *cis* $[PtCl_2(en)_2]^{2+}$

22. $CH_3-\underset{}{\overset{CH_3}{\overset{|}{C}}}H-CH=CH_2 \xrightarrow[CCl_4]{Br_2} P$, Product P is

a. $CH_3-\overset{CH_3}{\overset{|}{\underset{Br}{\underset{|}{C}}}}-CH_2-CH_2-Br$

b. $CH_3-\overset{CH_3}{\overset{|}{CH}}-\overset{Br}{\overset{|}{\underset{Br}{\underset{|}{C}}}}-CH_3$

c. $CH_3-\overset{CH_3}{\overset{|}{\underset{Br}{\underset{|}{C}}}}-\underset{Br}{\underset{|}{CH}}-CH_3$

d. $CH_3-\overset{CH_3}{\overset{|}{CH}}-\underset{Br}{\underset{|}{CH}}-\underset{Br}{\underset{|}{CH_2}}$

23. The pressure exerted by 1 mol of CO_2 at 273 K is 34.98 atm. Assuming that volume occupied by CO_2 is negligible, the value of van der Waal's constant for attraction of CO_2 is

a. 3.59 dm^6 atm mol^{-2} *b.* 2.59 dm^6 atm mol^{-2}

c. 1.25 dm^6 atm mol^{-2} *d.* 1.59 dm^6 atm mol^{-2}

24. The pyrometallurgical process used for reduction of highly electropositive metal (such as Ag, Au) is known as cyanide process. Why this process is chosen instead of carbon reduction method?

P. Because this method is economically favourable

Q. Because carbon reduction method require high temperature

R. Because this process involves displacement reaction

S. Because this is done by using electrolysis

Choose the correct pair of statement(s) which explain the exact answer of the above question.

a. P and Q *b.* P and S *c.* P, Q and S *d.* R and Q

25. Acidic or basic impurity can be removed by using $[X]$. When a mixture of concentrated ore, $[X]$ and carbon is heated, it produces a fusible slag. This process is known as $[Y]$

Here $[X]$ and $[Y]$ can be filled by

a. slag and roasting *b.* flux and smelting *c.* flux and calcination *d.* gangue and roasting

26. Which of the following is used for distinguishing carbinol and methyl carbinol?

a. NaOH *b.* Na *c.* NaOI *d.* CH_3COCl

27. Helium crystallizes in FCC arrangement and the density of solid and liquid He is 1.59 g/cm^3 and 1.42 g/cm^3 respectively. Find the percentage of empty space in liquid He.

a. 33.92% *b.* 40.92% *c.* 38.92% *d.* None of these

28. Which of the following set of species have planer structures?

a. $I_3^+, ICl_4^-, Al_2Cl_6, TeCl_4, AlF_3$ *b.* $I_3^-, CH_3^-, ClO_3^-, SiF_2^{2-}$

c. $SCl_2, N_2O_5, XeOF_2, XeOF_4, SF_4$ *d.* $I_2Cl_6, XeF_2, BrF_4^-, XeF_5^-$

29. The test used for detection of protein is

a. Silver mirror test *b.* Molisch test *c.* Biureate test *d.* Iodoform test

30. If π back bonding involves the vacant orbital of the central atom, then bond angle gets centered due to

a. the increased bp-bp repulsions for the enhanced bond multiplicity

b. the decreased of lp-lp and lp-bp repulsion(s)

c. Both (a) and (b)

d. None of the above

Section 2 More Than One Correct Option

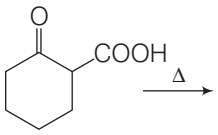
This section contains **5 *multiple choice questions***. *Each question has four choices, (a), (b), (c) and (d) out of which* ***one*** *or* ***more than one*** *is/are correct.*

31. The metal carbides of calcium and aluminium on hydrolysis produces

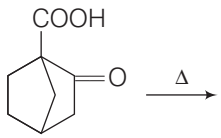
a. methane and ethane respectively *b.* ethyne and methane respectively

c. ethyne and ethyne respectively *d.* ethyne and propyne respectively

32. Which of them liberate a gas and turns lime water milky?

a. 

b. $CH_3-\underset{\underset{COOH}{|}}{CH}-COOH \xrightarrow{\Delta}$

c. 

d. $HOOC-(CH_2)_5-COOH \xrightarrow[\Delta]{Ca(OH)_2}$

33. Which of the following represents wrong order of properties, that are written in bracket?

a. $NH_3 < PH_3 < AsH_3 < SbH_3 < BiH_3$ (acidic strength)

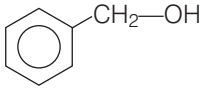
b. $NH_3 < PH_3 < AsH_3 < SbH_3 < BiH_3$ (reducing power)

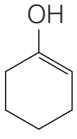
c. $NH_3 < PH_3 < AsH_3 < SbH_3 < BiH_3$ (basicity order)

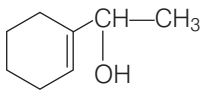
d. $NH_3 < PH_3 < AsH_3 < SbH_3 < BiH_3$ (bond angle)

34. Which of the following gives white turbidity with anhydrous $ZnCl_2$ + conc. HCl more faster than ?

a. 

b. 

c. 

d. 

35. Which of the following statement is/are correct regarding $[Fe(CN)_6]^{3-}$ complex?

a. It is an inner orbital complex
b. It is an outer orbital complex
c. It is paramagnetic
d. It is diamagentic

Section 3 Integer Answer Type

This section contains ***5 multiple choice questions****. The answer to each question is a single-digit integer, ranging from* 0 *to* 9 *(both inclusive).*

36. How many equivalent are there per mole of H_2S in its oxidation to SO_2?

37. A person takes 6.1 g of an antacid tablet comprising bicarbonate ions at 20.8%. The volume of CO_2 evolved at 1 atm and 25°C in the stomach on neutralisation, multiplied by a factor of 10 will be x L. Calculate approximate integer value of x.

38. Number of atoms bonded in cyclic fashion in the monomeric unit of nylon-6.

39. A solution of 1L has 0.6 g non-radioactive Fe^{3+} with mass number 56. To this solution, 0.209 g of radioactive Fe^{2+} is added with mass number 57, and the following exchange reaction occured.

$$^{57}Fe^{2+} + {}^{56}Fe^{3+} \rightleftharpoons {}^{57}Fe^{3+} + {}^{56}Fe^{2+}$$

At the end of 1h, it was found that 10^{-5} moles of non-radioactive $^{56}Fe^{2+}$ was obtained and that the rate of the reaction was 3.38×10^{-7} $mol^{-1}h^{-1}$. The rate constant of the reaction is neglecting any change in volume, calculate the activity of the sample at the end of 1 h $t_{1/2}$ (^{57}Fe) = 4.62 h and the unit digit of answer will be

40. Calculate the solubility in 10^{-4} mol/L of TlN_3 in a solution prepared by shaking excess of TlN_3 and $TlPO_4$. The solution contains/m mol of PO_4^{3-} per 200 mL solution. K_{sp} (TlN_3) = 5.6875×10^{-4}. Suppose the answer comes as x then the value $x/10^6$ will be approximately

Part III

Section 1 Single Correct Option Type

This section contains ***10 multiple choice questions****. Each question has four choices, (a), (b), (c) and (d) out of which* ***only one*** *is correct.*

41. The number of values of y in $[-2\pi, 2\pi]$ satisfying the equation $|\sin 2x| + |\cos 2x| = |\sin y|$ are

a. 2 *b.* 3 *c.* 4 *d.* 5

42. A square $ABCD$ is inscribed in a circle of radius 4 units. A point P moves inside the circle such that $d(P, AB) \leq \min[d(P, BC), d(P, CD), d(P, DA)]$ where $d(P, AB)$ denotes the distance of the point P from the line AB. The area of the region covered by the point P is equal to

a. 4π *b.* 8π *c.* $8\pi - 16$ *d.* None of these

43. The sides of triangle ABC are in AP (order being a,b,c) and satisfy $\frac{2!}{1!9!}+\frac{2!}{3!7!}+\frac{1}{5!5!}=\frac{8^a}{(2b)!}$, then the value of $\cos A+\cos B$ is

a. $\frac{12}{7}$ *b.* $\frac{13}{7}$ *c.* $\frac{11}{7}$ *d.* $\frac{10}{7}$

44. Let S_n be the sum of first n terms of an AP with non-zero common difference. If $\frac{S_{n_1n_2}}{S_{n_1}}$ is independent of n_1. The ratio of the first term and common difference is

a. $\frac{1}{3}$ *b.* $\frac{1}{2}$ *c.* $\frac{1}{4}$ d. $\frac{1}{5}$

45. If a vector $\mathbf{r}$ is equally inclined with the vectors $\mathbf{a}=\cos\theta\,\hat{\mathbf{i}}+\sin\theta\,\hat{\mathbf{j}}$, $\mathbf{b}=-\sin\theta\,\hat{\mathbf{i}}+\cos\theta\,\hat{\mathbf{j}}$ and $\mathbf{c}=\hat{\mathbf{k}}$, then the angle between $\mathbf{r}$ and $\mathbf{a}$ is

a. $\cos^{-1}\left(\frac{1}{\sqrt{2}}\right)$ *b.* $\cos^{-1}\left(\frac{1}{\sqrt{3}}\right)$ *c.* $\cos^{-1}\left(\frac{1}{3}\right)$ *d.* $\cos^{-1}\left(\frac{1}{2}\right)$

46. The value of $\frac{1}{81^n}-\frac{10}{81^n}{}^{2n}C_1+\frac{10^2}{81^n}{}^{2n}C_2-\frac{10^3}{81^n}{}^{2n}C_3+\ldots+\frac{10^{2n}}{81^n}$ is

a. 2 *b.* 0 *c.* $\frac{1}{2}$ *d.* 1

47. If $\alpha_1,\alpha_2,\alpha_3,\ldots,\alpha_{100}$ are the 100 roots of unity. The numerical value of $\sum_{1\le i\le j}(\alpha_i\alpha_j)^5$ is

a. 0 *b.* $(20)^{1/20}$ *c.* 20 *d.* None of these

48. The number of real solution of the equation $(x+4)^3+(x+3)^3+(x+2)^3+(x+1)^3+(x-5)^3+180=0$ is

a. 0 *b.* 1 *c.* 2 d. 3

49. If $\Delta=\begin{vmatrix} \frac{1}{z} & \frac{1}{z} & -\frac{x+y}{z^2} \\ -\frac{y+z}{x^2} & \frac{1}{x} & \frac{1}{x} \\ \frac{-y(y+z)}{x^2z} & \frac{x+2y+z}{xz} & \frac{-y(x+y)}{xz^2} \end{vmatrix}$, where x, y and $z\in R$, then

a. $\Delta=1$ *b.* $\Delta=2$ *c.* $\Delta=0$ *d.* $\Delta=3$

50. If $f(x)-f(4020-x)=0$ and $\int_{2009}^{2011}x\,f(x)dx=k\int_{2011}^{2009}f(x)dx$, then $10-k$ will be

a. 2010 *b.* 2020 *c.* 2030 *d.* 2040

Section 2 More Than One Correct Option

This section contains **5 multiple choice questions.** *Each question has four choices, (a), (b), (c) and (d) out of which* ***one*** *or* ***more than one*** *is/are correct.*

51. Let $f(x)=\begin{cases}\int_0^x(1+|1-t|)\,dt, & x>2\\ 5x+1, & x\le 2\end{cases}$ if $y=f(x)=|x|=\begin{cases}x \text{ if } x>0\\ -x \text{ if } x<0\end{cases}$, then

a. f(x) is not continuous at x = 2. *b.* f(x) is continuous but not differentiable at x = 2

c. f(x) is differentiable everywhere *d.* the right derivative of f(x) at x = 2 does not exist

52. The line $\frac{x-2}{3}=\frac{y+1}{2}=\frac{z-1}{-1}$ intersects the curve $xy=c^2, z=0$ if c equals to

a. $\sqrt{5}$ *b.* $-\sqrt{5}$ *c.* 1 *d.* -1

53. If two tangents can be drawn to the different branches of hyperbola $\frac{x^2}{1}-\frac{y^2}{4}=1$ from the point (α,α^2), then α can become equal to each and every point of which of the following intervals.

a. $(-2, 0)$ *b.* $(0, 2)$
c. $(-\infty, -2)$ *d.* $(2, \infty)$

54. Let **a, b** and **c** be three non-coplanar vectors and **d** be a non-zero vector, which is perpendicular to $(\mathbf{a}+\mathbf{b}+\mathbf{c})$. Now, $\mathbf{d}=(\mathbf{a}\times\mathbf{b})\sin x+(\mathbf{b}\times\mathbf{c})\cos y+2(\mathbf{c}\times\mathbf{a})$. Then,

a. $\frac{\mathrm{a}\cdot(\mathrm{a}+\mathrm{c})}{[\mathrm{a\,b\,c}]}=-2$ *b.* $\frac{\mathrm{d}\cdot(\mathrm{a}+\mathrm{c})}{[\mathrm{a\,b\,c}]}=2$

c. minimum value of x^2+y^2 is $\frac{\pi^2}{4}$ *d.* minimum value of x^2+y^2 is $\frac{5\pi^2}{4}$

55. $\sin 3x+\sin x$ can be equal to (where $x=\sin\theta, \theta\in\left[\frac{-\pi}{2},\frac{\pi}{2}\right], x\in[-1,1]$)

a. $-\frac{\mathrm{e}}{2}$ *b.* $-\frac{\pi}{2}$ *c.* $\frac{\pi}{2}$ *d.* $\frac{\mathrm{e}}{2}$

Section 3 Integer Answer Type

This section contains ***5 multiple choice questions****. The answer to each question is a single-digit integer, ranging from* 0 *to* 9 *(both inclusive).*

56. Evaluate $\int_{-1/3}^{1/3}\sqrt{\cos x}\,\ln\left[\frac{2-x}{2+x}\right]dx$

57. The minimum distance of the point (1, 1, 1) from the plane $x+y+z=1$ measured parallel to the line $\frac{x-2}{1}=\frac{y-3}{2}=\frac{z-4}{3}$ is k, where $\frac{3k}{\sqrt{14}}$ is

58. AB is a chord of the parabola $y^2=8x$ with the vertex at A and BC is drawn perpendicular to AB meeting the axis of parabola at C. The projection of BC on axis of parabola is equal to

59. If $\alpha,\beta\,(\alpha<\beta)$ are the two roots of the equation $\frac{1-8(\log_{10}x)^2}{\log_{10}x-2(\log_{10}x)^2}=1$, then find the value of $\frac{1}{10}\left[\frac{(\alpha^2\beta^3+1)^2}{\alpha^4}-30000\,\alpha^4\right]$.

