

Test **RIDER**

JEE Advanced

A Single Door Entry to the Various IITs...

Paper 1

Duration : 3 Hours

Max. Marks : 180

Please read the instructions carefully. You are allotted 5 minutes specially for this purpose.

- ▶ This booklet is your question paper. Attempt all the questions.
- ▶ Blank papers, clipboards, log tables, slide rules, calculators, cameras, cellular phones, pagers and electronic gadgets are not allowed.
- ▶ Write your name and roll number in the space provided on the bottom of this page.

Question Paper Format

- ▶ The question paper consists of three parts (Physics, Chemistry and Mathematics). Each part consists of three sections.
- ▶ **Section 1** contains 10 multiple choice questions. Each question has four choices (a), (b), (c) and (d) out of which only one is correct.
- ▶ **Section 2** contains 5 multiple choice questions. Each question has four choices (a), (b), (c) and (d) out of which one or more than one is/are correct.
- ▶ **Section 3** contains 5 questions. The answer to each question is a single-digit integer, ranging from 0 to 9 (both inclusive)

Marking Scheme

- ▶ For each question in Section 1, you will be awarded **2 marks** for correct answer and zero mark for unattempted. No negative marks will be awarded for incorrect answers in this section.
- ▶ For each question in Section 2, you will be awarded **4 marks** for correct answer(s) and zero mark for unattempted questions. In all other cases, minus one (–1) mark will be awarded.
- ▶ For each question in Section 3, you will be awarded **4 marks** for the correct answer and zero mark for unattempted questions. In all other cases, minus one (–1) mark will be awarded.

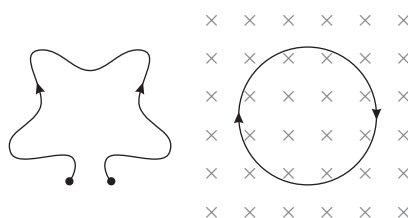
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Part I

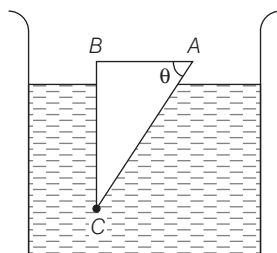
Section 1 Single Correct Option Type

This section contains **10 multiple choice questions**. Each question has four choices, (a), (b), (c) and (d) out of which **only one** is correct.

1. A thin flexible wire of length L is connected to two fixed points and carries a current I in the clockwise direction as shown in the figure. When the system is put in a uniform magnetic field of strength B going into the plane of the paper, the wire takes the shape of a circle. The tension in the wire is



- a. $\frac{IBL}{16\pi}$ b. $\frac{IBL}{\pi}$ c. $\frac{IBL}{2\pi}$ d. $\frac{IBL}{4\pi}$
2. A glass prism of refractive index 1.5 is immersed in water (refractive index is $4/3$). A light beam incident normally on the face AB is totally reflected to reach the face BC if



- a. $\sin\theta > 8/9$ b. $2/3 < \sin\theta < 8/9$
 c. $\sin\theta < 2/8$ d. None of these
3. The variation of induced emf (e) with time (t) in a coil if a short bar magnet is moved along its axis with a constant velocity is best represented as

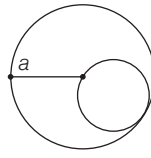


- a. b. c. d.

10. Two equal point charges are fixed at $x = -a$ and $x = +a$ on the x -axis. Another point charge Q is placed at origin. The change in electrostatic potential energy of the system when Q is displaced by a small distance x along the x -axis is approximately proportional to
- a. x b. x^2 c. x^3 d. $\frac{1}{x}$

Section 2 More Than One Correct Option

This section contains 5 multiple choice questions. Each question has four choices, (a), (b), (c) and (d) out of which one or more than one is/are correct.

11. If the tension in a stretched string fixed at both ends is changed by 20%, the fundamental frequency is found to increase by 15 Hz. Then, the
- a. original frequency is 157 Hz
 b. velocity of propagation of transverse wave along the string changes by 5%
 c. velocity of propagation of transverse wave along string changes by 10%
 d. fundamental wavelength of string does not change
12. A point isotropic source of sound power 1 mW emits sound of frequency 170 Hz in all directions. Velocity of sound is 340 m/s, then
- a. intensity at any point at distance r depends on $1/r^2$.
 b. at $r = \sqrt{\frac{250}{\pi}}$ m, the loudness of source is 60 dB
 c. amplitude of sound wave depends on $1/\sqrt{r}$
 d. amplitude of oscillation of a point at 4 m is 2A and at 55 m it is $-\frac{8A}{55}$ at $t = 0$
13. A spherical hole is made in a solid sphere of radius. The mass of the sphere before hollowing was M_0 . The gravitational field at the centre of the hole due to the remaining mass is
- a. $\frac{GM_0}{10a^2}$ b. zero c. $\frac{GM_0}{a^2}$ d. $\frac{GM_0}{2a^2}$
- 
14. A golf club hit a golf ball and that collision last for only Δt . Mass of golf ball is 45 g and maximum range corresponding to given shot is 160 m. [Take $g = 10 \text{ m/s}^2$] [Distance covered by ball during collision is 2 cm].
- a. impulse imparted by golf club is 1.8 N-s
 b. average force applied is 1800 N
 c. estimated time of collision is 0.001 s
 d. average acceleration of ball during collision is 40000 m/s^2
15. A long straight wire along the z -axis carries a current i in the negative z -direction. The magnetic field \mathbf{B} at a point having coordinates (x, y) on the $z = 0$ plane is
- a. $\frac{\mu_0 i (x\hat{i} - y\hat{j})}{2\pi(x^2 + y^2)}$ b. $\frac{\mu_0 i (x\hat{j} - y\hat{i})}{2\pi(x^2 + y^2)}$ c. $\frac{\mu_0 i (x\hat{i} + y\hat{j})}{2\pi(x^2 + y^2)}$ d. $\frac{\mu_0 i (y\hat{i} - x\hat{j})}{2\pi(x^2 + y^2)}$

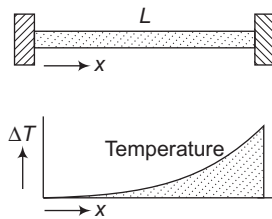
Section 3 Integer Answer Type

This section contains 5 multiple choice questions. The answer to each question is a single-digit integer, ranging from 0 to 9 (both inclusive).

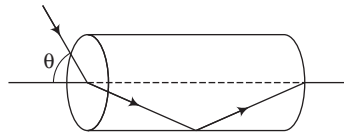
- 16.** A spring of force constant $k = 300 \text{ N/m}$ connects two blocks having masses 2 kg and 3 kg , lying on a smooth horizontal plane. If the spring block system is released from a stretched position, the number of complete oscillations in 1 min is $6n$. Find the value of n . Take $\pi = \sqrt{10}$.
- 17.** A steel bar of length L is held between rigid supports and heated non-uniformly in such a manner that the temperature increases ΔT at distance x from one end is given by $\Delta T = T_0 x^2 / L^2$ as shown in the figure.

The stress in the bar, (assume that modulus of elasticity for steel is E and thermal expansion coefficient is α) is given as $\sigma = \frac{E \alpha T_0}{x}$. Find the value of x .

- 18.** When the voltage applied to an X-ray tube is increased from 10 kV to 20 kV , the wavelength interval between the K_α line and the short wave cut off of the continuous X-ray spectrum increases by a factor 3. Find the atomic number of element of which the tube anode is made. Fill second digit (tenth) of your answer.



- 19.** Light incidence at an angle θ w.r.t. the axis on one plane end, of a transparent cylindrical fiber of refractive index $n = \sqrt{1.25}$. Determine the maximum value of θ , so that the light entering the rod does not come out of curved surface. Answer the value $\theta/5$.



- 20.** A parallel beam of nitrogen molecules moving with velocity $v = 400 \text{ m/s}$ impinges on a wall at an angle $\theta = 30^\circ$ to its normal. The concentration of molecules in the beam $n = 0.9 \times 10^{19} \text{ cm}^{-3}$. Find the pressure exerted by the beam on the wall assuming the molecules to scatter in accordance with the perfectly elastic collision law.

Part II

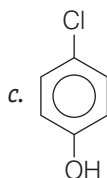
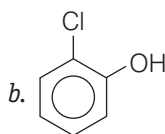
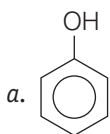
Section 1 Single Correct Option Type

This section contains **10 multiple choice questions**. Each question has four choices, (a), (b), (c) and (d) out of which **only one** is correct.

21. Among the hydroxides of period 3 elements, the element whose oxide is amphoteric has value of quantum numbers are

a. $n = 3, l = 1, s = 1/2$ b. $n = 3, l = 1, s = 3/2$ c. $n = 3, l = 1, s = 1$ d. $n = 3, l = 0, s = 1/2$

22. An aromatic organic compound A having molecular mass 112.5 containing C, H and chlorine only, have 4 degree of unsaturation what will be the product when A undergoes reaction with sodium hydroxide?



d. None of these

23. x mole KIO_3 is treated with excess of KI , liberated I_2 which was dissolved in freshly prepared starch solution. Which was neutralised by 60 mL 0.1 N $\text{Na}_2\text{S}_2\text{O}_3$ until white ppt. was obtained. What is x ?

a. 10^{-3} mol b. 10^{-5} mol c. 5 mol d. 6 mol

24. Some oxides of p -block elements show colour while some do not among following, which of the following pair of oxides are coloured?

a. NO_2 and N_2O_3 b. NO_2 and N_2O_5 c. N_2O_4 and NO_2 d. NO and N_2O

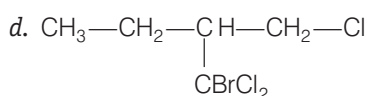
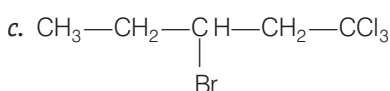
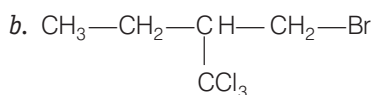
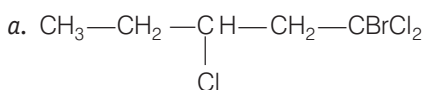
25. 1 mol of an ideal gas A ($C_V = 3R$) and 2 moles of an ideal gas B ($C_V = \frac{3}{2}R$) taken in a container and expanded reversibly and adiabatically from 1 L to 4 L starting from initial temperature of 320 K.