60. The number of real solutions of the equation $4^{\log_2\log_e x}=\log_e x-(\log_e x)^2+1$ is

Analytical Explanations

1. (*a*) **Idea** This problem is based on hydrogen spectrum. When an electron jumps from first excited state to ground state. Then extra energy $E_m - E_n$ is emitted as a photon of electromagnetic radiation.

i.e., $$\frac{1}{\lambda} = RZ^2\left[\frac{1}{n^2} - \frac{1}{m^2}\right]$$

where, R is Rydberg constant and Z is atomic number.

$\Delta E =$ Energy exchanged between states $\times m_e$

$$\frac{1}{\lambda} = 2\left[RZ^2\left(\frac{1}{n_1^2} - \frac{1}{n_2^2}\right)\right]$$

$$= 2R(1)^2\left[\frac{1}{1} - \frac{1}{4}\right] = \frac{3R}{2}$$

$$\therefore \quad \lambda = \frac{2}{3R}$$

TEST Edge It is important according to JEE Advanced. Every year, one question is asked from this topic. Students should concentrate on hydrogen spectrum, binding energy, ionisation potential and wave function of an electron.

Remember Ionisation energy of the hydrogen is the amount of energy required to remove the electron completely. In ground state ($n = 1$), energy of atom is –13.6 eV and energy corresponding to $n = \infty$ is zero.

Hence, energy required to remove the electron from the ground state is 13.6 eV.

2. (*a*) **Idea** It is based on the concept of varition in the value of g due to rotation of the earth.

i.e., $g'_\lambda = g - R\omega^2\cos^2\lambda$

Where g is the value of acceleration due to gravity at a latitude.

At equator $g' = g - \omega^2 R$

At depth, d on the pole $g'' = g\left(1 - \frac{d}{R}\right)$

Both should be equal, then

$$g'' = g'$$

$$g\left(1 - \frac{\omega^2 R}{g}\right) = g\left(1 - \frac{d}{R}\right)$$

So, the value of $d = \frac{\omega^2 R^2}{g}$

TEST Edge This concept is important according to JEE Advanced. Every year, two questions are asked from this topic. Students should focus on variation of g with altitude and depth, gravitational potential energy and escape velocity of particle.

Case 1 At the equator, $\lambda = 0°$

then $g'_\lambda = g - R\omega^2$ [minimum value]

Case 2 At the pole, $\lambda = 90°$

then $g'_\lambda = g$ [maximum value]

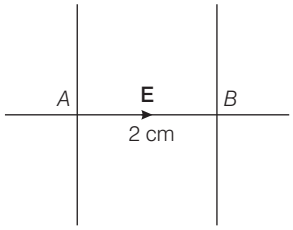
3. (*d*) **Idea** It is based on the relation $\mathbf{E} = -\frac{\Delta V}{\Delta r}$, Δr is the shortest distance between the two equipotential surfaces. Potential decreases steeply in the direction of electric field.

So, every one must understand the relation between ΔV and $\mathbf{E}$.

$$\mathbf{E} = \frac{-\Delta V}{\Delta r}$$

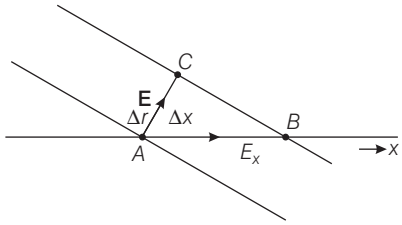
Two cases are possible

Case 1



For this case, $\mathbf{E} = \frac{-20\text{ V}}{\text{cm}}$ along $A \to B$

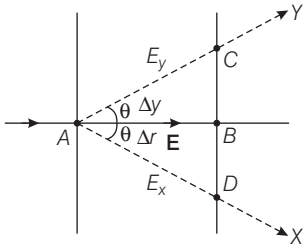
Case 2



In this case $\Delta r < \Delta x$, so $E_x < \mathbf{E}$.

$$\therefore \quad \mathbf{E} > \frac{20\text{ V}}{\text{cm}}.$$

TEST Edge This concept is generally asked in JEE Advanced/IIT. Student should concentrate on this idea and relate with electric field and potential energy. It is given that two points A and B having potential difference between them is + 40 V.



$$\mathbf{E} = \frac{-V_{AB}}{\Delta r} \text{ and } E_y = \frac{-V_{AC}}{\Delta y}$$

Here $V_{AB} = V_{AC}$ but $\Delta r < \Delta y$

So, $|\mathbf{E}| > |\mathbf{E}_y|$

In the same way, we can take the component of electric field along the x-axis

$$\mathbf{E}_x = \frac{-V_{AD}}{\Delta x}$$

$$|\mathbf{E}_x| < |\mathbf{E}|$$

4. (a) **Idea** This problem is based on a parallel plate capacitor with plate area A and separation d between the plates. Then, dielectric slab of dielectric constant K is inserted in the space between the plates.

i.e., $C = \frac{Q}{V} = \frac{K\varepsilon_o A}{d} = KC_o$

where $C = \frac{\varepsilon_o A}{d}$ is the capacitance without the dielectric.

Consider a small strip of width dx at a separation x from the left as figure (b).

This strip form a small capacitor of plate area $x\,dx$. Its capacitance is

$$dC = \frac{(K_o + \alpha x)\,\varepsilon_o a dx}{d}$$

The given capacitor may be divided into such strips with x varying from 0 to a.

All these strips are connected in parallel. The capacitance of the given capacitor is,

$$C = \int_o^a \frac{(K_o + ax)\,\varepsilon_o a dx}{d}$$

$$= \frac{\varepsilon_o a^2}{d}\left[K_o + \frac{a\alpha}{2}\right]$$

TEST Edge Students should concentrate on the energy stored in a capacitor, energy density in electric field and force between the plates of a capacitor.

Remember If instead of two plates, n uniform plates are placed at the same distance from each other and connected successively, then capacitance of this arrangement

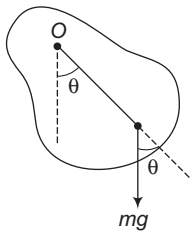
i.e., $C = \frac{(n-1)\varepsilon_o A}{d}$

5. (d) **Idea** This problem is asked about the concept of moment of inertia, torque and time period of SHM. When the rigid body is oscillating about point O but it's weight is concentrated on centre of gravity. Here, it is not a simple pendulum, it is a physical pendulum, so first we have to find it's time period and then by comparing it by time period formula, we can find equivalent length.

Consider rigid body oscillates about O. Torque due to gravity $\tau = -mgL\sin\theta$

For small θ

$$\tau = -(mgL)\theta$$



So, time period

$$T = 2\pi\sqrt{\frac{I_O}{mgL}}$$

where, I_O is moment of inertia about O

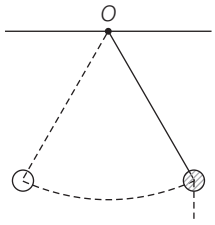
$$I_O = I_{COM} + mL^2$$

$$= mK^2 + mL^2$$

$$T = 2\pi\sqrt{\frac{\frac{K^2}{L} + L}{g}}$$

So, $L_{eq} = \frac{K^2}{L} + L$

TEST Edge The oscillation of pendulum is an important topic in SHM. So one should practice these types of questions as well. Let us consider an example



A hollow bob initially filled with water is oscillating with a small hole at it's bottom. Find the variation in it's time period, if any?

The length of simple pendulum is considered to solve from point O to the centre of gravity of bob. Now try to solve this question.

6. (a) **Idea** This problem is based on Young's Double Slit Experiment in order to find out minima and maxima of the interference pattern.

i.e., $\lambda = \frac{d}{4}$

As λ is large in comparison with d so only 1st assumption of YDSE is valid, so Δx of any point

$$\Delta x = d\sin\theta = 4\lambda\sin\theta$$

For $\theta = 60°$, $\Delta x = 4\lambda \times \frac{\sqrt{3}}{2} = 2\sqrt{3}\,\lambda$

Neither maxima nor minima.

For $\theta = 30°$, $\Delta x = 4\lambda \times \frac{1}{2} = 2\lambda$ maxima at P.

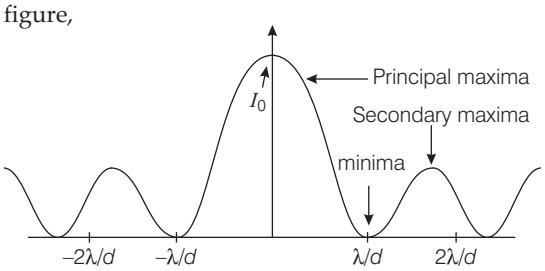
TEST Edge Above concept is important specially for JEE Advanced. Every year, one or two questions are asked. So, students should focus on different cases of superposition of waves and YDSE.

Remember When a monochromatic light of wavelength λ is incident on a circular aperture of diameter d, then angular width of dark fringe.

$$d\sin\theta = 1.22\,\lambda \quad \text{or} \quad \sin\theta = \frac{1.22\,\lambda}{d}$$

Angular radius of central maximum

i.e., $\sin\theta = \frac{1.22\lambda}{d}$ diffraction pattern as shown in figure,



Where I_o is intensity of monochromatic light.

7. (*c*) **Idea** It is based on the concept of first law of thermodynamics and thermodynamics processes.

i.e., $\Delta Q = \Delta U + \Delta W$

where, ΔQ is change in heat energy, ΔU is change in internal energy and ΔW is the amount of work done.

As initial and final condition is same that's why ΔU will be same for all three.

For *abe*, $\Delta Q = \Delta U + \Delta W$

Overall work done is positive as we can see that in upper part of graph, net volume increases.

So, ΔQ will be sum of $\Delta U\,(>0)$ and $\Delta W\,(>0)$.

In *ae* part, $W = 0$ so $\Delta Q = \Delta U$

In *ade* part, $W < 0$, $\Delta Q = \Delta U - \Delta W$

So, heat absorbed by gas is greater in process (i) than process (iii).

TEST Edge It is important according to JEE Advanced. Every year two or three questions are asked in this concept. Student should relate this idea with second law of thermodynamics, heat engines and Carnot cycle.

Remember

1. When work is done by the system, ΔW is positive. If work is done on the system, ΔW is negative.
2. When heat is given to the system, then ΔQ is positive. If heat is given by the system, ΔQ is negative.
3. A positive ΔW, decreases the internal energy and positive ΔQ increase it.

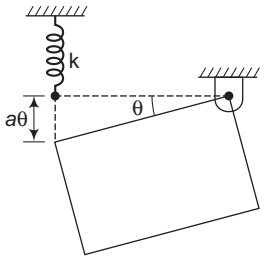
8. (*b*) **Idea** This problem is based on time period of simple pendulum and parallel axes theorem on moment of inertia.

i.e., $T = 2\pi\sqrt{\frac{I}{mgL}}$

where, I is moment of inertia, and T is time period of simple pendulum.

For small value of θ, spring force will be $= k\,(a\theta)$

Restoring torque about hinge $\tau = (ka\,\theta)\,a = (ka^2)\,\theta$



So, tension on spring $T = 2\pi\sqrt{\frac{I_H}{ka^2}}$

I_H = Moment of inertia about hinge

$$I_H = I_{COM} + m\left(\frac{\sqrt{a^2+b^2}}{2}\right)^2$$

$$= \frac{m(a^2+b^2)}{12} + \frac{m(a^2+b^2)}{4} = \frac{m(a^2+b^2)}{3}$$

$$T = 2\pi\sqrt{\frac{m(a^2+b^2)}{3ka^2}}$$

TEST Edge This idea is generally asked in JEE Advanced. It relates with conservation of energy in SHM, damped harmonic motion and resonance.

Remember If a simple pendulum is in carriage which is accelerating with an acceleration a, then

i.e., $g' = g - a$

Case 1 If the acceleration a is upwards then

$$|g'| = g + a \text{ and } T = 2\pi\sqrt{\frac{l}{g+a}}$$

Case 2 If the acceleration a is downward, then $(g > a)$

i.e., $|g'| = g - a$ and $T = 2\pi\sqrt{\frac{l}{g-a}}$

Case 3 In a freely falling left, $g' = 0$ and $T = \infty$,

i.e., the pendulum will not oscillate.

9. (*d*) **Idea** This problem is based on SHM of a particle in a straight line. When a particle displaced from its original position, then to calculate value of its amplitude.

By general equation of SHM with ω frequency

$$x = A\cos(\omega t)$$

At $t = 1$ s $\quad x = A - d_1$

At $t = 2$ s $\quad x = A - d_1 - d_2$

From equation,

$$A - d_1 = A\cos(\omega)$$

$$A - d_1 - d_2 = A\cos(2\omega)$$

$$= A[2\cos^2\omega - 1]$$

$$A - d_1 - d_2 = A\left[2\left(\frac{A - d_1}{A}\right)^2 - 1\right]$$

$$A = \frac{2d_1^2}{3d_1 - d_2}$$

TEST Edge Some important points related to SHM/oscillation.