ΔE for the process is

a. $-240R$ b. $240R$ c. $480R$ d. $-960R$

26. $\text{CH}_3\text{—CH}_2\text{—CH=CH}_2 + \text{CBrCl}_3 \xrightarrow{(\text{C}_6\text{H}_5\text{CO})_2\text{O}_2} M$

M will be



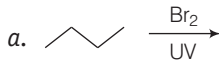
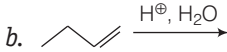
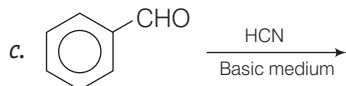
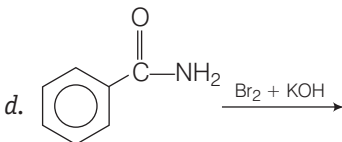
27. What is the magnetic moment of coordination compound formed during brown ring test?

a. 3.87 BM b. 4.92 BM c. 5.92 BM d. 2.83 BM

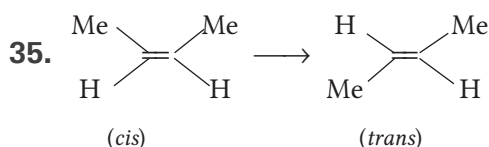
28. Which of the following will produce a stable non-planar carbocation?
- Addition of antimony pentafluoride to cycloheptatrienyl fluoride
 - Addition of bromonium ion to propene
 - Addition to acid to propene
 - Addition of argenticum ion to triphenylmethyl chloride
29. Which of the following is optically inactive?
- Trioxalato chromate (III)
 - Trans*-dichlorobis ethylenediamine platinum (II) chloride
 - Trans*-diamine dichlorobis ethylenediamine chromium (I)
 - Both a and c.
30. Derived name of iso-octane is
- dimethyl n-pentyl methane
 - iso-butyl trimethyl methane
 - trimethyl iso-butyl methane
 - iso-propyl trimethyl methane

Section 2 More Than One Correct Option

This section contains 5 multiple choice questions. Each question has four choices, (a), (b), (c) and (d) out of which only one or more than one is/are correct.

31. Decomposition of $3A(g) \longrightarrow 2B(g) + 2C(g)$ follows first order kinetics. Initially, only A is present in the container pressure developed after 20 min and infinite time are 3.5 atm and 4 atm respectively. Which of the following is true?
- $t_{50\%} = 20$ min
 - $t_{75\%} = 40$ min
 - $t_{99\%} = 64/3$ min
 - $t_{87.5\%} = 60$ min
32. Choose that which of the following statement (s) is/are correct regarding given equations written below?
- $\text{FeO} + \text{SiO}_2 \longrightarrow \text{Fe}_2\text{SiO}_3$
 - $\text{ZnO} + \text{C} \longrightarrow \text{Zn} + \text{CO}$
 - $2\text{Al}_2\text{O}_3 + 3\text{C} \longrightarrow 4\text{Al} + 3\text{CO}_2$
- A represents formation of slag and B represents reduction of zinc
 - A represents formation of flux and B represents reduction of zinc
 - A represents formation of slag and C represents Hall Heroult process
 - B represents reduction of zinc and C represents Hall Heroult process
33. Which of the following reactions complete with racemisation?
- 
 - 
 - 
 - 

34. Out of the following the correct statement(s) is/are
- the structure of diamond and corundum is same.
 - a mixture of 5-10% CO₂ and O₂ is known as carbogen which is used for artificial respiration in pneumonia patient.
 - SnCl₂ is a strong oxidising agent.
 - PbO is a yellow orange coloured powder commonly known as litharge.



Correct sequence of reagents used for above conversion are

- (i) H[⊕], H₂O (ii) H[⊕]/Δ
- (i) Ni (ii) Br₂/H (ii) also KOH Δ
- (i) Br₂/CCl₄ (ii) 2NaNH₂, Δ (iii) liq. NH₃, Na
- (i) Br₂/CCl₄ (ii) NaI + Acetone, Δ

Section 3 Integer Answer Type

This section contains 5 multiple choice questions. The answer to each question is a single-digit integer, ranging from 0 to 9 (both inclusive)

36. Consider the equation $A(g) + 2B(g) \rightleftharpoons C(g)$ when the reaction was carried out at 120°C, the equilibrium concentrations of A and B were 3 M and 4 M respectively. When the volume of the vessel was doubled and system is allowed to reach equilibrium, the concentration of B was found to be 3 M. The original concentration of C will be
37. Electrons in a sample of H-atom make transition from state $n = x$ to some lower excited state. The emission spectrum from the sample is found to contain only the lines belonging to a particular series. If one of the photons had energy of 0.6375 eV. Then, find the value of x .
(Take $0.6375 \text{ eV} = \frac{3}{4} \times 0.85 \text{ eV}$)
38. What will be the change in oxidation state during transformation of KMnO₄ to manganese dioxide?
39. Oxidation state of Ti in Ziegler Natta catalyst is
40. In the acid-base titration [H₃PO₄(0.1M) + NaOH(0.1 M)] emf of the solution is measured by coupling this electrodes with suitable reference electrodes. When alkali is added, pH of the solution is in accordance with equation.

$$E_{\text{cell}}^{\circ} = E_{\text{cell}}^{\circ} \times 0.059 \text{ pH}$$

For H₃PO₄ $K_1 = 10^{-3}$, $K_2 = 10^{-8}$, $K_3 = 10^{-13}$

What is the cell emf at the 2nd end point of titration if E_{cell}° at the this stage is 1.3805 V?

48. If $|\sin^2 x + 26 - x^2| = \cos^2 x + 2 \sin^2 x + |25 - x^2|$, then x lies in
 a. $[-6, 6]$ b. $[-7, 7]$ c. $[-5, 5]$ d. None of these
49. If three squares are selected at random from chessboard, then the probability that they form the letter L is
 a. ${}^{49/64}C_3$ b. ${}^{196/64}C_3$ c. ${}^{36/64}C_3$ d. ${}^{98/64}C_3$
50. $x(x+1)\frac{dy}{dx} + (x+1)y = \frac{f(xy)}{f'(xy)}$, where $x \neq -1$, then $f(xy)$ is equal to
 a. $f(xy) = c(x-y)$ b. $f(xy) = \frac{cx}{y}$ c. $f(xy) = c(x+1)$ d. $f(xy) = c(x-1)$

Section 2 More Than One Correct Option

This section contains 5 multiple choice questions. Each question has four choices, (a), (b), (c) and (d) out of which only one or more than one is/are correct.

51. Let \mathbf{u} and \mathbf{v} be unit vectors inclined at an angle θ such that for some vector \mathbf{w} ?
 $\mathbf{w} + \mathbf{w} \times \mathbf{u} = \mathbf{v}$
 If $\beta = [\mathbf{u} \ \mathbf{v} \ \mathbf{w}]$ then,
 a. $\mathbf{v} \cdot \mathbf{w} = \cos \theta$ b. $\beta = 1/2 \sin^2 \theta$ c. $\mathbf{u} \cdot \mathbf{w} = \cos \theta$ d. $\mathbf{u} \cdot \mathbf{w} = 1 - \beta$
52. A straight line through the vertex P of a triangle PQR intersect the side QR at the point S and the circumcircle of the triangle PQR at point T . If S is not the centre of the circumcircle, then
 a. $\frac{1}{PS} + \frac{1}{ST} < \frac{2}{\sqrt{QS \times SR}}$ b. $\frac{1}{PS} + \frac{1}{ST} > \frac{2}{\sqrt{QS \times SR}}$
 c. $\frac{1}{PS} + \frac{1}{ST} < \frac{4}{QR}$ d. $\frac{1}{PS} + \frac{1}{ST} > \frac{4}{QR}$
53. Let $f: R \rightarrow R$ be a function defined by $f(x+1) = \frac{f(x)-5}{f(x)-3} \forall x \in R$. Then, which of the following statement (s) is/are true?
 a. $f(2008) = f(2004)$ b. $f(2006) = f(2010)$ c. $f(2006) = f(2002)$ d. $f(2006) = f(2018)$
54. Two sides of rhombus $OABC$ (lying entirely in first quadrant or third quadrant) are $y = \frac{x}{\sqrt{3}}$ and $y = \sqrt{3}x$. If the area of rhombus $OABC$ is equal to 2sq units. Then, the possible coordinates of B is/are (where O being the origin)
 a. $(1 + \sqrt{3}, 1 + \sqrt{3})$ b. $(-1 - \sqrt{3}, -1 - \sqrt{3})$
 c. $(\sqrt{3} - 1, \sqrt{3} - 1)$ d. None of these
55. CF is the internal bisector of angle C of ΔABC , then CF is equal to
 a. $\frac{2ab}{a+b} \cos(C/2)$ b. $\frac{a+b}{2ab} \cos(C/2)$
 c. $\frac{b \sin A}{\sin(B+C/2)}$ d. None of these

Section 3 Integer Answer Type

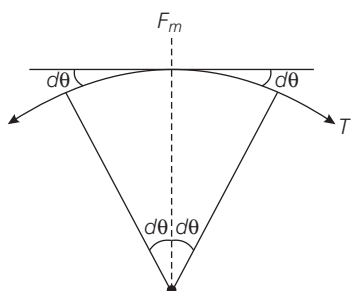
This section contains 5 multiple choice questions. The answer to each question is a single-digit integer, ranging from 0 to 9 (both inclusive)

56. Find the value of $\int_0^2 (x^2 + 1) d[x]$, where $[x]$ represent Gf
57. If the triangle ABC whose vertices are $A(-1, 1, 1)$ $B(1, -1, 1)$ and $C(1, 1, -1)$ is projected on xy -plane. Then, the area of the projected triangle is
58. The locus of the centre of a variable circle touching two circles of radius r_1 and r_2 externally, which also touch each other externally is a conic. If $\frac{r_1}{r_2} = \frac{7}{5}$ and the eccentricity of the conic is k , then k is equal to
59. Let $\mathbf{a}, \mathbf{b}, \mathbf{c}$ be three unit vectors such that \mathbf{a} is perpendicular to the plane of \mathbf{b} and \mathbf{c} . If the angle between \mathbf{b} and \mathbf{c} is $\pi/3$ then, $|\mathbf{a} \times \mathbf{b} - \mathbf{a} \times \mathbf{c}| =$
60. Number of solutions of the equation $|\sin x \cos x| + \sqrt{2 + \tan^2 x + \cot^2 x} = \sqrt{3}$, $x \in [0, 6\pi]$ are

Analytical Explanations

1. (b) **Idea** When a current carrying wire is placed in a magnetic force $I(\mathbf{L} \times \mathbf{B})$ will act on it. Where I is current, L is length and B is magnetic field. By symmetry, in this question the shape will be circular.

Consider a small arc as below



Clearly, net force $2T \sin(d\theta)$ will be by magnetic force on the wire hence,

$$2T \sin(d\theta) = F_m = I dLB \sin 90^\circ$$

$$2T(d\theta) \approx I dLB$$

$$T = \frac{IdLB}{2d\theta} = IB$$

$$= IBR = IB \times \frac{L}{2\pi} \quad (\because L = 2\pi R)$$

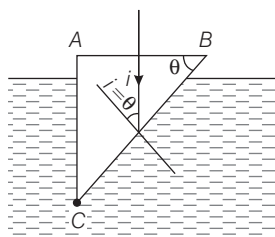
$$T = \frac{IBL}{2\pi}$$

TEST Edge Questions are framed very often in JEE Advanced, on this concept wire could be of any shape. The wire may be moving also. The points should be kept in mind.

- The net force on the circular wire will be zero.
- To find tension we have to consider a small arc element.

2. (b) **Idea** The problem is based on Snell's law and concept of TIR is used. As the ray incident normal to the face AB , hence the ray will pass undeviated and will incident on side BC as shown in figure and there total internal reflection will take place.

Consider the diagram below



Clearly, the angle of incidence at the face BC will also be θ as shown in above diagram.

Now for total internal reflection

$$i > \theta_c$$

$$\sin \theta > \sin \theta_c$$

$$\sin \theta > \frac{\mu_w}{\mu_g} \quad (\because \mu_g \sin \theta = \mu_w \sin 90^\circ)$$

$$\sin \theta > \frac{4/3}{3/2} = \frac{8}{9}$$

TEST Edge In every year, questions are asked in JEE Advanced based on this concept. Variety of questions can be framed by varying the angle of incidence of light as well as varying refractive indices.

- Note**
- For total internal reflection to take place light ray must travel from denser to rarer medium.
 - At just critical angle, refractive angle will be 90°
 - At angle greater than critical angle then total internal reflection will take place.

3. (d) **Idea** The problem is asked about the bar magnet is moved towards the coil then flux linked with the coil is changing, hence emf will be induced according to Faraday's law.

$$E_{\text{ind}} = \frac{-d\phi}{dt}$$

When the magnet is leaving the coil then flux decreases hence current flow tend to flow in opposite direction.

When the bar magnet is entering and leaving the coil, the polarity of emf should be opposite in two cases.

Only in option (b) polarity is different.

TEST Edge Questions are framed in JEE Advanced by changing the magnetic flux by different modes like by rotation, by translation, combination of movement of both coil and magnet or by the variation of the field. *The points should be kept in mind*

- Variation of flux (by any means) is necessary for the emf to be induced.
- For induced current circuit must not be incomplete.

4. (d) **Idea** To solve this problem we have to consider the field at axial point of the ring i.e.,

$$E = \frac{-Gmx}{(R^2 + x^2)^{3/2}}, \text{ field will be directed}$$

towards the centre of the ring.

We will treat sphere as a point mass placed at its centre.

For all point out the sphere we can treat it as point mass at its centre so, effectively it will be the force between ring and point mass.

Gravitation field at $x = \sqrt{3}a$ on axis of ring is

$$E = -\frac{Gm x}{(R^2 + x^2)^{3/2}} = -\frac{Gm(\sqrt{3}a)}{[a^2 + 3a^2]^{3/2}}$$

$$= -\frac{\sqrt{3} Gm}{8a^2}$$

$$F = ME = -\frac{\sqrt{3}GMm}{8a^2}$$

TEST Edge Questions can be framed by using different configurations. In place of ring it can be other shaped objects like shell, disc etc. *The points should be kept in mind*

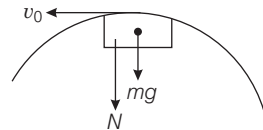
- (i) We should calculate the field at axial points.
- (ii) We should always be aware about symmetry.

5. (b) **Idea** To solve these type of questions, the concept of circular motion will be used. We will draw the FBD of the system and then we will write equation according to net centripetal force. At the highest point we will have

$$N + mg = \frac{mv^2}{R}$$

As the block rises to same height in all cases, hence speed of the highest point will be same at all points according to conservation of energy.

We can write



$$N + mg = \frac{mv_0^2}{R}$$

$$N = \frac{mv_0^2}{R} - mg$$

$$N_{\max} = \frac{mv_0^2}{R_{\min}} - mg$$

R is minimum for first case hence (b) is correct.

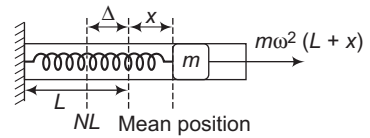
TEST Edge These types of questions are generally asked in JEE Advanced IJ-JEE. Questions could be framed by varying the radius of curvature of the track. *The points should be kept in mind*

- (i) We should be careful in applying the direction of normal force acting on the system.
- (ii) Sometimes the block will leave the track in that case we have to put $N = 0$.

6. (b) **Idea** We will use the concept of centripetal force to solve the question. The required force for the circular motion will be provided by the spring *i.e.*, when elongation in the spring is x then,

$$kx = m\omega^2 L$$

The free body diagram of system at x displacement from mean position.



At mean position,

$$m\omega^2 L = k\Delta$$

At x displacement,

$$m\omega^2(L+x) = k(\Delta+x) - m\omega^2(L+x)$$

So,
$$\omega_n = \sqrt{\frac{k}{m} - \omega^2}$$

TEST Edge Question can be asked by changing the course of motion of the system. We can use combination of springs also. We should be sure about the centripetal force which is providing the circulatory motion.

7. (b) **Idea** We have to calculate the image formed by the concave lens. The image so formed will act as object for the convex lens. Then, we will apply the lens formula

$$\frac{1}{v} - \frac{1}{u} = \frac{1}{f}$$

We can also use the formula for focal length of combination of lens

$$\frac{1}{f_{eq}} = \frac{1}{f_1} + \frac{1}{f_2} + \dots$$

Image formation from 1st lens

$$\frac{1}{v} - \frac{1}{u} = \frac{1}{f}$$

$$\frac{1}{v} + \frac{1}{80} = \frac{1}{-40}$$

$$\Rightarrow v = \frac{-80}{3} \text{ cm}$$

For second lens,

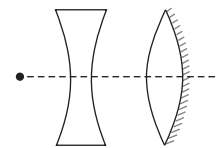
$$\frac{1}{v} - \frac{1}{u} = \frac{1}{f}$$

$$\frac{1}{v} - \frac{1}{-80/3} = \frac{1}{40}$$

$$v = -80 \text{ cm}$$

TEST Edge Different questions can be formed by changing the configuration of the lens system.

i.e., we can use convex-concave, one lens can also be polished



The points should be kept in mind

- (i) Image formed by first lens or mirror will be object for the second lens or mirror
- (ii) Paraxial rays will be considered.

8. (c) **Idea** To solve these types of questions we have to apply equation for energy of photon

$$i.e., \quad E = hv = \frac{hc}{\lambda}$$

$$\lambda_{\min} = \frac{hc}{eV}$$

Wavelength λ_k is independent of the accelerating voltage (V), while the minimum wavelength λ_c is inversely proportional to v . Therefore as v is increased, λ_k remains unchanged whereas λ_c decreases or $\lambda_k - \lambda_c$ will increase.

TEST Edge Questions could be framed by varying the potential of the used tube. If potential changes then accordingly λ will vary and different types of questions can be formed.

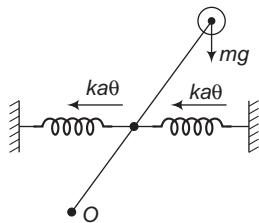
- (i) We should be aware about the wavelength and the potential applied.
- (ii) According to the frequency observed we will decide the type like K_{α_1} K_{β_1} K_{γ} ...

9. (c) **Idea** It is based on standing waves on a string fixed at both ends and to calculate velocity, fundamental wavelength of string.

$$i.e., \quad f = \frac{\pi}{2L} \sqrt{\frac{F}{\mu}}$$

where f is frequency of standing wave and μ is mass per unit length.

Free body diagram at angle θ (small)



So, restoring torque about O

$$\tau_R = ka^2\theta + ka^2\theta - mgL\theta$$

$$= (2ka^2 - mgL)\theta$$

For unstable system

$$2ka^2 - mgL \leq 0$$

because then torque will not be restoring

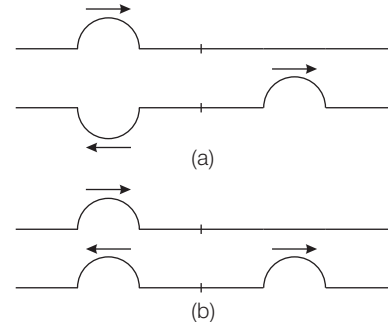
$$So, \quad 2ka^2 = mgL$$

$$\frac{L}{a^2} = \frac{2k}{mg}$$

TEST Edge This idea is important according to JEE Advanced point of view. Students should focus on this concept and relate to laws of vibration of stretched string.

If a wave enter a region where the wave velocity is smaller, the reflected wave is inverted as shown in Fig. (a). If it enters a region where the wave velocity is larger, then the reflected wave is not inverted.

The transmitted wave is never inverted as shown in Fig. (b)



10. (b) **Idea** This problem is based on electrostatic potential energy *i.e.*, amount of work done in bringing the unit positive charge from one point to the other against the electric field.

$$i.e., \quad V_B - V_A = \frac{W_{AB}}{q_0}$$

$$U_i = \frac{1}{4\pi\epsilon_0} \frac{2Qq}{a} + \frac{q^2}{4\pi\epsilon_0 \times 2a}$$

$$U_f = \frac{1}{4\pi\epsilon_0} \frac{Qq}{a-x} + \frac{1}{4\pi\epsilon_0} \frac{Qq}{a+x} + \frac{q^2}{4\pi\epsilon_0 \times 2a}$$

$$= \frac{1}{4\pi\epsilon_0} \frac{2Qqa}{a^2 - x^2} + \frac{q^2}{4\pi\epsilon_0 \times 2a}$$

$$= \frac{2qQ}{4\pi\epsilon_0 a} \left(1 - \frac{x^2}{a^2}\right)^{-1} + \frac{q^2}{4\pi\epsilon_0 \times 2a}$$

$$\therefore \Delta U = U_f - U_i = \left(\frac{2qQ}{4\pi\epsilon_0 a}\right) x^2$$

TEST Edge This concept is important according to JEE Advanced. In every year, one question is asked from this topic. So, student should concentrate on this idea and relate to electric dipole, torque and potential energy of a dipole placed in a uniform electric field.

Just as the electric field due to a collection of point charges is the vector sum of the fields produced by each charge, the electric potential due to a collection of point charge is the scalar sum of the potential due to each charge.

$$i.e., \quad V = \frac{1}{4\pi\epsilon_0} \sum_i \frac{q_i}{r_i}$$

11. (a,c,d) **Idea** As we know, this problem is based on standing wave on a string. We have to calculate frequency, wavelength and velocity of propagation of transverse wave along the string end.

General equations for standing wave on string are

$$f = \frac{n}{2l} \sqrt{\frac{T}{\mu}} \Rightarrow L = n \frac{\lambda}{2}$$

where, n = mode of vibration,

For fundamental mode $n = 1$

$$L = 1 \times \frac{\lambda}{2},$$

Now that is clear that λ depend on length that's why λ will not change with change in tension.

$$f = \frac{l}{2L} \sqrt{\frac{T}{\mu}} \quad \text{i.e., } f \propto \sqrt{T}$$

$$T_1 = T + \frac{20}{100} T$$

$$\frac{f_1}{f} = \sqrt{\frac{T_1}{T}} = \sqrt{\frac{T \left(1 + \frac{20}{100}\right)}{T}} = \left(1 + \frac{1}{5}\right)^{1/2}$$

$$\frac{f_1}{f} = 1.0954$$

$$\therefore f = \frac{15}{0.0954} = 157.23 \text{ Hz}$$


\therefore Velocity of propagation

$$\frac{v_1}{v} = \sqrt{\frac{T_1}{T}} = \sqrt{\frac{T(1+1/5)}{T}} = \sqrt{1 + \frac{1}{5}} = 1 + \frac{1}{10}$$

So, 10% increment in velocity.