1. The time period of SHM does not depend on amplitude.
2. As UCM and SHM are connected that is why ω (angular frequency) comes in the expression of SHM as $x = A\cos\omega t$.
3. There is no basic difference between oscillation and vibration. In oscillation, frequency is low while in vibration frequency is high.

10. (*b*) **Idea** This problem is based on projectile motion of a particle *i.e.,* equation of trajectory of projectile and to calculate the time of flight of a projectile.

Suppose the coordinate of the point where projectile strikes the inclined plane is (x, y).

Now, $\quad y = (u\sin\theta)t - \frac{1}{2}gt^2 \quad \ldots(i)$

and $\quad x = (\cos\theta)t \quad \ldots(ii)$

From the geometry of the figure,

$$d = x + y\cot\alpha \quad \ldots(iii)$$

From Eqs. (i), (ii) and (iii)

$$d = (u\cos\theta)t + \left((u\cos\theta)t - \frac{1}{2}gt^2\right)\cot\alpha \quad \ldots(iv)$$

For t_{min}, $\frac{dt}{d\theta} = 0$ which, gives $\tan\theta = \cot\alpha$

$$\Rightarrow \quad \theta = \frac{\pi}{2} - \alpha$$

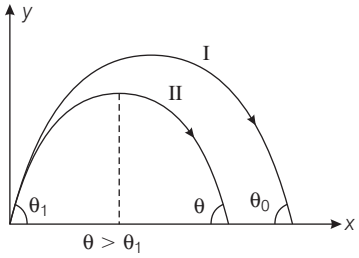
Now, substituting this value of θ in Eq. (iv)

$$t_{min} = \left(\frac{u - \sqrt{u^2 - gd\sin 2\alpha}}{g\cos\alpha}\right)$$

TEST Edge Above idea is important according to JEE Advanced. Every year, one question is asked from this concept. Students should focus on time of flight, range, maximum height and applications of projectile motion on an inclined plane.

Remember If air resist the projectile motion, then

1. Time taken by projectile during upward motion < time taken during downward motion.
2. The values of height attained and range of a projectile decreases.



3. The projectile returns to the ground with less speed. At its trajectory, horizontal velocity also decreases.

11. (*a,d*) **Idea** The concept is based on the difference between the frequency of the standing waves produced by one closed and one open organ pipe will make a beat. Apply the formula, for the frequency of both closed and open organ pipes and solve it.

First overtone of open organ pipe is

$$f_1 = \frac{N}{2L_o}v_s = \frac{2}{2L_o} \times v_s = \frac{v_s}{L_o}$$

First overtone of close organ pipe

$$f_2 = \frac{3v_s}{4L_c}$$

$\because \quad f_1 - f_2 = \pm 10$

Fundamental frequency of close organ pipe is 110 Hz, so

$$110 = \frac{v_o}{4L_c} = \frac{330}{4L_c}$$

$$L_c = \frac{3}{4}\text{m} = 0.75\text{ m}$$

$\because$ So, its first overtone frequency must be 330 Hz and frequency of open organ pipe should be 340 or 320 Hz.

$$\frac{v_o}{L_o} = 340 = \frac{330}{L_o}$$

$$\Rightarrow \quad L_o = \frac{34}{33}\text{m}$$

$$\frac{v_o}{L_o} = 320 = \frac{330}{L_o}$$

$$\Rightarrow \quad L_o = \frac{33}{32}\text{m}$$

TEST Edge In wave, the topics of standing waves, beats and Doppler's effect are important and questions may come from these topics in JEE Advanced from this topic.

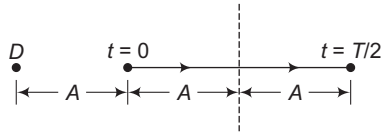
Remember The beat means intensity of the resultant wave will vary with time when two waves of nearly same frequency will be superimposed. The frequency of the variation of beat amplitude is half of the frequency of the variation of beat intensity.

12. (*a, c, d*) **Idea** The given question is based on Doppler's effect. So, the observer is stationary but the source is oscillating.

So, in the motion of oscillating source, the frequency observed by the observer will also vary. Just observing the motion of source and you can easily solve it.

Maxima will be detected when source will be at mean position with maximum speed and after every time interval it will happen. Minima is detected, when source is at maximum speed and moving away from source, this will also happen after every time interval.

Detector will having frequency less than actual, when source is moving away as shown in figure.



Sound generated by source between $\left(0, \frac{T}{2}\right)$ will be corresponding to less frequency.

Sound generated at $t = 0$, will reach D at $t_1 = \frac{A}{v}$

and sound generated at $t = \frac{T}{2}$ will reach D at $t_2 = \frac{T}{2} + \frac{3A}{v}$

So, time interval in which it hear less frequency is

$$t_2 - t_1 = \frac{T}{2} - \frac{2A}{v} > \frac{T}{2}$$

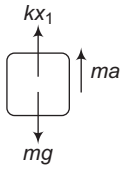
Similarly, greater frequency will be heared for time interval $\left(\frac{T}{2} - \frac{2A}{v}\right)$.

TEST Edge Doppler's effect is an important topic from sounds and waves. In doppler's effect there are same important points that one should notice.

1. If the source is at rest then the wavelength of the wave will not change but if the source is moving then its wavelength and frequency (observed) both will change.
2. The motion of the medium will not affect variation of frequency (observed), it is just the relative motion between source and observer due to which Doppler's effect occur.

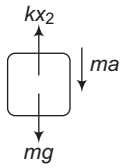
13. (*a, c*) **Idea** The problem is based on the concept of pseudo forces. When the body is on an accelerating platform, then weight of the body appears to be changed.

For **upward** moving elevator, $kx_1 - mg = ma$ ...(i)



For **downward** moving elevator,

$$mg - kx_2 = ma \quad \text{...(ii)}$$



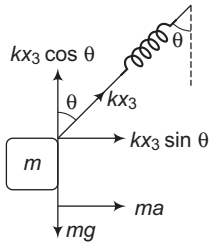
Dividing Eq. (i) by Eq. (ii), we get

$$\frac{g + a}{g - a} = \frac{x_1}{x_2}$$

$$= \frac{4\sqrt{2}}{3\sqrt{2}}$$

$$\therefore \quad a = \frac{g}{7}$$

For horizontal motion of elevator, spring will be inclined at some angle as given in figure.



$$kx_3 \cos\theta = mg \quad \text{...(iii)}$$

$$kx_3 \sin\theta = ma \quad \text{...(iv)}$$

Squaring and adding Eqs. (iii) and (iv), we get

$$(kx_3)^2 = m^2 (g^2 + a^2)$$

$$x_3 = \frac{m}{k}\sqrt{g^2 + a^2} \quad \text{...(v)}$$

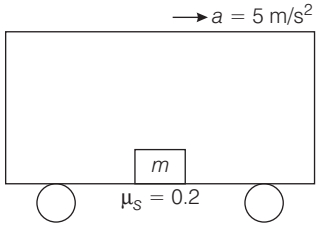
From Eqs. (i) and (v), we get

$$\frac{x_3}{x_1} = \frac{\sqrt{g^2 + a^2}}{g + a}$$

$$x_3 = \left(\frac{\sqrt{g^2 + a^2}}{g + a}\right), \text{ hence, } x_3 = 5\,\text{mm}$$

TEST Edge Pseudo force is an important concept in laws of motion and questions may come on pseudo force in JEE Advanced.

Let us consider one example based on pseudo force.

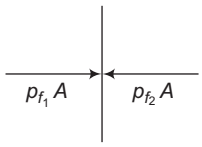


A box is placed on the floor of the train. The train starts accelerating with an acceleration 5 m/s ^{2}. The relative acceleration of block with respect to train can be find out.

Here, we can observed from the train's frame, then apply pseudo force and real friction force to solve that problem.

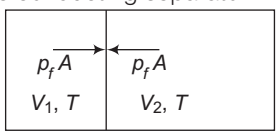
14. (*a,b,c,d*) **Idea** The number of moles in each container will not change. The final pressures in both containers will be same. Final temperature will also be same. Now this question could easily be solved.

At final equilibrium force must be equal from two side, so



$$p_{f_1} A = p_{f_2} A \Rightarrow p_{f_1} = p_{f_2}$$

In final condition final temperature will also be same because conducting separator.



Conservation of moles in I and II components

$$\frac{pV}{RT} = \frac{p_f (V_1)}{RT} \quad \ldots(i)$$

$$\frac{2p(2V)}{RT} = \frac{p_f V_2}{RT} \quad \ldots(ii)$$

$$V_1 = \frac{pV}{p_f}$$

$$V_2 = \frac{4pV}{p_f} \quad \ldots(iii)$$

$$V_1 + V_2 = 3V$$

$$\frac{pV}{p_f} + \frac{4pV}{p_f} = 3V$$

$$\frac{pV}{p_f} \times 5 = 3V \Rightarrow p_f = \frac{5p}{3}$$

So, $$V_1 = \frac{3V}{5}, \quad V_2 = \frac{12V}{5}$$

TEST Edge Above question is based on kinetic theory of gases. In these types of questions, one must understand if the partition is conducting then temperature of both parts will be same, if the partition is not fixed, then their final pressures will be same. But concentrate on the number of moles if they are same or not for both the compartments.

15. (*b, d*) **Idea** The energy of photon must be greater than the work function of the material. If the wavelength is doubled then energy of photon will be halved but the KE_{max} will not halved because work function will remain constant.

$$E_{photon} = \frac{12400}{\lambda \text{ (in Å)}} = \frac{12400}{4000} = 3.1 \text{ eV}$$

As W_0 for Zn, Fe and Ni > 3.1 eV, there will be no photoelectric emission from any surface.

To emit photoelectrons from all the three metals, λ_{max} should corresponds to λ_{max} for Ni (as it has highest W_0)

$\Rightarrow \lambda_{max}$ (to start ejection from Ni)

$$= \frac{12400}{W_0 \text{ (eV)}} \text{ Å} = \frac{12400}{5.9} \text{ Å} = 2101.7 \text{ Å}$$

If wavelength of the radiation is less than 2000 Å, then photoelectrons from all the metal surface will be emitted.

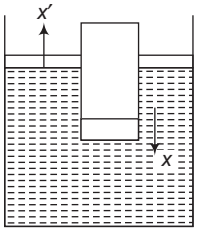
TEST Edge Photoelectric effect is an important part of modern physics. To solve the questions based on photoelectric effect, it is necessary to understand the graphs of intensity-current, current-stopping potential and frequency-stopping potential. Then one should understand the photoelectric equation.

Remember In photoelectric effect, the kinetic energy of all the electrons are not same then KE_{max} is obtained by surface electrons only.

16. (*6*) **Idea** As the cylinder goes inside the liquid, the liquid also comes up, so we have to see the net displacement of cylinder with respect to the liquid. Net displacement of the cylinder will be equal to the length of cylinder outside the liquid for maximum amplitude.

Cylinder can perform SHM only till it is partially submersed. When cylinder goes down by x inside the liquid level comes up by x' (say)

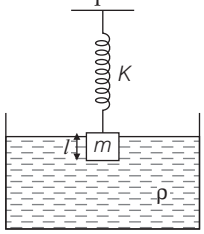
$$(4a - a)x' = xa \Rightarrow x' = \frac{x}{3}$$



So, the centre of the cylinder goes down by (w.r.t. the liquid surface).

$$(x + x') = \frac{4}{3}x \le \frac{l}{10} \quad \Rightarrow \quad x \le \frac{3l}{40} = 6 \text{ cm}$$

TEST Edge From mechanical properties of fluids some good questions could be made. Let us consider one such example



A block is attached with a spring (vertical) and submerged in a liquid. If it oscillates inside the liquid. One can ask to find the time period of the oscillation.

Here, spring force and Buoyant force both are variable with l, so both will help in SHM.

17. (3) **Idea** It is based on conservation of mechanical energy for the two springs and the non conservative force *i.e.,* friction will dissipate same part of mechanical energy.

Velocity of block at A is according to energy conservation

$$\frac{mv^2}{2} = \frac{kx^2}{2}$$

$$\frac{1 \times v_A^2}{2} = \frac{1350 \times (0.1)^2}{2}$$

$$v_A^2 = \frac{1350}{100}$$

$$v_A = \sqrt{\frac{1350}{100}} \text{ m/s}$$

During motion of a block from A to B, we get

$$v_B^2 = v_A^2 + 2as$$

[Here, $a = -\mu g = -0.3 \times 10 = -3 \text{ m/s}^2$]

$$= \frac{1350}{100} - 2 \times 3 \times 2$$

$$= \frac{1350}{100} - 12$$

$$= \frac{1350 - 1200}{100} = \frac{150}{100} \text{ m/s}^2$$

Now with this KE, it will compress right spring.

$$\frac{mv_B^2}{2} = \frac{kx_f^2}{2}$$

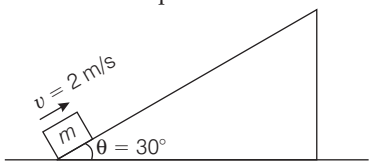
$$\frac{1}{2} \times \frac{150}{100} = \frac{1350}{2} \times x_f^2$$

$$x_f^2 = \frac{150}{1350} \times \frac{1}{100} = \frac{1}{900}$$

$$x_f = \frac{1}{30} = \frac{100}{30} = 3.33 \text{ cm} \simeq 3 \text{ cm}$$

TEST Edge Questions on this concept involving conservative and non-conservative forces were asked in IIT-JEE/JEE Advanced. Students should focus on this concept and relate with laws of motion and its applications.