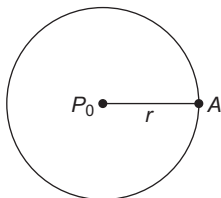
TEST Edge This idea is important according to JEE Advanced. In every year, this concept is asked and students should relate with stationary waves in air column (i.e., open and closed organ pipes) and Doppler's effect.

Note We know that $v_1 = 2v_0$ is the first overtone, $v_2 = 3v_0$ is the second overtone. Thus, for a string fixed at both the ends, all the overtones are harmonics of the fundamental frequency and all the harmonics of the fundamental frequency are overtones. Above property is unique to the string and make it so valuable in musical instruments such as **violin, sitar and sarod**.

12. (a,b,d)  **Idea** This problem is based on wave motion, intensity and loudness of a wave.

$$\text{i.e., } L = 10 \log_{10} \frac{I}{I_0}$$

where, I_0 is the intensity of minimum audible sound which is 10^{-12} Wm^{-2} .



For point source, sound will spread in the spherical region. So, intensity at any point A is

$$I = \frac{P_0}{4\pi r^2} \frac{W}{\text{m}^2}$$

$$\text{So, } I \propto \frac{1}{r^2}.$$

Loudness of sound in decible will be

$$L \text{ (dB)} = 10 \log_{10} \left(\frac{I}{I_0} \right)$$

[where, I_0 = reference intensity]

$$60 = 10 \log \left(\frac{I}{I_0} \right) \Rightarrow I = I_0 \times 10^6$$

$$I = 10^{-12} \times 10^6 = 10^{-6} \text{ W/m}^2 \quad [I_0 = 10^{-12} \text{ W/m}^2]$$

$$\frac{P}{4\pi r^2} = I = 10^{-6}$$

$$r^2 = \frac{P}{4\pi \times 10^{-6}} = \frac{1 \times 10^{-3}}{4 \times 3.14 \times 10^{-6}} = \frac{250}{\pi}$$

$$r = \sqrt{\frac{250}{\pi}} \text{ m,}$$

$$I \propto A^2 \propto \frac{1}{r^2}, \text{ so } A \propto \frac{1}{r}$$

At a given time phase difference between 2 points.

$$\Delta\theta = \frac{2\pi}{\lambda} \Delta x \quad \left[\lambda = \frac{v}{f} \right]$$

$$\lambda = \frac{340}{170} = 2 \text{ m}$$

$$\Delta\theta = \frac{2\pi}{2} \times (55 - 4) = 51\pi$$

$$\Delta\theta = 25(2\pi) + \pi$$

i.e., when both points will oscillates in opposite phase.

$$A \propto \frac{1}{r} \text{ so}$$

$$\frac{A_1}{A_2} = \frac{r_2}{r_1}, \quad \frac{2A}{A_2} = \frac{55}{4}, \quad A_2 = \frac{8A}{55}$$

\therefore Due to opposite phase that would be written as $\frac{8A}{-55}$

TEST Edge In every year atleast one question is asked in JEE Advanced. IIT-JEE students should concentrate on this idea and interrelate with sound waves, speed of sound in a gas, and intensity of sound waves.


Note The intensity of sound waves is given by

$$I = \frac{p_{\text{max}}^2}{2\rho v}$$

where, p_{max} is the maximum change of pressure in the medium.

The intensity of waves emitting in all directions due to point source

$$\text{i.e., } I \propto \frac{1}{r^2}$$

13. (a,b,c,d)  **Idea** In solving these types of questions, we have to find the electric fields individually due to both parts then we will calculate the net field by principle of superposition.

$$\text{i.e., } \mathbf{E}_{\text{net}} = \mathbf{E}_1 + \mathbf{E}_2 + \dots$$

By the principle of superposition of fields

$$\mathbf{E} = \mathbf{E}_1 + \mathbf{E}_2$$

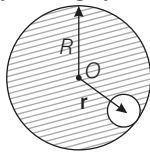
where, \mathbf{E} = net field

\mathbf{E}_1 = field due to remaining mass

\mathbf{E}_2 = field due to mass in hole = v

$$\mathbf{E}_1 = E = \left(\frac{GM_0}{a^3}\right)r = \left(\frac{GM_0}{a^3}\right)\frac{a}{2} = \frac{GM_0}{2a^2}$$

TEST Edge Questions can be asked by forming hole of different shape. The variety in question can also be improved by taking system of varying density.



Consider the situation where in a solid sphere of radius R a hole is made at position r as shown.

That the field will be as follows

$$E_{\text{inside}} \propto r \Rightarrow E_{\text{outside}} \propto \frac{1}{r^2}$$

14. (a,b,c,d) **Idea** It is based on the concept of elastic collision, of a body and to calculate average force and impulse imparted by particle.

$$R_{\text{max}} = \frac{v^2}{g}, v^2 = Rg = 160 \times 10, v = 40 \text{ m/s}$$

$$(a) I = \Delta P = mv = (45 \times 10^{-3}) \times (40) = 1.8 \text{ Ns}$$

$$(b) v^2 = 2 a_{\text{av}} x, a_{\text{av}} = \frac{v^2}{2x} = \frac{(40)^2}{2 \times 2 \times 10^{-2}} = 40000 \text{ m/s}^2$$

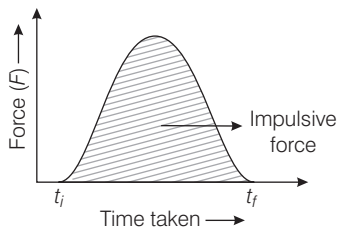
$$F_{\text{av}} = m a_{\text{av}} = 45 \times 10^{-3} \times 40000 = 1800 \text{ N}$$

$$(c) \Delta t = \frac{I}{F_{\text{av}}} = \frac{1.8}{1800} = 0.001 \text{ s}$$

TEST Edge As we know, when two bodies collide, they exert force on each other while in contact such that large forces acting for a very short duration are called impulsive forces.

The change in momentum produced by such an impulsive force is

$$\mathbf{p}_f - \mathbf{p}_i = \int_{P_i}^{P_f} d\mathbf{p} = \int_{t_i}^{t_f} \frac{d\mathbf{p}}{dt} dt = \int_{t_i}^{t_f} \mathbf{F} dt$$

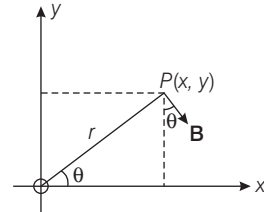


This quantity $\int_{t_i}^{t_f} \mathbf{F} dt$ is known as the impulse of the force \mathbf{F} during the time interval t_i to t_f .

15. (a,b,d) **Idea** When a wire is carrying a current there will be magnetic field surrounding the wire. The value of magnetic field due to a infinite length wire of a distance r will be

$$B = \frac{\mu_0 I}{2\pi r}$$

Magnetic field at P is \mathbf{B} , perpendicular to OP in the direction shown below.



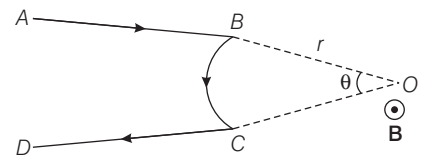
$$\mathbf{B} = B \sin\theta \hat{i} - B \cos\theta \hat{j}$$

$$B = \frac{\mu_0 I}{2\pi r}, \sin\theta = \frac{y}{r}, \cos\theta = \frac{x}{r}$$

$$\mathbf{B} = \frac{\mu_0 I}{2\pi r^2} (y\hat{i} - x\hat{j}) = \frac{\mu_0 I}{2\pi (x^2 + y^2)} (y\hat{i} - x\hat{j})$$

$$r^2 = x^2 + y^2$$

TEST Edge Questions can be asked by varying the shape of the wire. Suppose we have the shape as shown below

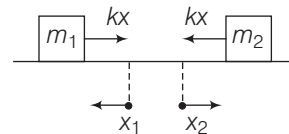


So, the electric field due to the parts AB and AC will be zero and we will calculate the net field due to the part BC which will be outward.

16. (4) **Idea** This problem is based on conservation of momentum and frequency in simple harmonic motion. *i.e.*, Frequency (ν) = $\frac{1}{2\pi} \sqrt{\frac{k}{m}}$

where k is the force constant and m is the mass of the particle.

The total momentum of the system in the horizontal direction is conserved. We draw the FBD, assuming the displacement of the block to be x_1 and x_2 in opposite directions, and the total extension x is given by



$$x = x_1 + x_2$$

and

$$m_1 x_1 = m_2 x_2$$

$$\therefore m_1 \frac{d^2 x_1}{dt^2} = -k(x_1 + x_2)$$

and $m_2 \frac{d^2x_2}{dt^2} = -k(x_1 + x_2)$

After suitably manipulating the equations, we get

$$\frac{d^2x_2}{dt^2} = -\frac{k(m_1 + m_2)}{m_1 m_2} \cdot x$$

i.e., frequency = $\frac{1}{2\pi} \sqrt{\frac{k(m_1 + m_2)}{m_1 m_2}}$
 $= \frac{1}{2} \sqrt{\frac{300(2 + 3)}{2 \times 3}}$
 $\approx 2.5 \text{ Hz.}$

∴ Number of complete oscillations in 1 min
 $= \frac{60}{2.5} = 24.$

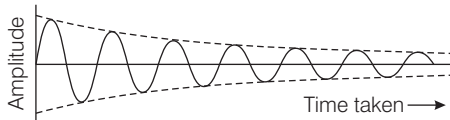
As given $24 = 6n$
 $n = 4$

TEST Edge Above idea is important according to JEE Advanced. Every year, two or three questions are asked in this concept and relate with torque, energy conservation and damped harmonic motion.

Note When the system oscillates with almost the natural angular frequency $\sqrt{k/m}$ (with which the system will oscillate if there is no damping) and with amplitude decreasing with time according to the equation

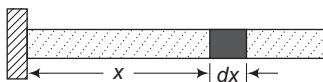
i.e., $A = A_0 e^{-bt/2m}$

Therefore, the amplitude decreases with time and finally becomes zero as shown in figure.



17. (3) Idea It is based on the thermal conductivity of the material and bulk modulus of elasticity. We can directly relate change in length of steel bar with stress applied in bar and easily calculate the thermal coefficient of expansion (α).

Considering an element of the bar of the length dx shown in figure



Change in length of element length dx due to temperature difference is

$$du = dx \times \alpha \times \Delta T$$

or $du = dx \times \alpha \times \frac{T_0 x^2}{L^2}$

Integrating, we get

$$\int du = \int_0^L \frac{\alpha T_0 x^2}{L^2} dx$$

$$\Delta = \frac{\alpha T_0 L^3}{3L^2}$$

Bar is rigidly field

Stress in bar $\sigma = E \times \text{strain} = \frac{\Delta}{L} \times E$

or $\sigma = \frac{\alpha T_0 L^3}{3L^2 \cdot L} \times E = \frac{E \alpha T_0}{3}$

Stress in bar $\sigma = \frac{E \alpha T_0}{3}$

As given in question

$$\Rightarrow \frac{E \alpha T_0}{x} = \frac{E \alpha T_0}{3}$$

So, $x = 3$

TEST Edge In thermal conduction, it is found that in steady state the heat current is proportional to the area of cross-section A . Proportional to the change in temperature ($T_1 - T_2$).