Let us consider an example



A box starts moving up a fixed inclined plane with a speed 2 m/s. So, it has moved a distance 0.2 m along the inclined before coming to rest. The work done by gravity can be find out.

Note, observe one thing that the mechanical energy is not conserved.

18. (3) **Idea** This problem is based on the relation between refractive index (μ) and the angle of minimum deviation (δ_m).

i.e., $\delta_m = (\mu - 1)\, A$

$$\phi = \frac{d\delta}{d\lambda}$$

This can be written as,

$$\phi = \frac{d\delta}{d\mu} \times \frac{d\mu}{d\lambda}$$

$$\delta = (\mu - 1)\, A_0$$

$$\frac{d\delta}{d\mu} = A_0$$

$$\mu = A + \frac{B}{\lambda^2} \quad \Rightarrow \quad \frac{d\mu}{d\lambda} = -\frac{2B}{\lambda^3}$$

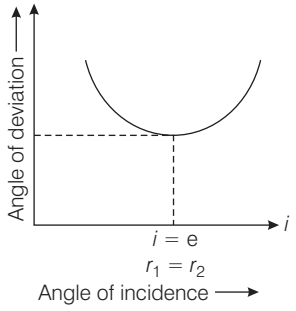
So, $$\phi = -\frac{2BA_0}{\lambda^3} \quad \Rightarrow \quad \phi \propto \frac{1}{\lambda^3}$$

So, $$N = 3$$

TEST Edge It is important from the JEE Advanced point of view. Students should concentrate on prism, refraction at spherical surface, lens maker's formula and its magnification.

Remember It is found that the angle of deviation (δ) varies with the angle of incidence

i.e., $\delta_D,\ r_1 = r_2 = r$



$\therefore$ $$r = \frac{A}{2}$$

So, $\delta = \delta_m = (i + i) - A$

$$i = \frac{A + \delta_m}{2}$$

$$\Rightarrow \quad \mu = \frac{\sin i}{\sin r} = \frac{\sin\left(A + \frac{\delta_m}{2}\right)}{\sin\left(\frac{A}{2}\right)}$$

where, μ is a refractive index of a prism

Note For this prism, $\delta_m = (\mu - 1)\, A$

19. (7) **Idea** It is based on the concept of Newton's law of cooling in which temperature of body varies from T_1 to T_2.

i.e., $$\frac{T_1 - T_2}{t} = K\left[\frac{T_1 + T_2}{2} - T_o\right]$$

where t is the time in which temperature of body changes from T_1 to T_2

From Newton's law of cooling, we get

$$\frac{T_1 - T_2}{t} = K\left[\frac{T_1 + T_2}{2} - T_0\right]$$

$$\frac{50 - 40}{5 \times 60} = K\left[\frac{50 + 40}{2} - 20\right] \quad \ldots(i)$$

$$\frac{40 - T}{5 \times 60} = K\left[\frac{40 + T}{2} - 20\right] \quad \ldots(ii)$$

Dividing Eq. (i) by Eq. (ii), we get

$$\frac{10}{40 - T} = \frac{25}{T/2}$$

$$5T = 25 \times 40 - 25T$$

$$T = \frac{25 \times 40}{30} = 33.33°$$

So, $\Delta T \simeq 40° - 33.33 = 6.67° \approx 7°$

TEST Edge This idea is important according to JEE Advanced. Every year, one or two questions are asked from this topic. Students should focus on this idea and relate to perfectly black body, Stefan's law and ideal gas equation.

Remember A surface or a medium which transmits most of the radiation is called diathermanous, substance.

e.g., dry air, rock salt etc.

A surface or a medium which does not transmit radiation at all is called opaque or a diathermanous medium.

e.g., water, wood and solid.

20. (6) **Idea** This problem is based on the energy emission of hydrogen atom and to calculate the excited state of an electron.

From the given conditions, we get

$$E_n - E_2 = (10.2 + 17)\,\text{eV} = 27.2\,\text{eV} \quad \ldots(i)$$

$$E_n - E_3 = (4.25 + 5.95)\,\text{eV} = 10.2\,\text{eV} \quad \ldots(ii)$$

So, Eq. (i) – Eq. (ii), we get,

$$E_3 - E_2 = 17.0\,\text{eV}$$

$$Z^2(13.6)\left[\frac{1}{4} - \frac{1}{9}\right] = 17.0$$

$$Z^2(13.6)\left(\frac{5}{36}\right) = 17.0$$

$$Z^2 = 9 \text{ or } Z = 3$$

From Eq. (i) we get

$$Z^2(13.6)\left[\frac{1}{4} - \frac{1}{n^2}\right] = 27.2$$

$$3^2(13.6)\left[\frac{1}{4} - \frac{1}{n^2}\right] = 27.2$$

$$\frac{1}{4} - \frac{1}{n^2} = 0.222$$

$$\frac{1}{n^2} = 0.0278$$

$$n^2 = 36$$

$$n = 6$$

TEST Edge It is important according to JEE Advanced. One question is asked in every year. So, students should concentrate on Bohr's model, energy of hydrogen atom, hydrogen spectra and binding energy.

Remember Total energy E_n, is the sum of the kinetic and potential energies.

i.e., $$E_n = K_n + U_n = \frac{-me^4}{8\varepsilon_0{}^2 n^2 h^2}$$

or $$E_n = \frac{-13.6\,\text{eV}}{n^2}$$

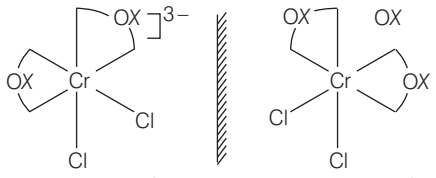
where, n is the shell number.

21. (b) **Idea** This problem includes conceptual mixing of structure and isomerism in coordination compound. While solving the problem, students are advised to draw the structures and mirror images of given isomers of coordination compound. Then analyse them either by the given pairs are superimposable or not, if they are non-superimposable mirror image then both are optically active as well as chiral.

Keep in mind that almost all *cis* isomers are optically active.

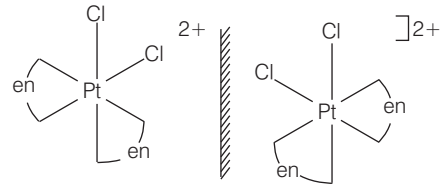
This problem involves the concept of structure and isomerism in coordination compound.

Structure of coordination compound



Cis $[CrCl_2(OX)_2]^{3-}$ *Cis* $[CrCl_2(OX)_2]^{3-}$

Both are non-superimposable mirror image isomers. Hence, it will show optical activity and chirality.



Cis $[PtCl_2(en)_2]^{2+}$ *Cis* $[Pt\,Cl_2(en)_2]^{2+}$

Both isomers are non-superimposable mirror image isomer, hence are optically active and will show chirality.

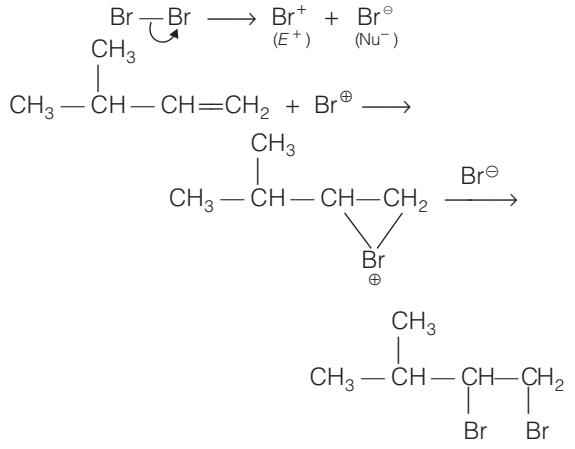
Overall, we can say all isomers of coordination compound will show optical activity due to non-superimposable mirror relation with each other.

TEST Edge Similar problems based on the concept of analytical study of coordination compound and isomerism in coordination compound can also be asked in JEE Advanced, such as

What is the type of isomerism exhibited by the product when cis-diaminodichloronickel(I) bromide adds up with aqueous solution of silver bromide?

After following proper stepwise approach to solve this question *i.e.,* writing structural formula of complex ion on reaction of coordination compound with silver bromide, you will get optical isomerism as answer.

22. (*d*) **Idea** This problem is based on the concept of addition reaction to alkene, while solving this problem students are advised to go through proper mechanism *i.e.,* electrophilic addition reaction to alkene *via* formation of cyclic bromonium ion.



TEST Edge In JEE Advanced this type of problems are included to judge the knowledge of students, regarding mechanism of organic reaction.

Similar problems based on nucleophilic addition reaction, Markovnikov addition reactions can also be asked.

e.g., What will be the product when 2-methyl prop-1-ene is treated with deuteriumbromide in presence of hydrogenperoxide?

This problem can be solved by using the concept of free radical addition reaction through anti-Markovnikov's addition and one can get answer as tertiary butyl bromide.

23. (*a*) **Idea** This problem is based on conceptual mixing of van der Waal's equation and calculation of van der Waal's constant. The problem can be solved by undergoing following sequential approach.

- Write van der Waal's equation and put the value of b as zero then calculate the value of V using concept of quadratic equation.
- Calculate the value of a using consideration that V is constant at given T and p followed by putting the value R, T and p.

$$\left(p + \frac{a}{V^2}\right)(V - b) = RT$$

$$\therefore \quad \left(p + \frac{a}{V^2}\right)V = RT$$

$$\left(\frac{pV^2 + a}{V^2}\right)\cdot V = RT$$

$$V^2p - RTV + a = 0$$

$$V = \frac{RT \pm \sqrt{R^2T^2 - 4pa}}{2p}$$

Since V is constant at given p and T, V can have only one value or discriminant = 0

$$\therefore \quad R^2T^2 = 4pa \quad \text{or} \quad a = \frac{R^2T^2}{4p}$$

$$a = \frac{(0.821)^2 \times (273)^2}{4 \times 34.98} = 3.59\,\text{dm}^6\,\text{atm mol}^{-2}$$

TEST Edge Similar problem based on concept of van der Waal's equation, gas laws and unit transformation can also be asked, students are also recommended to go through in depth study of these topics. While solving these type of problems, students are advised to be careful regarding unit conversions.

24. (*a*) **Idea** This problem includes reduction of metal ore with its economic value. While solving the problem students are advised to go through the concept used in cyanide process and characteristics of cyanide process.

The reduction of metal by carbon requires high temperature which is economically not viable but the reduction using cyanide process requires low temperature and hence economically favourable.

Hence, statements *P* and *Q* correctly explain the exact reason.

TEST Edge Similar process including the concept of cyanide process, along with its drawbacks, advantages using chemical transformations can also be asked in JEE Advanced.

25. (*b*) **Idea** This problem includes purification of metal using carbon reduction method and concept of smelting. While solving problems, students are suggested to keep in mind the concept of carbon reduction method and smelting.

Impurities present in any ore may be acidic or basic. Impurity combines with flux to produce fusible slag, flux is chosen depending upon behaviour of impurity. If impurity is basic in nature then acidic flux is chosen and if impurity is acidic then basic flux is chosen.

Acidic flux SiO_2, P_2O_5, B_2O_3 etc.

Basic flux CaO, MgO etc.

$$\text{Flux} + \text{Impurity} \longrightarrow \text{Fusible slag}$$

This process of removing impurity by means of conversion of impurity to fusible slag on treatment with flux is known as smelting.

TEST Edge Similar problems based on concept of leaching, roasting, Baeyer's process, Serpeck's process, Hall and Heroults process can also be asked in JEE Advanced. In general the purpose to ask these type of question is to judge the knowledge regarding various viable metallurgical process and economical value of separate processes. Let us see one such problem

Leaching of Ag_2S *is carried out by heating* Ag_2S *with a dilute solution of*

(a) NaOH
(b) HCl
(c) NaOH in presence of O_2
(d) NaCN only

NaOH in presence of O_2

26. (*c*) **Idea** This problem is based on concept of naming of alcohol and haloform test. While solving the problem students are suggested to keep in mind the key points (characteristics) of haloform reaction. Student must be careful during answering such question that substrate undergoing haloform test must have secondary alcoholic group.

(NaOI) with

CH_3OH	$CH_3 - CH_2 - OH$
Carbinol	methyl carbinol
↓	↓
it does not give haloform reaction.	It gives haloform reaction with NaOI.

TEST Edge In JEE Advanced, these type of questions are asked to judge the knowledge of students regarding the naming of organic compound and test to distinguish them.

27. (*a*) **Idea** This problem is based on concept of FCC crystal lattice structure, packing fraction and empty space determination. While solving the problem, students are advised to calculate the packing fraction of liquid He. By equating ratio of packing fraction of liquid He to density of liquid He and packing fraction of solid He to density of solid He first. Then calculate empty space by using formula

empty space = 100–packing fraction of liquid He

$$\frac{\text{Packing fraction of Liquid He}}{\text{Density of Liquid He}} = \frac{\text{Packing fraction of solid He}}{\text{density of solid He}}$$

$$\frac{x}{1.42} = \frac{0.74}{1.59} \Rightarrow x = 66.8\%$$

Empty space = (100 – 66.08) = 33.92%

TEST Edge In JEE Advanced, these type of problem are asked to judge the basic subject knowledge as well as analytical thinking of students. So while solving such problems in JEE Advanced, students are advised to keep their analytical thinking as well as basic need to solve the problem in front of him. Similar problem based on packing efficiency, size of void, can also be asked in JEE Advanced *e.g.,*

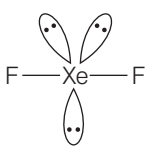
Which of the following expression represent the packing fraction of NaCl *correctly if ions along an edge diagonal are absent*?

By using the ratio of volume of effective number of cations and anions to volume of unit cell. We can get answer as

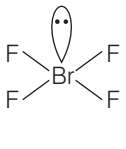
$$\text{P.F} = \frac{\frac{4}{3}\pi\left(\frac{5}{2}r_+^3 + 4r_-^3\right)}{16\sqrt{2}r_-^3}$$

28. (*b*) **Idea** This problem is based on concept of hybridisation and structure of inorganic species. While solving this problem, students are advised to keep in mind the concept of VSEPR theory and bent rule.

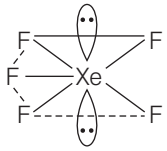
Only set (d) has all planar structures



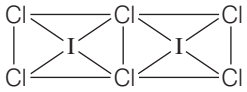
Linear



square pyramidal



Pentagonal pyramidal



sp^3d^2 planar

Look at the atoms only, all the atoms in above structures are in same plane, Hence set (d) denote planar species.

TEST Edge Similar problems based on concept of isoelectronic, isostructural and isolobal species can also be asked in JEE Advanced, such as

Which is the correct pair of isoelectronic and isostructural species.

(a) BF_3 and $\overset{\ominus}{C}F_3$ (b) NO_3^-, CO_3^{2-}

(c) NH_3 and PH_3 (d) BF_4^-, NH_4^+

After using concept of isoelectronic and isostructural species one can get answer as (b).

29. (*c*) **Idea** This problem is based on the principle involved in detection of protein and can be solved by using test for the detection of peptide bond.

Biureate test is used for test of protein.

All compounds containing $\left(\begin{array}{c} O \\ \| \\ -C-NH \end{array}\right)$ peptide bond

gives this test.

$$\text{Protein} \xrightarrow[\Delta]{CuSO_4, NaOH} \text{Violet colour}$$

TEST Edge Similar problems based on distinguishing test for primary, secondary and tertiary amine can also be asked. So, students are advised to go through study of these topics and remember the key point used to distinguish these three.

30. (*c*) **Idea** This problem is based on conceptual mixing of back bonding, VSEPR theory and bent rule. While solving this problem students are advised to look at the problem with thinking that how many reasons are possible for bond angle to be centered.

Since during back bonding transfer of electron takes place from filled orbital to vacant orbital which create a double bond character between linking atoms.

The linking between atoms create double bond character mainly due to two reasons

(a) increased bp-bp repulsion and

(b) decreases in lp-lp repulsion and lp-bp repulsion

TEST Edge Similar problems based on concept of π back bonding to explain reactivity, adduct forming ability structural deformation etc can also be asked. So students are advised to go through in depth study of these topics in comparative manner.

31. (*b*) **Idea** This problem is based on hydrolysis of different metal carbides.

Aluminium carbide *i.e.*, Al_4C_3 on hydrolysis produces methane.

$$Al_4C_3 + H_2O \longrightarrow 4Al(OH)_3 + 3CH_4$$

Calcium carbide *i.e.*, CaC_2 on hydrolysis produces ethyne.

$$CaC_2 + H_2O \longrightarrow Ca(OH)_2 + CH{\equiv}CH$$

TEST Edge Similar problems based on nature of by product on hydrolysis of carbides and their chemical reaction can also be asked, so students are advised to go through the study and analysis of chemical reaction involved in hydrolysis of metal carbides and metal halides, such as

A tetrachloride of molecular mass 170 *when undergoes hydrolysis produces*

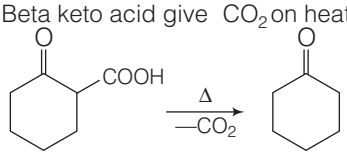
(a) silicon (b) silicone

(c) silicate (d) silicic acid

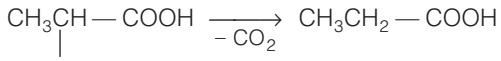
The answer will be silicic acid.

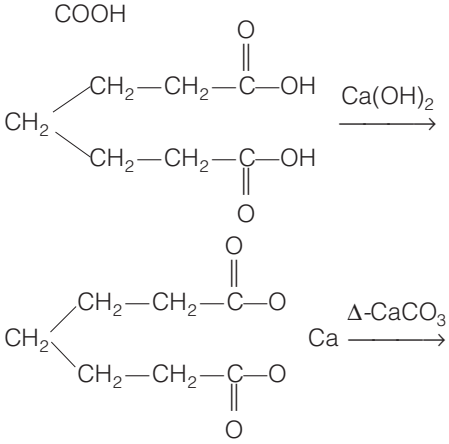
32. (*a,b,d*) **Idea** This problem is based on concept of decarboxylation of carboxylic acid. While solving this problem students must have two key points that which gas turns lime water milky and which molecule will produce this gas on heating.

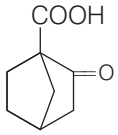
Beta keto acid give CO_2 on heating.



α, α-dicarboxylic acid gives CO_2 on heating.



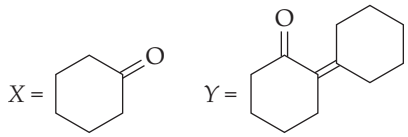


According to Bridth rule  does not evolve CO_2.

TEST Edge Similar type of problems including conceptual mixing of nature of product and the chemical properties of product can also be asked, such as

When an organic dicarboxylic acid is treated with calcium hydroxide, its salt is produced which when heated produces a cyclic compound [*X*] *having molecular formula* $C_6H_{10}O$. *X on heating with dilute alkali produces Y. What are X and Y?*

After making of all steps required one can get answer as



33. (*c,d*) **Idea** Problem is based on conceptual mixing of acidic strength, reducing power, basicity order and bond angle of hydrides of nitrogen family. While solving these type of problems students must have the knowledge of acidic strength, reducing power, basicity order and bent rule.

This problem includes concept of various characteristics of hydrides of nitrogen.

NH_3	• removal of H capacity increases
PH_3	• acidic strength increases
AsH_3	• basic strength decreases
SbH_3	• bond angle decreases
BiH_3	(H—A—H bond angle)

TEST Edge Similar problems based on characteristics of hydrides of chalcogens, icosagens can also be asked.

34. (*a,b,d*) **Idea** This problem is based on concept of Lucas, test, while solving this problem students are advised to understand the key concept through which alcohols undergo formation of turbidity during Lucas test.

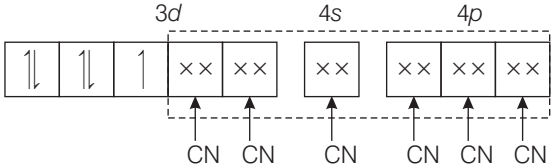
Rate of reaction towards Lucas reagent ∝ stability of carbocation

TEST Edge Similar problems based on the concept of victor Mayer test, oxidation test, dehydration test etc. to distinguish 1°, 2° and 3° alcohol can also be asked.

35. (*a,b,d*) **Idea** This problem involves conceptual mixing of hybridisation of coordination compound and their magnetic properties.

Hints Hybridisation in $[Fe(CN)_6]^{3-}$ complex ion

GSVSEC of Fe in $[Fe(CN)_6]^{3-}$



Hybridisation = d^2sp^3

Here, there is one unpaired electron, hence it is a paramagnetic complex.

TEST Edge Similar problems based on conceptual mixing of hybridisation, isomerism, magnetic properties and characteristics of coordination compounds can also be asked in JEE Advanced.

36. (*6*) **Idea** Problem is based on concept of oxidation and change in oxidation number. While solving this problem, students are advised to go through calculation of change in oxidation number then to calculate number of moles of H_2S used.

$$\underset{-2}{H_2S} \longrightarrow \underset{+4}{SO_2}$$

Change in oxidation number = +4 – (– 2) = + 6

∴ 1 mole H_2S = 6 equivalents of H_2S

TEST Edge Similar problems based on redox reaction and its application can also be asked. So students are advised to go through study of redox reaction and its application in calculation of equivalent weight.

37. (*7*) **Idea** This problem is based on stoichiometry and general concept used in quantitative calculations. While solving this problem, students are advised to follow the given stepwise approach

Calculate number of moles of CO_2 first

Calculate total value of CO_2 produced

$$HCO_3^- \longrightarrow CO_2$$

$$\text{mole of } HCO_3^- = \frac{6.1}{61} \times \frac{20.8}{100} = 0.0286 \text{ mole of } CO_2$$

1 mole of CO_2 at 25° and 1 atm = 24.4 L

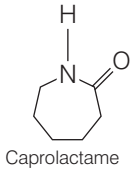
∴ 0.0286 mol CO_2 = 24.4 × 0.0286

= 0.6947 = 0.7 L

V_{CO_2} = 0.7 × 10 = 7 L

TEST Edge Similar problems based, on conceptual mixing of gaseous law, stoichiometry, limiting reagent can also be asked.

38. (*7*) **Idea** This problem is based on the concept of monomeric unit of polymer Nylon 6. To solve these types of problems, students must be careful because the question is asked about number of atoms present on cycle of monomer unit *i.e.*, excluding H. Hence count the members of the cycle including N and excluding H and O.



Caprolactame

TEST Edge Similar problems, with slight modification can also be asked in JEE Advanced, so students are advised to go through the study of monomeric unit present in various polymers.

39. (*2*) **Idea** This problem is based on concept of kinetic study of radioactive reaction. While solving this problem, student must have knowledge of rate and activity of radioactive decay.

Initial moles of non-radioactive

$$^{56}Fe^{3+} = \frac{0.6}{56} = 0.0107$$

Initial moles of radioactive $^{57}Fe^{2+} = \frac{0.209}{57} = 0.0036$

Moles of non-radioactive $^{56}Fe^{2+} = 10^{-5}$

$$R = k\,[^{57}Fe^{2+}]\,[^{56}Fe^{3+}]$$

$3.38 \times 10^{-7} = 10^{-2}\,(0.0036 - 10^{-5} - x)\,(0.0107 - 10^{-5})$

where x is the amount of $^{57}Fe^{2+}$ decayed in one hour due to radioactivity

$$x = 4.4 \times 10^{-4} \text{ moles}$$

$(0.0036 - x - y) = 0.0036\, e^{-\lambda_c 1}$

(where y is the number of moles of nuclei disintegrated from $^{57}Fe^{3+}$)

$$\lambda = 0.693 / 4.62 = 0.15\,h^{-1}$$

$$\Rightarrow \quad y = 7.14 \times 10^{-5} \text{ moles}$$

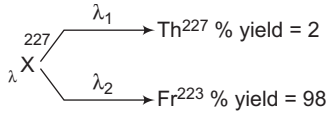
Activity $= \lambda N = 0.15\,(0.0036 - 4.4 \times 10^{-4} - 7.14 \times 10^{-5}) \times 6.023 \times 10^{23}$

$$A = 2.74 \times 10^{20} \text{ dis/h}$$

Hence, the unit number is 2.

TEST Edge Similar problems including conceptual mixing of radioactive disintegration, rate law of radioactive decay, half life period, average life period etc. can also be asked, such as

X^{227} has a half life of 22 *years, the decay of X^{227} follows two parallel paths*



What will be the value of decay constant for Th *and* Fr *respectively*?

Ans. $\lambda_1 = 0.00063\ yr^{-1}$

$$\lambda_2 = 0.03087\ yr^{-1}$$

Which comes *via* taking the ratio of yields.

40. (*2*) **Idea** This problem is based on solubility and solubility principle. While solving this problem, students must have the knowledge of solubility product principle.

$$TlN_3 = Tl^+ + N_3^-$$

$$(s + 1.5 \times 10^{-2})\ s$$

$$Tl_3PO_4 \rightleftharpoons \underset{(1.5 \times 10^{-2} + s)}{3Tl^+} + \underset{(5 \times 10^{-3})}{PO_4^{3-}}$$

For TlN_3, $s(s + 1.5 \times 10^{-2}) = 5.6875 \times 10^{-4}$

$$\Rightarrow \quad s^2 + 0.015\,s - 5.6875 \times 10^{-4} = 0$$

$$\Rightarrow \quad s = \frac{-0.015 \pm \sqrt{(0.015)^2 + 22.75 \times 10^{-4}}}{2}$$

$$= 175 \times 10^{-4} \text{ mol/L}$$

As per the given condition

$$1.75 \times 10^{-4} / 10^{-6} = 1.75 \approx 2$$

TEST Edge In JEE Advanced, generally similar types of problems with slight conceptual mixing of electrochemistry can also be asked.

The value of the molar solubility of Ag_2CO_3 in 0.1 M. Na_2CO_3 will be ..... . If

$$K_{sp} = 16 \times 10^{-13}$$

On solving you will get $= 4 \times 10^{-6}$

41. (*c*) **Idea** Here apply concept of modulus function, periodic function such as $y = |x| = \begin{cases} x \text{ if } x > 0 \\ -x \text{ if } x < 0 \end{cases}$

$\sin x \in [-1, 1]$ and $\cos x \in [-1, 1]$ and draw the graph of $y = |\sin x|$ and $y = |\sin 2x| + |\cos 2x|$

Given function is

$$f(x) = |\sin 2x| + |\cos 2x|$$

Now,

$$f\left(\frac{\pi}{4} + x\right) = \left|\sin 2\left(\frac{\pi}{4} + x\right)\right| + \left|\cos 2\left(\frac{\pi}{4} + x\right)\right|$$

$$= |\cos 2x| + |\sin 2x|$$

$$= f(x)$$

Thus, the function is periodic with period $\frac{\pi}{4}$.