Then, $\frac{\Delta Q}{\Delta t} = \frac{K A (T_1 - T_2)}{x}$

where, K is a constant for material of the slab and is called thermal conductivity of the material.

18. (2) Idea This problem is based on Moseley's law in order to calculate atomic number of element.

i.e., $\Delta E = h\nu = Rhc(Z - b)^2 \left[\frac{1}{n_1^2} - \frac{1}{n_2^2} \right]$

where ΔE is the change in energy (state).

K_α line wavelength remains same irrespective of the accelerating voltage

Now, $\frac{1}{\lambda_{K_\alpha}} = R(Z - 1)^2 \left(\frac{1}{1^2} - \frac{1}{2^2} \right) = \frac{3}{4} R(Z - 1)^2 \dots (i)$

and $\lambda_{\text{cut off (at } V=10 \text{ kV)}} = \frac{12400}{V} \text{ \AA} = \frac{12400}{10 \times 10^3} \text{ \AA}$
 $= 1.248 \text{ \AA} = \lambda_{\text{min.1}}$

and $\lambda_{\text{cut off (at } V=20 \text{ kV)}} = \frac{12400}{20 \times 10^3} \text{ \AA} = 0.62 \text{ \AA} = \frac{\lambda_{\text{min.1}}}{2}$

Given, $\frac{\lambda_{K_\alpha} - \lambda_{\text{min.2}}}{\lambda_{K_\alpha} - \lambda_{\text{min.1}}} = 3$

$\Rightarrow \lambda_{K_\alpha} - \lambda_{\text{min.2}} = 3\lambda_{K_\alpha} - 3\lambda_{\text{min.1}}$

$\Rightarrow \lambda_{K_\alpha} = \frac{5}{4} \lambda_{\text{min.1}} = 1.55 \text{ \AA} \dots (ii)$

Using, Eq. (ii) in (i), we get

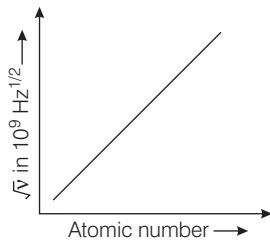
$$Z = 1 + \sqrt{\frac{4}{3\lambda_{K_\alpha} \cdot R}} = 1 + \sqrt{\frac{4}{3} \times \frac{911.5}{1.55}} \Rightarrow Z \approx 29$$

TEST Edge According to JEE Advanced, this concept is important and it is generally asked in exams. So, students should focus on Moseley's law and relate with Bragg's law and properties of X-rays.

Note Moseley's observation can be mathematically expressed as

$$\sqrt{\nu} = a(Z - b)$$

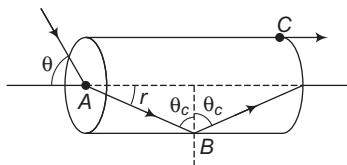
where a and b are constants. This relation is known as Moseley's law.



In figure, show a graph $\sqrt{\nu}$ of X-rays against the atomic number.

19. (0006) **Idea** As, we know whenever a ray of light goes from a denser medium to a rarer medium, it bends away from the normal *i.e.*, based on total internal reflection of light.

Ray enters at A just reflects at B in TIR.



So, $\theta_c = \sin^{-1}(1/n)$

\therefore Snell's law at A

$$\sin \theta = n \sin r$$

$$r + \theta_c = 90^\circ$$

$$r = 90^\circ - \theta_c$$

$$\sin r = \sin(90^\circ - \theta_c) = \cos \theta_c$$

If rays does not come out from second surface than incidence angle at curved surface should be $\geq \theta_c$

So, r should be $\leq 90^\circ - \theta_c$

$$\sin r \leq \sin(90^\circ - \theta_c)$$

$$\sin r \leq \sqrt{1 - \sin^2 \theta_c} \leq \sqrt{1 - \frac{1}{n^2}}$$

From Snell's law at A

$$\sin r = \frac{\sin \theta}{n}$$

$$\sin \theta \leq \sqrt{n^2 - 1} \leq \sqrt{1.25 - 1}$$

$$\sin \theta \leq \frac{1}{2}$$

$\theta \leq 30^\circ, \quad \theta_{\max} = 30^\circ$

So, $\theta/5 \Rightarrow 0006$

TEST Edge In every year, one or two are asked in JEE Advanced. Student should focus on this idea and related to Lens maker's formula and its magnification.

For total internal reflection of light take place, then following conditions must be obeyed.

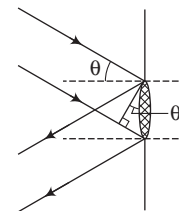
(i) The ray must travel from denser to rarer medium.

(ii) Angle of incidence (L_i) must be greater than critical angle (L_c) *i.e.*, $c = \sin^{-1} \left[\frac{\mu_{\text{rarer}}}{\mu_{\text{denser}}} \right]$

where, c is a critical angle.

20. (1) **Idea** It is based on the conversation of total momentum in perfectly elastic collision and to calculate the pressure exerted by beam on the wall.

Let say mass of each molecule is m , momentum change for each molecules



$$= 2mv \cos \theta$$

Total number of molecules colliding in unit time or passing through cross-section of beam

$$= n(v)(A')$$

[A' = Area of cross-section of beam]

$$A' = A \cos \theta = n(v)(A \cos \theta)$$

So, total momentum exchange per unit line per unit area.

$$\frac{\Delta p}{\Delta t} = 2mv^2 nA \cos^2 \theta$$

$$p = F/A = 2mv^2 n \cos^2 \theta = 1.0043 \times 10^5 \text{ N/m}^2$$

TEST Edge According to the principle of conservation of linear momentum, if no external force is acting on a system, then momentum of the system remains constant.

\Rightarrow If no force is acting, then $F = 0$

i.e., $\frac{d\mathbf{p}}{dt} = 0$

$\therefore \mathbf{p} = \text{constant}$

or $m_1 v_1 = m_2 v_2 = \text{constant}$

21. (a) **Idea** This problem is based on the concept of nature of oxide of 3rd period elements. To solve this problem students are suggested to analyse the reactivity of aluminium oxide with acid or bases.

Nature of oxides

A set of quantum numbers

- (a) denotes $3p^1$ which is valence shell electronic configuration of Al.
 (b) denotes $3p^3$ which is valence shell electronic configuration of P.
 (c) denotes $3p^2$ which is valence shell electronic configuration of Si.
 (d) denotes $3s^1$ which is valence shell electronic configuration of Na.

The elements of period 3 forms following respective oxides as

Element	Oxides
Na	Na_2O (Alkaline)
Mg	MgO (Alkaline)
Al	Al_2O_3 (Amphoteric)
Si	SiO_2 (Acidic)
P	$\text{P}_2\text{O}_5 \equiv \text{P}_4\text{O}_{10}$ (Acidic)
S	SO_3, SO_2 (Acidic)
Cl	Cl_2O_7 (Acidic)

out of above oxides the Al_2O_3 is the only one which produces amphoteric solution after hydrolysis.

TEST Edge Similar problems based on the concept of nature and reaction of various oxides of metal and non-metals can also be asked in JEE Advanced. Here, the generalisation is

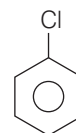
Metal oxide (ionic) \longrightarrow Alkaline
 Non-metal oxide (covalent) \longrightarrow Acidic.

Semi metal oxide (polar covalent) \longrightarrow Amphoteric
 However, precisely we can say alkaline \propto ionic character in oxide. Acidic nature of oxide \propto covalent character oxide.

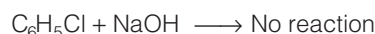
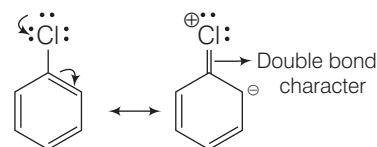
22. (d) **Idea** This problem is based on concept of determination of molecular structure of organic compound and reaction of phenol with NaOH. To solve this problem students are advised to search the most polar bond which is to be broken. It is also advised to be careful as the Ph-Cl bond has double bond character. Hence it will not undergo nucleophilic substitution reaction. While solving this problem students are advised to undergo following sequential steps.
 Determine molecular structure of organic compound using degree of unsaturation.
 Then use the concept of resonance to answer the question asked.

Determination of molecular structure

Degree of unsaturation is 4 and it is an aromatic compound. So possible structure of compound is



Alkyl halide are extremely less reactive towards $\text{S}_{\text{N}}2$ reaction due to following reason.

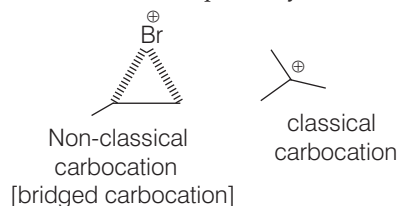
Resonance effect

TEST Edge JEE Advanced, includes these types of questions to judge the basic knowledge of students regarding nucleophilic substitution reaction. Similar questions based on basic concept of electrophilic addition reaction, nucleophilic addition reaction etc, can also be asked such as

Which type of intermediate is obtained when bromine add of propene?

After undergoing proper mechanism of reaction one can get 'non-classical carbocation' as answer.

Note Non-classical and classical carbocation The cyclic and acyclic carbocation are two types of carbocation which are known as non-classical and classical carbocation respectively.



23. (a) **Idea** This question is based on conceptual mixing of mole concept, neutralisation reaction and stoichiometry, while solving this type of problem students are advised to go through making road map of given problem and then use the concept of stoichiometry and mole concept to solve this problem.

As 6 mole of I_2 is formed during the reaction hence,

$$x \times 6 = \frac{60}{1000} \times 0.1$$

$$x = \frac{6}{6 \times 1000} = 10^{-3} \text{ mole}$$

TEST Edge Similar problems based on conceptual mixing of mole concept, stoichiometry and titration reaction can also be asked such as

Generally hard water contains Ca^{2+} and Mg^{2+} , if 1 L of pond water contains 20 mg of Ca^{2+} and 12 mg of Mg^{2+} ions. What is the volume of 2N, Na_2CO_3 solution required to soften 5000 L of pond water?

After solving problem one can get answer as 5 litre (use the concept of pH and stereochemistry)

24. (a) **Idea** This problem is based on the concept of molecular orbital electronic configuration of oxides of nitrogen. While solving this problem students are advised to keep in mind the presence of unpaired electron in given oxide, if molecule/compound contain unpaired electron then excitation becomes possible.

Cause of color of NO_2

Molecular orbital electronic configuration of NO_2 confirms the presence of unpaired electron which can undergo easy transition from ground state energy level to excited state level by absorbing light of suitable wavelength.

Cause of color of N_2O_3

Due to low difference between occupied and unoccupied energy levels of electrons of molecules of N_2O_3 it absorbs a part of visible light spectrum which causes color of N_2O_3 .

TEST Edge Similar problems based on colour magnetic properties, dimerisation reaction, disproportionation reaction and MOEC (Molecular Orbital Electronic Configuration) can also be asked in JEE Advanced.