Let $y = |\sin 2x| + |\cos 2x|$

$$y^2 = \sin^2 2x + \cos^2 2x + 2|\sin 2x||\cos 2x|$$

$$= 1 + |\sin 4x|$$

Now, $|\sin 4x| \in [0, 1]$

$1 + |\sin 4x| \in [1, 2]$

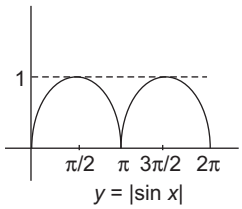
$y^2 \in [1, 2]$

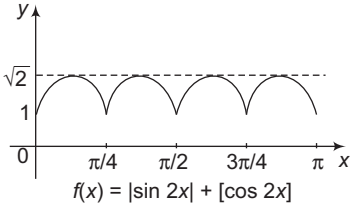
$y \in [1, \sqrt{2}]$

Thus, $|\sin 2x| + |\cos 2x| \in [1, \sqrt{2}]$

and $|\sin y| \le 1$

So, solution is possible if both the sides are equal to 1.





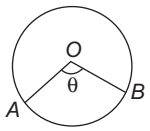
From the graphs, solutions are

$$x = \pm\frac{\pi}{2}, \pm\frac{3\pi}{2}$$

$\therefore$ Number of solutions are 4.

TEST Edge Domain and range of algebraic functions, greatest integer function and solution of trigonometric function, related questions are asked. To solve such type of questions, students are advised to understand the concept of the functions such as, the function $f: R \to R$ defined as $f(x) = \begin{cases} \frac{|x|}{x} \text{ for } x \neq 0 \\ 0 \text{ for } x = 0 \end{cases}$ is called the signum function

42. (*a*) **Idea** To solve this problem, use the concept of area of given region in 2D, such as the area of sector $(OABO)$ is $\frac{\theta}{360}\pi r^2$

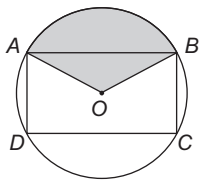


If the point P lies in the sector created by $\angle AOB$, then

$$d(P, AB) \le d(P, BC)$$
$$d(P, AB) \le d(P, CD)$$
$$d(P, AB) \le d(P, DA)$$

$\Rightarrow \quad d(P, AB) \le \min [d(P, BC), d(P, CD), d(P, DA)]$

Hence, the required area is equal to sector created by $\angle AOB = \left(\frac{\pi}{2}\right)$.



So, the required area = Shaded area

$$= \frac{\pi r^2}{4} = \frac{\pi (2^4)^2}{4} = 4\pi$$

TEST Edge Generally in JEE Advanced, area of the given region such as area of sector, area of polygon related questions are asked to solve such type of questions. Students are advised to understand the concept of two dimensional geometry, such as, the area of a polygon of n sides with vertices. $A_1(x_1, y_1), A_2(x_2, y_2) \ldots A_n(x_n, y_n)$ is

$$= \frac{1}{2}[\{(x_1y_2 + x_2y_3 + \ldots + x_ny_1) - (y_1x_2 + y_2x_3 + \ldots + y_nx_1)\}]$$

43. (*a*) **Idea** Here apply the concept of binomial coefficient, cosine formula and AP to solve this problem. Such as ${}^nC_r = {}^nC_{n-r} = \frac{n!}{r!(n-r)!}$, In ΔABC, $\cos A = \frac{b^2 + c^2 - a^2}{2bc}$, $\cos B = \frac{a^2 + c^2 - b^2}{2ac}$ and if a, b, c are in AP then $2b = a + c$

$$\frac{2!}{1!\ 9!} + \frac{2!}{3!\ 7!} + \frac{1!}{5!\ 5!}$$

$$= \left[\frac{2}{10!}\left(\frac{10!}{1!\ 9!} + \frac{10!}{3!\ 7!}\right) + \frac{1}{10!}\frac{10!}{5!\ 5!}\right]$$

$$= \frac{2}{10!}\left({}^{10}C_1 + {}^{10}C_3 + \frac{1}{10!}{}^{10}C_5\right)$$

$$= \frac{1}{10!}[2({}^{10}C_1) + 2({}^{10}C_3) + ({}^{10}C_5)]$$

$$= \frac{1}{10!}[{}^{10}C_1 + {}^{10}C_9 + {}^{10}C_3 + {}^{10}C_7 + {}^{10}C_5]$$

$[\because {}^nC_r = {}^nC_{n-r}]$

$$= \frac{1}{10!}(2^{10-1}) = \frac{2^9}{10!} = \frac{8^a}{(2b)!}$$

$\Rightarrow \quad \frac{2^9}{10!} = \frac{2^{3a}}{(2b)!}$

$\Rightarrow \quad 9 = 3a, 2b = 10 \Rightarrow \ a = 3, \ b = 5$

Since a, b, c are in AP

$$2b = a + c$$
$$10 = 3 + c$$
$$c = 7$$
$$\cos A = \frac{b^2 + c^2 - a^2}{2bc} = \frac{25 + 49 - 9}{2 \times 5 \times 7} = \frac{13}{14}$$
$$\cos B = \frac{c^2 + a^2 - b^2}{2ca} = \frac{11}{14}$$
$$\cos A + \cos B = \frac{13 + 11}{14} = \frac{24}{14} = \frac{12}{7}$$

TEST Edge Binomial expression, properties of triangle and AP, GP related question are asked. To solve such type of questions, students are advised to understand the concept of binomial expression properties of triangle also acquainted yourself with the properties of triangle such as, the area of triangle is $\Delta = \frac{1}{2}bc \sin A = \frac{1}{2}ca \sin B$

$$= \frac{1}{2}ab \sin c = \sqrt{s(s-a)(s-b)(s-c)} = \frac{abc}{4R} = rs$$

where R and r are the radii of the circumcircle and in circle of the ΔABC respectively.

44. (*b*) **Idea** $\because$ The first term of an AP be a and common difference be d, then $S_n = \frac{n}{2}[2a + (n-1)d]$ and apply the condition for $\frac{Sn_1n_2}{Sn_1}$ is independent of n_1.

Given that S_n is sum of first n terms of an AP with non-zero common difference.

So, $\quad S_{n_1n_2} = \frac{n_1n_2}{2}[2A + (n_1n_2 - 1)d]$

and $\quad S_{n_1} = \frac{n_1}{2}[2A + (n_1 - 1)d]$

$$\Rightarrow \quad \frac{S_{n_1n_2}}{S_{n_1}} = \frac{\frac{n_1n_2}{2}[2A + (n_1n_2 - 1)d]}{\frac{n_1}{2}[2A + (n_1 - 1)d]}$$

$$\Rightarrow \quad \frac{S_{n_1n_2}}{S_{n_1}} = n_2 \frac{(2A - d) + n_1n_2d}{(2A - d) + n_1d}$$

$\frac{S_{n_1n_2}}{S_{n_1}}$ will be independent from n_1, if $2A - d = 0$

$$\Rightarrow \quad 2A = d \quad \Rightarrow \quad \frac{A}{d} = \frac{1}{2} \quad \Rightarrow \quad 1 : 2$$

TEST Edge In JEE Advanced, properties of AP, n^{th} terms of an AP, arithmetic mean related questions are asked. To solve such type of questions, students are advised to understand the concept of AP such as, if common difference d, number of terms n and the last term of AP are given then $S_n = \frac{n}{2}[2l - (n-1)d]$

45. (*b*) **Idea** If three vectors $\mathbf{a}, \mathbf{b}$ and $\mathbf{c}$ are mutually perpendicular to each other then

$$\mathbf{a}.\mathbf{b} = \mathbf{b}.\mathbf{c} = \mathbf{c}.\mathbf{a} = 0$$

$\because \quad \mathbf{a}.\mathbf{b} = |\mathbf{a}||\mathbf{b}|\cos\theta$

Given, $\mathbf{a} = \cos\theta\,\hat{\mathbf{i}} + \sin\theta\,\hat{\mathbf{j}}$

$\mathbf{b} = -\sin\theta\,\hat{\mathbf{i}} + \cos\theta\,\hat{\mathbf{j}}, \mathbf{c} = \hat{\mathbf{k}}$

It can be easily observed that,

$|\mathbf{a}| = |\mathbf{b}| = |\mathbf{c}| = 1$ and $\mathbf{a}\cdot\mathbf{b} = \mathbf{b}\cdot\mathbf{c} = \mathbf{c}\cdot\mathbf{a} = 0$

If $\mathbf{r}$ is equally inclined with the mutually perpendicular vectors $\mathbf{a} = \cos\theta\,\hat{\mathbf{i}} + \sin\theta\,\hat{\mathbf{j}}$, $\mathbf{b} = -\sin\theta\,\hat{\mathbf{j}} + \cos\theta\,\hat{\mathbf{j}}$ and $\mathbf{c} = \hat{\mathbf{k}}$, then $\mathbf{r}$ must be of type $\mathbf{r} = \lambda(\mathbf{a} + \mathbf{b} + \mathbf{c})$.

Now, substituting the values of $\mathbf{a}, \mathbf{b}, \mathbf{c}$ in above, we get

$\mathbf{r} = \lambda(\mathbf{a} + \mathbf{b} + \mathbf{c})$

$= \lambda[(\cos\theta - \sin\theta)\hat{\mathbf{i}} + (\sin\theta + \cos\theta)\hat{\mathbf{j}} + \hat{\mathbf{k}}]$

To find angle between $\mathbf{a}$ and $\mathbf{r}$, we find $\mathbf{a}\cdot\mathbf{r}$.

$$\therefore \quad \cos(\mathbf{r}, \mathbf{a}) = \frac{\mathbf{r}\cdot\mathbf{a}}{|\mathbf{r}||\mathbf{a}|} = \frac{1}{\sqrt{3}}$$

$$\therefore \quad (\mathbf{r}, \mathbf{a}) = \cos^{-1}\left(\frac{1}{\sqrt{3}}\right)$$

TEST Edge Properties of dot products of two vectors such as the vectors are perpendicular to each other and angle between two vectors related questions are asked. To solve such type of questions, students are advised to understand the concept of dot product of two vectors. Such as, the component of a vector $\mathbf{b}$ along to vector $\mathbf{a}$ is $\left(\frac{\mathbf{a}.\mathbf{b}}{|\mathbf{a}|^2}\right)\mathbf{a}$

46. (*d*) **Idea**

$\because \quad (1-x)^n = {}^nC_0 - {}^nC_1x + {}^nC_2x^2 - {}^nC_3x^3 + \ldots + {}^nC_nx^n$ and ${}^nC_r = \frac{n!}{r!(n-r)!}$. Now use the relation between binomial coefficient to solve this problem.

Given that

$$\frac{1}{81^n} - \frac{10}{81^n}\,{}^{2n}C_1 + \frac{10^2}{81^n}\,{}^{2n}C_2 - \frac{10^3}{81^n}\,{}^{2n}C_3 \ldots \frac{10^{2n}}{81^n}$$

So, we can write it as follow

$$\Rightarrow \quad \frac{1}{81^n}[{}^{2n}C_0 - {}^{2n}C_1\,10 + {}^{2n}C_2\,10^2 - {}^{2n}C_3\,10^3 + \ldots + {}^{2n}C_{2n}\,10^{2n}]$$

$$\Rightarrow \quad \frac{1}{81^n}[1-10]^{2n}$$

$$\Rightarrow \quad \frac{(-9)^{2n}}{81^n} \quad \Rightarrow \quad \frac{81^n}{81^n} = 1$$

TEST Edge In JEE Advanced, general term, greatest term and the properties of binomial coefficient related questions are asked. To solve such type of questions, students are suggested that to understand the concept of binomial coefficient such as, the coefficient of $x^{n_1}y^{n_2}z^{n_3}$ in the expansion of $(x+y+z)^n$ is $\frac{x!}{x_1!x_2!x_3!}$ where $n = n_1 + n_2 + n_3$

47. (*a*) **Idea** $Z = \cos\theta + i\sin\theta = (1)^{1/n}$

then, $Z = (\cos 0 + i\sin 0)^{1/n}$

$$(1)^{1/n} = \cos\frac{2\pi r + 0}{n} + i\sin\left(\frac{2\pi r + 0}{n}\right),$$

$r = 0, 1, 2 \ldots (n-1)$

$Z = e^{\frac{i2r\pi}{n}}, r = 0, 1 \ldots\ldots (n-1)$

$Z = 1, e^{i2\pi/n}, e^{i4\pi/n}\ e^{(i2(n-1)/n)}$

then $1, \alpha, \alpha^2, \alpha^3 \ldots \alpha^{n-1}$ where $\alpha = e^{i2\pi/n}$ are the n^{th} roots of unity

then, $1 + \alpha + \alpha^2 + \alpha^{n-1} = 0$

$$\sum\sum \alpha_i^5\alpha_j^5 = \frac{1}{2}[(\alpha_1{}^5 + \alpha_2{}^5 + \ldots + \alpha_{100}^5)^2 - (\alpha_1^{10} + \alpha_2^{10} + \alpha_3^{10} + \alpha_4^{10} + \ldots)]$$

$$= \frac{1}{2}[0 - 0] = 0$$

Because $(\alpha_1^P + \alpha_2^P + \ldots + \alpha_{100}^P) = \begin{cases} 100, & \text{if } P = 100K \\ 0, & \text{if } P \neq 100K \end{cases}$

TEST Edge Cube roots of unity, properties of n^{th} roots of unity related questions are asked. To solve such type of questions understand the concept of complex number such as, the product of n^{th} roots of unity *i.e.*, $1.\alpha.\alpha^2 \ldots \alpha^{n-1} = (-1)^{n-1}$.