25. (d) **Idea** This problem is based on the concept of molar specific heat capacity and Boyle's law. While solving the problem students are advised to go through calculation of total molar heat capacity first followed by use of Boyle's law to calculate final temperature after expansion and then calculate total energy changes during expansion.

$$C_V = \frac{1 \times 3R + 2 \times \frac{3}{2}R}{1 + 2} = 2R$$

$$T_1 V_1^{r-1} = T_2 V_2^{r-1}$$

$$\Rightarrow T_2 = 160 \text{ K}$$

$$\Delta E = nC_V(T_2 - T_1)$$

$$= 3 \times 2R(160 - 320) = -960R$$

TEST Edge In JEE Advanced these types of questions are asked to judge the knowledge of students regarding states of matter.

Similar problems with conceptual mixing of gaseous law, ideal gas equation and work done can also be calculated. Thus, it is advisable to go through connective as well as comparative study of these topics.

Remember

For adiabatic process change in internal energy can be calculated as

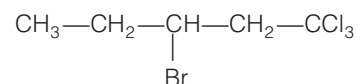
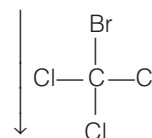
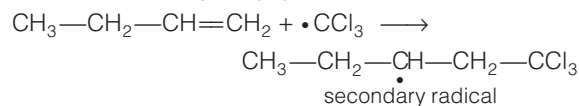
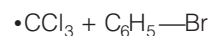
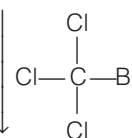
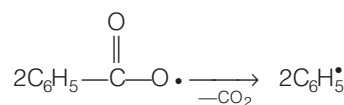
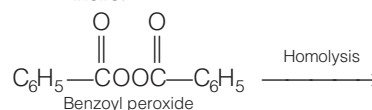
$$\Delta E = -nC_V(T_2 - T_1) \\ = \frac{(p_1 V_1 - p_2 V_2)}{(\gamma - 1)}$$

where, $\gamma = \frac{C_p}{C_V}$

Value of γ varies for different types of gases

Nature of gases	Value of γ
Monoatomic gases	1.66
Diatomic gases	1.40
Polyatomic gases	1.33

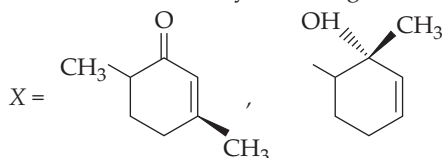
26. (c) **Idea** This problem is based on free radical addition reaction to the alkene. To solve this problem students are advised to undergo proper mechanism including free radical as an intermediate. The stability of free radicals generated during the reaction play key role here.



TEST Edge Similar problems based on conceptual mixing of nucleophilic addition reaction to α, β unsaturated carbonyl compound and function of Grignard and Gilman reagent can also be asked in JEE Advanced, such as

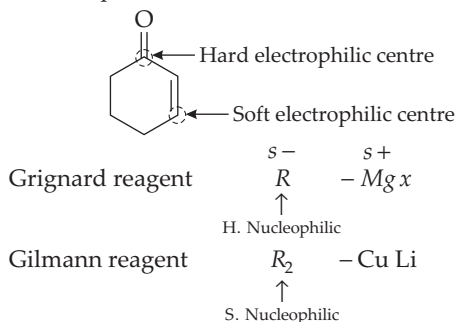
What will be the respective products X and Y when 5 methyl cyclohex-2-eneone is treated with Grignard reagent (RMgX) and Gilmann reagents respectively?

After using the concept of nucleophilic addition reaction and HSAB theory, one can get answer as



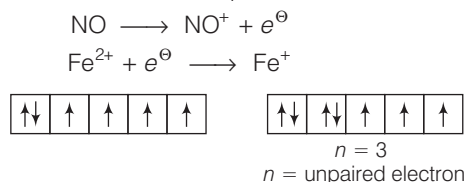
HSAB theory According to HSAB (Hard and Soft Acid Base) theory hard acid adds to hard base and soft acid adds to soft base leading to the formation of stable compound.

Here concept of hard electrophile E^+ with more positive charge density soft electrophile, (E^+ less positive charge density) hard nucleophile and soft nucleophile is used.



27. (a) **Idea** This problem includes conceptual mixing of brown ring test and determination of magnetic moment of coordination compound. While solving the problem students are advised to calculate number of unpaired electron in complex followed by magnetic moment.

Brown ring test When a freshly prepared FeSO_4 solution is added to aqueous solution of NO_3^- ion followed by addition of concentrated H_2SO_4 the brown ring is observed at junction between two liquids. This colour is due to charge transfer oxidation state of iron in this complex is + I.



Magnetic moment can be calculated as

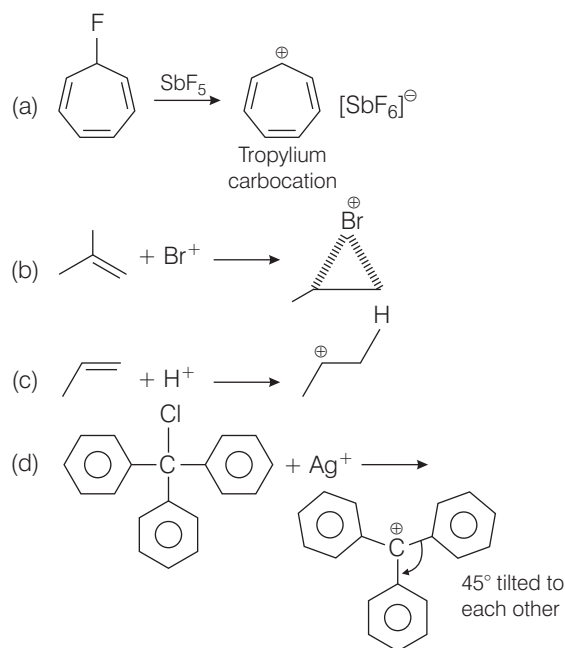
$$\mu = \sqrt{n(n+2)} \text{ BM} = \sqrt{3(3+2)} \text{ BM}$$

$$= \sqrt{15} \text{ BM} = 3.87 \text{ BM}$$

TEST Edge Similar problems based on the concept of complex formation during qualitative analysis of inorganic ions (atoms and anions) and their characteristics can also be asked in JEE Advanced,

so students are advised to go through study of qualitative analysis of inorganic compound and characteristics of complex.

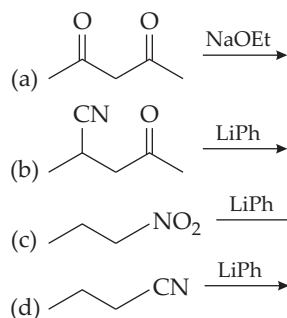
28. (d) **Idea** This problem is based on the concept of formation of carbocation, structure of carbocation and stability of carbocation. Complete the reaction given in options first followed by looking at the stability and structure of carbocation.



This is a non-planar carbocation as the cation is delocalized to all the three rings so this is stable too. The non-planar structure is due to steric hindrance of these three phenyl rings. All three phenyl ring is tilted to plane by 45 like wings of a fan.

TEST Edge Similar problems based on conceptual mixing of stability, structure and generation of carbon free radical and carbanion can also be asked, such as.

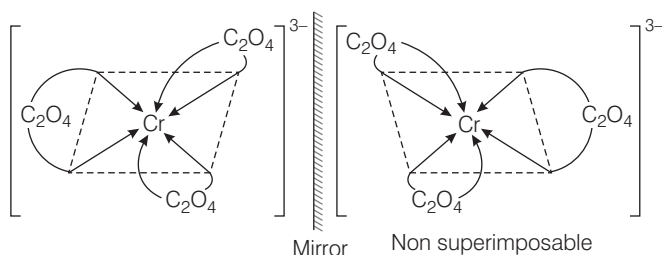
Which of the following produced carbanion has maximum stability?



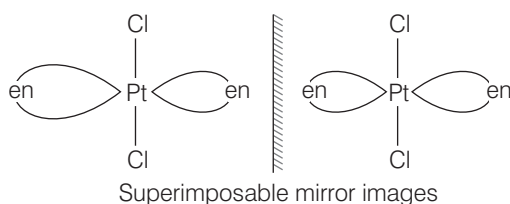
(c) After using the concept of electron withdrawing ability anyone can answer this question.

29. (b) **Idea** This problem is based on nomenclature and isomerism of coordination compound. Write the structure of coordination compound draw their mirror image so and then analyse either then are non-superimposable or superimposable, if they are superimposable then optically inactive otherwise optically active.

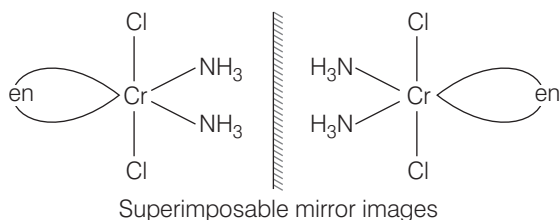
(a) trioxalato chromate (III) $[\text{Cr}(\text{C}_2\text{O}_4)_3]^{3-}$



(b) *trans* dichlorobis ethylene diamine platinum (II) chloride



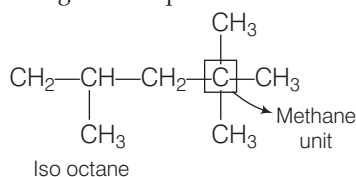
(c) *trans* diamine dichlorobis ethylene diamine chromium (I)



b and c both are optically inactive,

TEST Edge Similar problems based on concept of isomerism and optical active including magnetic moment and isomerism in coordination compound can also be asked in JEE Advanced, so students are advised to go through study of these topics.

30. (b) **Idea** This problem is based on nomenclature of organic compound. While solving the problem students are advised to choose the most commonly used term in derived name of organic compound.



So, name is iso-butyl trimethyl methane.

TEST Edge Similar problems based on nomenclature and isomerism of organic compound can also be asked. Remember the priority chart of functional group to tackle such problems.

31. (a,b,d) **Idea** This problem is based on conceptual mixing of kinetics of gaseous reaction and half-life time of reaction. Calculate partial pressure of substances in term of initial pressure of reactant taken first and then $t_{99\%}$.



$$t = 0; \quad p_0 \quad 0 \quad 0$$

$$t = 20; \quad p_0 - x \quad \frac{2x}{3} \quad \frac{2x}{3}$$

$$t = \infty \quad \quad \quad \frac{2p_0}{3} \quad \frac{2p_0}{3}$$

$$\Rightarrow \frac{4p_0}{3} = 4 \Rightarrow t_{50\%} = 20 \text{ min is the half-life.}$$

$$p_0 = 3 \text{ atm} \quad t_{75\%} = 2 \times 20 = 40 \text{ min}$$

$$p_0 + \frac{x}{3} = 3.5 \quad t_{87.5\%} = 3 \times t_{50\%}$$

$$= 3 \times 20 = 60 \text{ min}$$

$$x = 1.5 \quad t_{99\%} = \frac{2}{3} \times t_{99.9\%}$$

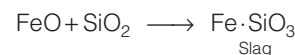
$$= \frac{2}{3} \times 10 \times t_{50\%}$$

$$= \frac{400}{3}$$

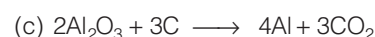
TEST Edge Similar problem based on concept of collision theory, absolute rate law and Arrhenius equation can also be asked.