48. (*b*) **Idea** To solve this problem, apply the concept of number of real solution in theory of equation and also use the concept of AM such as if $a_1a_2 \ldots a_5$ are positive real numbers then $\frac{a_1 + a_2 + a_3 + a_4 + a_5}{5}$ will be AM of these numbers.

$$(x+4)^3+(x+3)^3+(x+2)^3+(x+1)^3 + (x-5)^3+180=0$$

First AM of $\{x+4, x+3, x+2, x+1, x-5\}$

$$\Rightarrow \quad \frac{x+4+x+3+x+2+x+1+x-5}{5}=\frac{5x+5}{5}$$

$$\Rightarrow \quad \frac{5(x+1)}{5}=x+1$$

Let $y=x+1$

Now, $(y+3)^3+(y+2)^3+(y+1)^3+y^3 + (y-6)^3+180=0$

$(A+B)^3=A^3+B^3+3A^2B+3AB^2$

$y^3+27+27y+9y^2+y^3+12y+6y^2+8 + y^3+1+3y^2+3y+y^3+y^3-216 + 108y-18y^2+180$

$$5y^3+150y=0$$

$$5y(y^2+30)=0$$

$$y^2+30\neq 0$$

So, $5y=0 \quad y=0$

$y=x+1$, so $x+1=0$ or $x=-1$

TEST Edge Common roots, symmetric functions of the roots, nature of roots with respect to two real numbers related questions are asked. To solve such type of questions, students are advised to understand the concept of quadratic equations and expressions such as, if $a<0$, then the quadratic expression $y=ax^2+bx+c$ has no least value but it has greatest value $\frac{4ac-b^2}{4a}$ at $x=\frac{-b}{2a}$.

49. (*c*) **Idea** To solve this problem use the concept of properties of determinants. If each element of a column or row of determinant is zero, then its value is zero.

Given that

$$\Delta=\begin{vmatrix} \frac{1}{z} & \frac{1}{z} & -\frac{x+y}{z^2} \\ -\frac{y+z}{x^2} & \frac{1}{x} & \frac{1}{x} \\ \frac{-y(y+z)}{x^2z} & \frac{x+2y+z}{xz} & \frac{-y(x+y)}{xz^2} \end{vmatrix}$$

Multiply C_1 by x, C_2 by y and C_3 by z, we get

$$\Delta=\frac{1}{xyz}\begin{vmatrix} \frac{x}{z} & \frac{y}{z} & \frac{-(x+y)}{z} \\ \frac{-(y+z)}{x} & \frac{y}{x} & \frac{z}{x} \\ \frac{-y(y+z)}{xz} & \frac{y(x+2y+z)}{xz} & \frac{-(x+y)\cdot y}{xz} \end{vmatrix}$$

Now, $C_1+C_2+C_3 \longrightarrow C_1$

$$\Delta=\frac{1}{xyz}\begin{vmatrix} \frac{x+y-(x+y)}{z} & \frac{y}{z} & -\frac{(x+y)}{z} \\ \frac{-(y+z)+y+z}{x} & \frac{y}{x} & \frac{z}{x} \\ \frac{-y^2-yz+yx+2y^2+yz-xy-y^2}{xz} & \frac{y(x+2y+z)}{xz} & \frac{-(x+y)\cdot y}{xz} \end{vmatrix}$$

$$\Delta=\begin{vmatrix} 0 & y/z & -\frac{(x+y)}{z} \\ 0 & y/x & \frac{z}{x} \\ 0 & \frac{y(x+2y+z)}{xz} & \frac{-(x+y)y}{xz} \end{vmatrix}$$

If there is any one column is 0, then determinant will be zero.

So, $\Delta=0$

TEST Edge Product of determinants, solution of linear equation by determinants related questions are asked. To solve such type of questions, students are advised to understand the concept of determinants, such as

Let $\Delta(x)=\begin{vmatrix} a_1(x) & b_1(x) \\ a_2(x) & b_2(x) \end{vmatrix}$

then $\Delta'(x)=\begin{vmatrix} a_1'(x) & b_1'(x) \\ a_2(x) & b_2(x) \end{vmatrix}+\begin{vmatrix} a_1(x) & b_1(x) \\ a_2'(x) & b_2'(x) \end{vmatrix}$ where dash ($'$) denotes derivative with respect to x.

50. (*b*) **Idea** $\int_a^b f(x)dx=\int_a^b f(a+b-x)dx, \int_a^b kf(x)dx = k\int_a^b f(x)dx$

$\int_a^b f(x)dx=F(b)-F(a)+c$ also apply the concept of integration to solve this problem.

Let $I=\int_{2009}^{2011} x f(x)dx=\int_{2009}^{2011}(2009+2011-x) f(2009+2011-x)dx$

$$\Rightarrow \quad \int_{2009}^{2011}(4020-x)f(4020-x)dx$$

{From property $\int_a^b f(x)\,dx=\int_a^b f(a+b-x)dx$ $[\because f(x)=f(4020-x)]$

$$\Rightarrow \quad \int_{2009}^{2011}(4020-x)f(x)dx$$

$$\Rightarrow \quad 4020\int_{2009}^{2011} f(x)dx-\int_{2009}^{2011} x f(x)dx$$

$$=4020\int_{2009}^{2011} f(x)\,dx-I$$

$$2I = 4020\int_{2009}^{2011} f(x)dx$$

$$\Rightarrow I = -2010\int_{2011}^{2009} f(x)dx$$

$$\Rightarrow I = k\int_{2011}^{2009} f(x)dx$$

$$\therefore \quad k = -2010$$

$$\Rightarrow \quad 10 - k = 2020$$

TEST Edge Evaluation of definite integral by substitution, properties of definite integral and differentiation under integral sign related questions are asked. To solve such type of questions, students are advised to understand the concept of definite integral such as, for any two functions $f(x)$ and $g(x)$, integrable on the intervals $[a,b]$, the Schwarz - Bunyakovsky in equality holds.

$$\left|\int_a^b f(x)g(x)dx\right| \le \sqrt{\int_a^b f^2(x)dx - \int_a^b g^2(x)dx}$$

51. (a, d) **Idea**

$\int_a^b f(x)dx = \int_a^c f(x)dx + \int_c^b f(x)dx$, if $y = f(x) = |x|$ is a modulus function then, $|x| = \begin{cases} x \text{ if } x > 0 \\ -x \text{ if } x < 0 \end{cases}$. A function $f(x)$ is said to be continuous at $x = a$ if LHL = RHL = $f(a)$ and a function is said to differentiable if LHD = RHD.

For $x > 2$

$$f(x) = \int_0^1 (1 + |1 - t|)\,dt + \int_1^x (1 + |1 - t|)dt$$

$$= \int_0^1 \{1 + (1 - t)\}dt + \int_1^x (1 + t - 1)dt$$

$$= \int_0^1 (2 - t)dt + \int_1^x t\,dt$$

$$= \left[2t - \frac{t^2}{2}\right]_0^1 + \left[\frac{t^2}{2}\right]_1^x$$

$$= \left[\left(2 - \frac{1}{2}\right) - (0 - 0)\right] + \left[\frac{x^2}{2} - \frac{1}{2}\right]$$

$$= \frac{3}{2} + \frac{x^2}{2} - \frac{1}{2} = \frac{x^2}{2} + 1$$

$$\therefore \quad f(x) = \begin{cases} \frac{x^2}{2} + 1, & x > 2 \\ 5x + 1, & x \le 2 \end{cases}$$

$$f(2^+) = \frac{4}{2} + 1 = 3$$

$$f(2^-) = 5(2) + 1 = 11$$

$\therefore$ $f(x)$ is not continuous at $x = 2$.

Now, $f(2) = 11$

$f(2^+) = 3$

i.e., RHL $\neq f(2)$

$\therefore$ RHD at $x = 2$ does not exist.

TEST Edge Continuity of a function at a point, continuity in an open interval, continuity in a closed interval and continuity of composite functions, related questions are asked.

To solve such type of questions, students are advised to understand the concept of continuity of a function. Such as, a function $f(x)$ is said to be continuous in an open interval (a,b) if it is continuous at each point of (a,b).

52. (a,c) **Idea** To solve this problem, first of all find the point of intersection then put these values in the curve $xy = c^2$

For the points, where the line intersects the curve, we have $z = 0$.

$$\therefore \quad \frac{x-2}{3} = \frac{y+1}{2} = \frac{0-1}{-1}$$

$$\Rightarrow \quad \frac{x-2}{3} = \frac{y+1}{2} = 1$$

$$\Rightarrow \quad \frac{x-2}{3} = 1, \quad \frac{y+1}{2} = 1$$

$$\Rightarrow \quad x = 5, \quad y = 1$$

Putting these values in $xy = c^2$, we have

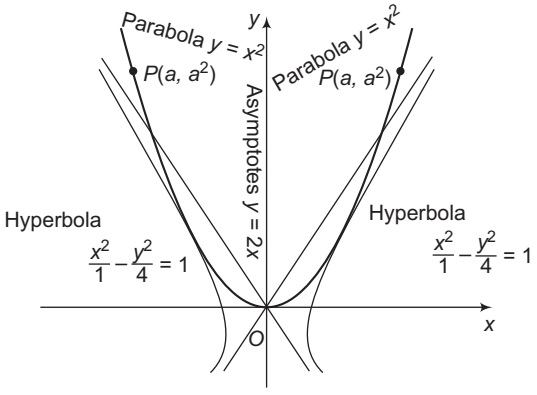
$$5 = c^2$$

$$\Rightarrow \quad c = \pm\sqrt{5}$$

TEST Edge Coplanar lines, the lines are perpendicular, parallel, equation of a line in various forms related questions are asked. To solve such type of questions, students are advised to understand the concept of line in 3-D such as, the angle between the two lines $\mathbf{r} = \mathbf{a}_1 + \mathbf{b}_1$ and $\mathbf{r} = \mathbf{a}_2 + \mathbf{b}_2$ is given by $\cos\theta = \frac{\mathbf{b}_1 \cdot \mathbf{b}_2}{|\mathbf{b}_1||\mathbf{b}_2|}$.

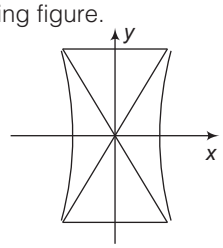
53. (c,d) **Idea** Draw the graph of hyperbola $\frac{x^2}{1} - \frac{y^2}{4} = 1$ and draw the tangent to it. The lines $\frac{x}{a} \pm \frac{y}{b} = 0$ are the asymptotes to the hyperbola $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$.

$P(\alpha, \alpha^2)$ lies on the parabola $y = x^2$



If two tangents can be drawn to the hyperbola $\frac{x^2}{1}-\frac{y^2}{4}=1$ from the point $P(\alpha,\alpha^2)$, then $P(\alpha,\alpha^2)$ must lie outside the hyperbola $\frac{x^2}{1}-\frac{y^2}{4}=1$

If two tangents can be drawn to the different branches of hyperbola $\frac{x^2}{1}-\frac{y^2}{4}=1$ from the point (α,α^2), then $P(\alpha,\alpha^2)$ must lie in the shaded region in the adjoining figure.



Since, the asymptotes are

$$y=\pm 2x \text{ or } (y^2-4x^2=0)$$

$\therefore \quad 2\alpha<\alpha^2$ and $-2\alpha<\alpha^2$

$\Rightarrow \quad \alpha(\alpha-2)>0$ and $\alpha(\alpha+2)>0$

$\Rightarrow \quad \alpha<0$ or $\alpha>2$ and $\alpha<-2$ or $\alpha>0$

Taking intersection of the above two results, we get the final range of α as

$$\alpha\in(-\infty,-2) \text{ or } \alpha\in(2,\infty)$$

TEST Edge Tangent and normal to the hyperbola, properties of hyperbola, conjugate hyperbola related questions are asked.

To solve such type of questions, students are advised to understand the concept of hyperbola such as the straight line $y=mx+c$ is a tangent to the curve, if $e^2=a^2m^2-b^2$

54. (a,d) **Idea** If $\mathbf{a,b,c}$ are given as $\mathbf{a}=a_i\hat{\mathbf{i}}+\mathbf{a}_2\,\hat{\mathbf{j}}+a_3\,\hat{\mathbf{k}}$, $\mathbf{b}=b_1\,\hat{\mathbf{i}}+b_2\hat{\mathbf{j}}+b_3\hat{\mathbf{k}}$ and $\mathbf{c}=c_1\,\hat{\mathbf{i}}+c_2\hat{\mathbf{j}}+c_3\hat{\mathbf{k}}$

then $(\mathbf{a}\times\mathbf{b})\cdot\mathbf{c}=[\mathbf{abc}]=\begin{vmatrix}a_1 & a_2 & a_3\\ b_1 & b_2 & b_3\\ c_1 & c_2 & c_3\end{vmatrix}$

$\because \mathbf{a}\times\mathbf{a}=\mathbf{b}\times\mathbf{b}=\mathbf{c}\times\mathbf{c}=0$

Also use the concept of maximum and minimum.