32. (a,c,d) **Idea** This problem includes conceptual mixing of Hall-Heroult process, formation of slag and reduction of zinc. Read all the statements very carefully then to answer the question.

Formation of slag When ore is treated with silica it form slag of iron silicates



(b) **Reduction of Zn** On reaction of ZnO with coke it converts ZnO into Zn and CO. This is due to reduction of ZnO into Zn.



This equation represents electrolysis of aluminium which is done by using electrolytic cell which contain steel cathode and graphite anode.

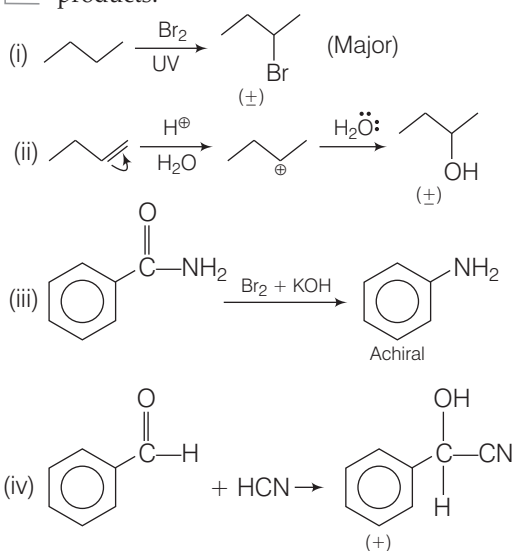
(d) Mond's process is used in refining of nickel which involves conversion of Ni to $\text{Ni}(\text{CO})_4$ and then $\text{Ni}(\text{CO})_4$ is subjected to higher temperature. So that it decomposes to give pure metals.

TEST Edge Similar problems based on the concept of purification, reduction and concentration of ore can also be asked in JEE Advanced, some care must be taken when roasting and calcination are studied together.

Remember

Roasting is heating of metal ores in presence of air below its melting point and calcination is heating of metallic ores in absence of air below its melting point.

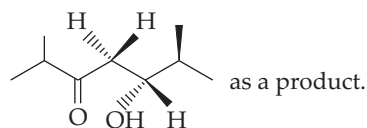
33. (a) **Idea** This problem is based on the concept of electrophilic addition reaction, free radical substitution reaction, Hoffman bromamide reaction including stereochemistry of the products.



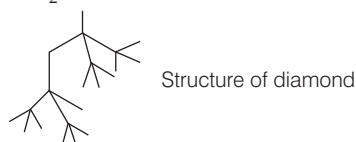
TEST Edge Similar problems based on Aldol condensation, cross-aldol reaction, Cannizzaro reaction, cross-cannizzaro reaction etc., including stereochemistry of products can also be asked.

What will the correct stereochemistry of product obtained by chemical reaction between sodium acetate and methyl butanone?

After undergoing proper mechanism you will get

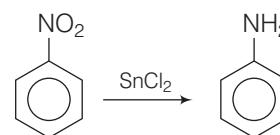


34. (a,b,d) **Idea** This problem involves conceptual mixing of the structure of diamonds and corundum, uses of carbogen and nature of SnCl₂.



Carbogen is a mixture of 0-5% CO₂ and O₂ which is used in artificial respiration of pneumonia patients.

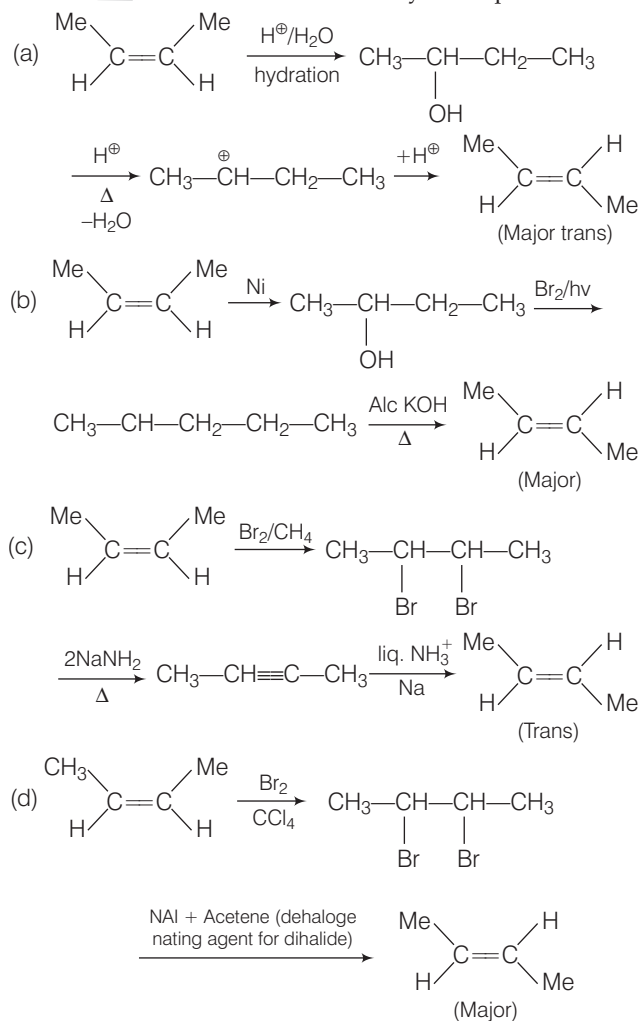
SnCl₂ is a powerful reducing agent



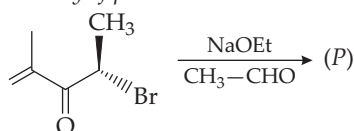
PbO is known as litharge which has yellow orange colour.

TEST Edge Similar problems based on conceptual mixing of preparations properties and uses of inorganic compounds can be asked in JEE Advanced, so students are advised to go through in depth study of preparation properties and uses of PH₃, O₃, SO₃, FeSO₄ etc.

35. (a) **Idea** This problem is based on the concept of function of reagents and stereochemistry. The conversion using, given reagents keeping in mind the stereochemistry of the product.



TEST Edge Similar problems, based on stereochemistry of product and addition reaction to carbonyl compounds such as aldehyde, ketone, carboxylic acid and derivatives of carboxylic acids can also be asked in JEE Advanced, related such as Which of the following is correct regarding stereochemistry of product?



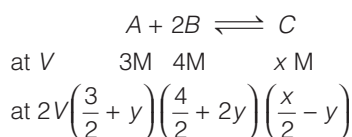
- (a) *P* exists in enantiomeric form
 (b) *P* exists in diastereomeric form
 (c) *P* exists in racemic form
 (d) None of the above

After undergoing proper mechanism to solve these question you will get (b) as a answer.

Remember

When there is presence of stereogenic centre at α carbon of carbonyl compound and it undergo addition reaction it will produce a diastereomer always.

36. (4) **Idea** This problem is based on concept of determination of equilibrium constant at different volume of reaction.



$$K_{eq} = \frac{x}{3 \times 4^2} = \frac{x}{428}$$

Given $2y + 2 = 3$
 $y = \frac{1}{2} = 0.5$

$$K_{eq} = \frac{\left(\frac{x-1}{2}\right)}{(1.5 + 0.5)3^2} = \frac{x}{48}$$

$$x = 4M$$

TEST Edge Similar problem based on the concept of equilibrium constant solubility product can also be asked. Go through in depth study of the relation between equilibrium constant and solubility product.

Remember

If $Q =$ Ionic product and

$k_{sp} =$ Solubility product

Then,

$Q > k_{sp}$, then precipitation takes place

$Q < k_{sp}$, then no precipitation takes place

$Q = k_{sp}$, then reaction is at equilibrium

37. (b) **Idea** This problem can be solved by using the concept of electronic transition in Brackett series and rydberg equation. While solving the problem students are advised to identify the type of series in which lines of spectrum is obtained. Use the energy boundary of each line to choose the correct choice.

$$\Delta E = \frac{3}{4} \times 0.85 \text{ eV}$$

Photon will be in Brackett series

($\because 0.31 \leq E \leq 0.85$) for Brackett

$$0.85 \left(1 - \frac{1}{4}\right) = 13.6 \left(\frac{1}{4^2} - \frac{1}{n^2}\right)$$

$$0.85 \left(1 - \frac{1}{4}\right) = \frac{13.6}{16} \left(1 - \left(\frac{4}{n}\right)^2\right)$$

$$\Rightarrow \frac{4}{n} = \frac{1}{2}$$

$$\Rightarrow n = 8$$

Hence, $n = 8$

TEST Edge Similar problems based on conceptual mixing of wavelength of light and energy of a particular transition can also be asked in JEE Advanced.

Extra care must be taken during calculation of energy levels regarding unit of energy conversion. e.g.,

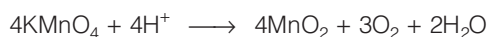
The ratio of the wave numbers for highest energy transition of electron in Lyman and Balmer series of atom is

Ans. 4 : 1 Calculate through the given formulae

38. (3) **Idea** This problem is based on oxidation state of conversion of KMnO_4 to MnO_2 while solving this problem students are advised to write the exact chemical reaction of given transformation followed by calculating the difference in oxidation state.

Extra care must be taken regarding medium of reaction.

The transformation of KMnO_4 to MnO_2 occurs in acidic medium as follows.



$$x - 4 = 0$$


$$x = +4$$

$$x - 8 + 1 = 0$$

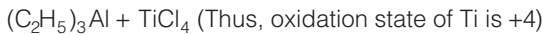
$$x = 7$$

Change in oxidation number = $7 - 4 = 3$

TEST Edge Similar problems and regarding transformation of KMnO_4 and $\text{K}_2\text{Cr}_2\text{O}_7$ in different medium of reaction can also be asked in JEE Advanced. Even sometimes equivalent mass questions can be asked also.

39. (4)  **Idea** Also this problem is based on the conceptual mixing of molecular formula of Ziegler natta catalyst and oxidation state of transition element present in the complex.
Ziegler Natta catalyst

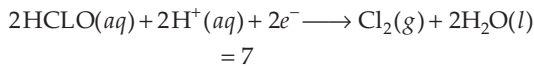
↓ is a mixture of




TEST Edge Similar problems based on redox reaction or oxidation and reduction reaction of inorganic and organic compound can also be asked. Such as

What will be the change in oxidation state when hypochlorous acid reacts with aqueous solution of acid?

Hints




40. (2)  **Idea** This problem is based on concept of measurement of emf during titration of acid and base at different end points. While solving the problems it is suggestive to calculate the pH first, followed by calculation of E_{cell} .

$$pH = \frac{pk_2 + pk_3}{2} = 10.5$$

$$E_{cell} = E_{cell}^\circ + 0.059 \cdot pH = 1.3805 + 0.059 \times 10.5 = 2V$$

TEST Edge Similar problem based on concept of electrochemical series and calculation of emf using Nernst equation for different chemical reaction at different concentration can also be asked.