Now,

$$\mathbf{d}\cdot\mathbf{a}=\{(\mathbf{a}\times\mathbf{b})\sin x+(\mathbf{b}\times\mathbf{c})\cos y+2(\mathbf{c}\times\mathbf{a})\}\cdot\mathbf{a}$$

$$[\because (\mathbf{a}\times\mathbf{b})\cdot\mathbf{a}=0,\ (\mathbf{c}\times\mathbf{a})\cdot\mathbf{a}=0]$$

$$=[\mathbf{a\,b\,c}]\cos y \quad \ldots(\text{i})$$

also $\mathbf{d}\cdot(\mathbf{a}+\mathbf{b}+\mathbf{c})=0$

$$\mathbf{d}\cdot\mathbf{a}=-\mathbf{d}\cdot(\mathbf{b}+\mathbf{c}) \quad \ldots(\text{ii})$$

From Eqs. (i) and (ii), we get

$$\cos y=\frac{-\mathbf{d}\cdot(\mathbf{b}+\mathbf{c})}{[\mathbf{a\,b\,c}]}$$

Similarly, $\sin x=-\frac{\mathbf{d}\cdot(\mathbf{a}+\mathbf{b})}{[\mathbf{a\,b\,c}]}$ and $-2=\frac{\mathbf{d}\cdot(\mathbf{a}+\mathbf{c})}{[\mathbf{a\,b\,c}]}$

$$\therefore \quad \sin x+\cos y+2=-\frac{\mathbf{d}\cdot(\mathbf{a}+\mathbf{b})}{[\mathbf{a\,b\,c}]}-\frac{\mathbf{d}\cdot(\mathbf{b}+\mathbf{c})}{[\mathbf{a\,b\cdot c}]}$$

$$-\frac{\mathbf{d}\cdot(\mathbf{a}+\mathbf{c})}{[\mathbf{a\,b\,c}]}=-\frac{-2\cdot[\mathbf{a}\cdot\mathbf{d}+\mathbf{b}\cdot\mathbf{d}+\mathbf{c}\cdot\mathbf{d}]}{[\mathbf{a\,b\,c}]}$$

also $\mathbf{d}\cdot(\mathbf{a}+\mathbf{b}+\mathbf{c})=0 \quad [\because \mathbf{d}\perp(\mathbf{a}+\mathbf{b}+\mathbf{c})]$

$\therefore \quad \sin x+\cos y+2=0$

$\Rightarrow \quad \sin x+\cos y=-2$

$\Rightarrow \quad \sin x=-1,\ \cos y=-1$

$\Rightarrow \quad x=-\frac{\pi}{2},\ y=\pi$

$[\because$ we want the minimum value of $x^2+y^2]$

$\therefore \quad x^2+y^2=\frac{\pi^2}{4}+\pi^2=\frac{5\pi^2}{4}.$

TEST Edge Scalar triple product, properties of scalar triple product and geometrical application of scalar triple product related questions are asked. To solve such type of questions, students are advised to understand the concept of scalar triple product, such as for any three vectors $\mathbf{a,b}$ and $\mathbf{c}$ $[\mathbf{a}\times\mathbf{b}\ \ \mathbf{b}\times\mathbf{c}\ \ \mathbf{c}\times\mathbf{a}]=[\mathbf{abc}]^2=|(\mathbf{a}\times\mathbf{b})\cdot\mathbf{c}|^2$

55. (a,d) **Idea** To solve this problem, use the concept of value of trigonometric expression. Also apply the concept of differentiation, maxima, minima and range of the function.

Let us assume that $y=\sin 3x+\sin x$

$\Rightarrow \quad y=(3\sin x-4\sin^3 x)+\sin x$

$\Rightarrow \quad y=4(\sin x-\sin^3 x)$

which is a cubic equation.

On differentiating both the sides w.r.t.x, we get

$$\frac{dy}{dx}=4(\cos x-3\sin^2 x\cos x)$$

$\Rightarrow \quad \frac{dy}{dx}=4\cos x(1-3\sin^2 x)$

$\therefore \quad \frac{dy}{dx}=0 \ \Rightarrow \cos x=0$

and $\quad \sin x=\pm\frac{1}{\sqrt{3}} \ \Rightarrow \ \sin x=\pm\frac{1}{\sqrt{3}}$ or ∓ 1

On substituting $\sin x=\pm 1$

$$y=4\sin x(1-\sin^2 x).$$

We get, $\quad y=\pm 4(1-1)=0$

On substituting $x=\pm\frac{1}{\sqrt{3}}$ in

$y=4\sin x(1-\sin^2 x)$, we get

$$y=\pm 4\frac{1}{\sqrt{3}}\left(1-\frac{1}{3}\right)=\pm\frac{8}{3\sqrt{3}}$$

$$y_{\max}=\frac{8}{3\sqrt{3}} \text{ and } y_{\min}=\frac{-8}{3\sqrt{3}}$$

$\Rightarrow$ Range of y is $\left[\frac{-8}{3\sqrt{3}}, \frac{8}{3\sqrt{3}}\right]$

$\Rightarrow$ $(\sin 3x + \sin x) \in \left[\frac{-8}{3\sqrt{3}}, \frac{8}{3\sqrt{3}}\right]$

Now, $\frac{8}{3\sqrt{3}} = 1.54$; $\frac{e}{2} \approx 1.36$; $\frac{\pi}{2} = 1.57$

TEST Edge Solution of trigonometric equation, principal value, range and domain, and solution of trigonometric inequalities, related questions are asked. To solve such type of questions, students are suggested to understand the concept of trigonometric equations and also acquainted yourself with the concept of maxima or minima. Such as, $f(x) = \text{cosec}\, x$, domain is $(-\infty, \infty) - \{n\pi : n \in I\}$ and range is $(-\infty, -1] \cup [1, \infty)$

56. (0) **Idea** $\because \int_{-a}^{a} f(x)dx = \begin{cases} 0 \text{ if } f(-x) = -f(x) \\ 2\int_0^a f(x)dx \text{ if } (f(-x) = f(x) \end{cases}$

Also apply the concept of limit to solve this problem.

Let $f(x) = \sqrt{\cos x}\ \ln \frac{2-x}{2+x}$

$f(-x) = \sqrt{\cos(-x)}\ \ln \frac{2+x}{2-x}$

$f(-x) = -\sqrt{\cos x} \cdot \ln \frac{2-x}{2+x}$

$\Rightarrow$ $f(-x) = -f(x)$

In function

$\int_{-1/3}^{1/3} \sqrt{\cos x}\ \ln\left(\frac{2-x}{2+x}\right) dx = 0$

Property If $\int_{-a}^{a} f(x)\,dx = \int_0^a f(x)\,dx + \int_0^a f(-x)\,dx$
So, in odd function, it will be zero.

TEST Edge Evaluation of limits of algebraic functions, modulus function and greatest integer functions related questions are asked. To solve such type of question, students are advised to understand the concept of limit such as,

$\lim_{x \to a} \frac{x^n - a^n}{x - a} = na^{n-1}$, where $n \in 9$

57. (1) **Idea** To solve this problem use the concept of the distance of point from plane. The distance between two points $A(x_1, y_1, z_1)$ and $\phi(x_2, y_2, z_2)$ is $\sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2 + (z_1 - z_2)^2}$

Let the given point is $A(1, 1)$ and the plane is $P : x + y + z = 1$ and the given line is

$L : \frac{x-2}{1} = \frac{y-3}{2} = \frac{z-4}{3}$

We have to find the minimum distance of the point A from the plane P measured parallel to line L. Suppose a line parallel to L passing through A intersects the plane P at B, then we have to find distance AB.

Let's find equation of AB. Now, AB passes through $A(1, 1, 1)$ and is parallel to L. So, direction ratios of AB should be same as L *i.e.*, $< 1, 2, 3 >$ Hence, equation of AB is $\frac{x-1}{1} = \frac{y-1}{2} = \frac{z-1}{3}$.

Now, we should find the coordinates of B to find AB.

Coordinates of any point on line $AB : \frac{x-1}{1} = \frac{y-1}{2} = \frac{z-1}{3} = k$ are of type $(k + 1, 2k + 1, 3k + 1)$.

also point $B(k + 1, 2k + 1, 3k + 1)$ lies on the plane $P : x + y + z = 1$.

$\therefore$ $k + 1 + 2k + 1 + 3k + 1 = 1$

$k = \frac{-2}{6} = \frac{-1}{3}$ $\Rightarrow$ $B\left(\frac{2}{3}, \frac{1}{3}, 0\right)$

Hence, the length

$AB = \sqrt{\left(\frac{2}{3}\right)^2 + \left(\frac{1}{3}\right)^2 + 1^2} = \frac{\sqrt{14}}{3}$

$\therefore$ $\frac{3k}{\sqrt{14}} = \frac{3}{\sqrt{14}} \frac{\sqrt{14}}{3} = 1$

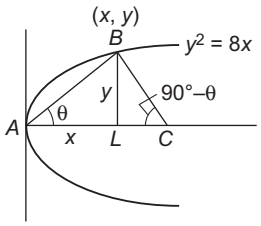
TEST Edge Distance between two plane, the plane are parallel or perpendicular the angle between two plane related questions are asked. To solve such type of questions, students are advised to understand the concept of plane, such as, the length of the perpendicular from a point having position vector $\mathbf{a}$ to the plane $\mathbf{r}.\mathbf{n} = d$ is given by $p = \frac{|\mathbf{a}.\mathbf{n} - d|}{|\mathbf{n}|}$

58. (8) **Idea** First of all draw the graph of the given curve then apply the concept of projection of line on another line, to solve this problem.

Let us assume that AB makes angle θ with positive x-axis.

Also, let the coordinates of B are (x, y)

$\therefore$ $\tan\theta = \frac{y}{x}$ and $y^2 = 8x$



$\therefore$ Projection of BC on the x-axis

$= LC = \frac{y}{\tan(90° - \theta)} = y \tan\theta = \frac{y^2}{x} = 8$

So, the required projection is 8 units.

TEST Edge The equation of the bisector of the angle which contains a given point, the equation of reflected ray and equation of a line in various forms, related questions are asked. To solve such type of questions, students are advised to understand the concept of straight line in 2-D . Such as, the homogeneous second degree equation $ax^2 + 2xhy + by^2 = 0$ represent a pair of straight line through the origin if $h^2 \geq ab$.

59. (*10*) **Idea** $\because \log_e x = 2 \Rightarrow x = e^2$

Apply the concept of quadratic equation to solve this problem, also use the concept of factorisation.

It is given that α, β are the roots of equation

$$\frac{1 - 8(\log_{10} x)^2}{\log_{10} x - 2[\log_{10} x]^2} = 1$$

Let $\log_{10} x = y$

So, $\frac{1 - 8y^2}{y - 2(y)^2} = 1$

$1 - 8y^2 = y - 2y^2$

$\Rightarrow -8y^2 + 2y^2 - y + 1 = 0$

$\Rightarrow -6y^2 - y + 1 = 0$

$\Rightarrow 6y^2 + y - 1 = 0$

$6y^2 + 3y - 2y - 1 = 0$

$3y(2y + 1) - 1(2y + 1) = 0$

$3y - 1 = 0 \qquad 2y + 1 = 0$

$y = \frac{1}{3} \qquad y = \frac{-1}{2}$

$\log_{10} x = -\frac{1}{2}$ and $\log_{10} x = \frac{1}{3}$

$x = 10^{-\frac{1}{2}}$ or $x = 10^{\frac{1}{3}}$

$x = \frac{1}{10^{1/2}}$ or $x = 10^{1/3}$

So, $\alpha = \frac{1}{10^{1/2}} \quad \beta = 10^{1/3}$

Now, $\alpha^2\beta^3 + 1 = \left(\frac{1}{10^{1/2}}\right)^2 \cdot (10^{1/3})^3 + 1$

$\Rightarrow \frac{1}{10} \times 10 + 1$

$\Rightarrow 1 + 1 = 2$

$\therefore (\alpha^2\beta^3 + 1)^2 = 2^2 = 4$

again $\alpha^4 = 10^{-2}$

$\Rightarrow \alpha^4 = \frac{1}{100}$ or $\frac{1}{\alpha^4} = 100$

Now, $\left[\frac{(\alpha^2\beta^3 + 1)^2}{\alpha^4} - 30000\,\alpha^4\right]$

$\Rightarrow \frac{1}{10}\left[(2)^2 100 - 30000 \times \frac{1}{100}\right]$

$\Rightarrow \frac{1}{10}[400 - 300]$

$\Rightarrow \frac{100}{10} = 10$

TEST Edge Interval in which the roots lie and quadratic in equations related question are asked. To solve such type of questions, students are advised to understand the concept of quadratic equation such as, let $f(x) = ax^2 + bx + c$, where $a, b, c \in R$ and $a \neq 0$. Suppose $N, N_1, N_2 \in R$ and $N_1 < N_2$, then interval N_1, N_2 will be contained between the roots of $f(x) = 0$ if a $f(N_1) < 0$, $a\ f(N_2) < 0$, $a\ f(N_2) < 0$

60. (*1*) **Idea** $\log_b a \frac{\log_c a}{\log_c b}$

$y = f(x) = \log_a x$ is defined when $x > 0$ an $a > 0$. Also apply the concept of factorisation, to solve this problem.

Now, $4^{\log_2 \log_e x} = 4^{\frac{\log_4 (\log_e x)}{\log_4 2}}$ (using $\log_b a = \frac{\log_c a}{\log_c b}$)

$= 4^{\frac{\log_4 (\log_e x)}{\log_2 2^2}} \quad \left(\because \log_a m^b = \frac{1}{m}\log_a b\right)$

$= 4^{\log_4 (\log_e x)^2}$

$= 4^{\log_4 (\log_e x)^2} = (\log_e x)^2$

$\therefore$ The equation is

$(\log_e x)^2 = \log_e x - (\log_e x)^2 + 1$

$2t^2 - t - 1 = 0 \qquad [\text{where}, t = \log_e x]$

$\Rightarrow t = 1, -\frac{1}{2}$

$\log_e x = 1$

$\Rightarrow x = e$

But $t = -\frac{1}{2}$

$\Rightarrow \log_e x = -\frac{1}{2}$

Then, $\log_2 (\log_e x)$ is not defined.

$\therefore$ There is only one real solution.

TEST Edge Solution of algebraic function, exponential functions, trigonometric function related questions are asked. To solve such type of questions, students are advised to understand the concept of solution of the function and also acquainted yourself with the properties of function.

e.g., Find the number of integral solution of the inequality $\frac{(x-1)^2(x-4)^3(x-7)}{(x-2)(x-6)} \leq 0$.

To solve this problem, use the concept of curve method and will get the answer *i.e.*, $f(x) \leq 0$ for $x = 1, 2, 3, 4$