41. (a)  **Idea** Angle between planes $a_1x + b_1y + c_1z + d_1 = 0$ and $a_2x + b_2y + c_2z + d_2 = 0$ is

$$\cos \theta = \left| \frac{a_1a_2 + b_1b_2 + c_1c_2}{\sqrt{a_1^2 + b_1^2 + c_1^2} \sqrt{a_2^2 + b_2^2 + c_2^2}} \right|$$

So, Two planes are perpendicular if $a_1a_2 + b_1b_2 + c_1c_2 = 0$ if $\mathbf{a} = a_1\hat{i} + a_2\hat{j} + a_3\hat{k}$ and

$$\mathbf{b} = b_1\hat{i} + b_2\hat{j} + b_3\hat{k} \text{ then } \mathbf{a} \times \mathbf{b} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \end{vmatrix}$$

Distance of a point $p(x_1, y_1, z_1)$ from a plane

$$ax + by + cz + d = 0 \text{ is } \left| \frac{ax_1 + by_1 + cz_1}{\sqrt{a^2 + b^2 + c^2}} \right|$$

To find equation of plane P , we have the following information available

P passes through the point $(1, -2, 1)$

P is perpendicular to planes

$$P_1 \equiv 2x - 2y + z = 0$$

and $P_2 \equiv x - y + 2z = 4$

The normal to the plane $P_1 \equiv 2x - 2y + z = 0$

lies along the vector $2\hat{i} - 2\hat{j} + \hat{k}$

The normal to the plane $P_2 \equiv x - y + 2z = 4$ lies along the vector $\hat{i} - \hat{j} + 2\hat{k}$.

The normal to both the plane P_1 and P_2 lies along the vector $(2\hat{i} - 2\hat{j} + \hat{k}) \times (\hat{i} - \hat{j} + 2\hat{k})$

The vector along the normal to the plane P is

$$\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & -2 & 1 \\ 1 & -1 & 2 \end{vmatrix}$$

The equation of the plane P is

$$\begin{vmatrix} x-1 & y+2 & z-1 \\ 2 & -2 & 1 \\ 1 & -1 & 2 \end{vmatrix} = 0 \Rightarrow x + y + 1 = 0$$

∴ Distance of the point $(1, 2, 2)$ from the plane

$P \equiv x + y + 1 = 0$ is

$$\frac{1 + 2 + 1}{\sqrt{1+1}} = \frac{4}{\sqrt{2}} = 2\sqrt{2}$$

$$\therefore k\sqrt{2} = 2\sqrt{2}$$

$$\Rightarrow k = 2$$

TEST Edge Questions based on distance between two planes, equation of planes in different form, distance between two parallel planes and cross product of vector are asked. To solve such type of questions students are advised to understand above mentioned topic such as distance between two parallel planes $ax + by + cz + d_1 = 0$ and $ax + by + cz + d_2 = 0$ is given by $d = \left| \frac{d_1 - d_2}{\sqrt{a^2 + b^2 + c^2}} \right|$.

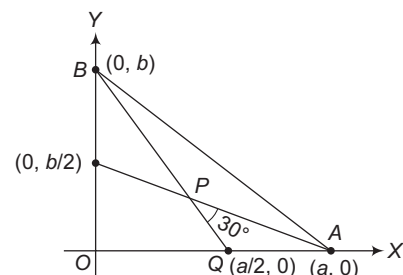
42. (a)  **Idea** Angle between two straight line in terms of their slopes

$$\tan \theta = \left| \frac{m_1 - m_2}{1 + m_1m_2} \right|$$

$$\text{Area of triangle} = \frac{1}{2} \times \text{base} \times \text{Altitude}$$

$$\text{Pythagoras theorem, (Hypotenuse)}^2 = (\text{Base})^2 + (\text{Altitude})^2$$

$$\text{Slope of } AP = \frac{0 - b/2}{a - 0} = \frac{-b}{2a}$$



$$\text{Slope of } PQ \text{ (BQ)} = \frac{0 - b}{a/2 - 0} = \frac{-2b}{a}$$

$$\begin{aligned} \therefore \tan 30^\circ &= \left| \frac{m_1 - m_2}{1 + m_1 m_2} \right| \\ &= \left| \frac{\frac{-b}{2a} + \frac{2b}{a}}{1 + \frac{b^2}{a^2}} \right| \\ &= \left| \frac{3ba}{2(a^2 + b^2)} \right| \\ &= \frac{1}{\sqrt{3}} \end{aligned}$$


$$\Rightarrow \frac{1}{2} ab = \frac{a^2 + b^2}{3\sqrt{3}}$$

$$\begin{aligned} \text{Now, } AB^2 &= a^2 + b^2 = 9 \\ \frac{1}{2} ab &= \frac{9}{3\sqrt{3}} \\ &= \sqrt{3} \end{aligned}$$

\Rightarrow Area of $\Delta = \sqrt{3}$ sq. units

TEST Edge Question based on basics of straight lines such as distance formula, slope formula, etc and geometrical concept such as area of different figures, intersecting lines, etc. To solve such type of questions students are advised to understand concept of straight line such as area of parallelogram = $\frac{p_1 p_2}{\sin \theta}$, where p_1 and p_2 are distance

of perpendicular between two pairs of opposite sides and θ is the angle between adjacent sides.

43. (d)  **Idea** If $y = [x]$ if $n \leq x < n + 1$, then $y = n$ for the cubic equation $ax^3 + bx^2 + cx + d = 0$ derivative $f'(x) = 3ax^2 + 2bx + c$, the roots is given by

$$x = \frac{-b \pm \sqrt{b^2 - 3ac}}{3a}$$

In the case where $b^2 - 3ac \leq 0$ cubic function is monotonic.

$$f(x) = x^3 - 3x + [a] = x^3 - 3x + k$$

where $[a] = k$,

$$\begin{aligned} f'(x) &= 3x^2 - 3 \\ &= 3(x - 1)(x + 1) \end{aligned}$$

For three roots $f(-1)f(1) < 0$

$$\Rightarrow (k + 2)(k - 2) < 0$$

$$\begin{array}{c} + \quad \quad \quad - \quad \quad \quad + \\ | \quad \quad \quad | \quad \quad \quad | \\ -2 \quad \quad \quad 2 \end{array}$$

$$\Rightarrow k \in (-2, 2)$$

$$\Rightarrow [a] \in (-2, 2)$$

$$\Rightarrow [a] = -1, 0, 1$$

$$a \in [-1, 2)$$

TEST Edge Questions based on domain and range of different functions such as polynomial function, modulus function, etc and concept of application of derivatives increasing and decreasing function are asked. To solve such type of question students are advised to learn definition of different functions and concept of increasing function such as function $f(x)$ is said to be increasing, if $x_1 < x_2 \Rightarrow f(x_1) \leq f(x_2)$ for this $f'(x) > 0$.

44. (a)  **Idea** By definition of definite integral

$$\int_a^b f(x) dx = \phi(b) - \phi(a)$$

where $\phi(x)$ in anti-derivative of $f(x)$ in $[a, b]$ If u and v are the differentiable function of x then

$$\int u \cdot v dx = u \int v dx - \int \left[\frac{d(u)}{dx} \right] \left[\int v dx \right] dx$$

$$\int_0^a x^2 f''(x) dx = [x^2 f'(x)]_0^a - \int_0^a 2x f'(x) dx$$

$$\Rightarrow a^2 f'(a) - 2 \left[\int_0^a x f(x) \right] - \int_0^a f(x) dx$$

$$\Rightarrow a^2 f'(a) - 2 \left[a f(a) - \int_0^a f(x) dx \right]$$

$$\Rightarrow a^2 f'(a) - 2a f(a) + 2 \int_0^a f(x) dx$$


$$\therefore \int_0^{2014} x^2 f''(x) dx = (2014)^2 f'(2014)$$

$$- 2(2014) f(2014) + 2 \int_0^{2014} f(x) dx$$

$$\Rightarrow (2014)^2 \cdot \frac{1}{2014} - 2 \cdot (2014) \cdot \frac{1}{2014} + 2 \cdot 1$$

$$\Rightarrow 2014 - 2 + 2 \Rightarrow 2014$$

TEST Edge In JEE Advanced question based on definite integral properties and various method of integration such as substitution method, etc., are asked. To solve such type of questions students are advised to learn concept and properties of definite integral such as if $f(x) \geq g(x)$ then $\int_a^b f(x) dx \geq \int_a^b g(x) dx$ (where $b > a$).

45. (b)  **Idea** We used trigonometric ratios of sub-multiple of an angle, $1 + \cos \theta = 2 \cos^2 \theta/2$

Sum of n terms of GP, $S_n = \frac{1 - r^n}{1 - r}$, $|r| < 1$ and

$\sin(90 - \theta) = \cos \theta$ to solve this question.

We know $1 + \cos \theta = 2 \cos^2 \theta/2$

$$\therefore 1 + \cos \frac{\pi}{4} = 2 \cos^2 \pi/8$$

$$\Rightarrow 1 + \frac{1}{\sqrt{2}} = 2 \cos^2 \pi/8$$

$$\Rightarrow 2 + \sqrt{2} = 4 \cos^2 \pi/8$$

$$\Rightarrow \sqrt{2 + \sqrt{2}} = 2 \cos \pi/8$$

Adding 2 to both sides

$$2 + \sqrt{2 + \sqrt{2}} = 2 + 2 \cos \pi/8$$

$$\Rightarrow 2 + \sqrt{2 + \sqrt{2}} = 2(1 + \cos \pi/8)$$

$$2 + \sqrt{2 + \sqrt{2}} = 2(2 \cos^2 \pi/16)$$

$$2 + \sqrt{2 + \sqrt{2}} = 4 \cos^2 \pi/16$$

$$\sqrt{2 + \sqrt{2 + \sqrt{2}}} = 2 \cos \pi/2^4$$

Similarly,

$$\sqrt{2 + \sqrt{2 + \sqrt{2}}} \dots n \text{ times} = 2 \cos \left(\frac{\pi}{2^{n+1}} \right)$$

Now, $1 + \frac{1}{2} + \frac{1}{2^2} + \frac{1}{2^3} + \dots + \frac{1}{2^{2005}}$

$$= \frac{1 - \left(\frac{1}{2}\right)^{2006}}{1 - \frac{1}{2}} = 2 - (1/2)^{2005}$$

$$\therefore 2 \sin \left[2 - \left(\frac{1}{2}\right)^{2005} \right] 45^\circ$$


$$= 2 \sin \left[90^\circ - 45^\circ \left(\frac{1}{2}\right)^{2005} \right]$$

$$= 2 \cos \left[\frac{\pi}{4} \cdot \left(\frac{1}{2}\right)^{2005} \right]$$

$$= 2 \cos \frac{\pi}{2^{2007}}$$

$$= X_{2006}$$

TEST Edge Question based on trigonometric ratios of complementary and supplementary angles, trigonometric ratios of multiples of an angle and sum of n term of AP series are also asked. To solve such type of questions students are advised to understand concept of trigonometric ratio such as $\cot 3\theta = \frac{\cot^3 \theta - 3 \cot \theta}{3 \cot^2 \theta - 1}$

46. (a)  **Idea** If $z = (1)^{1/n}$ then $z = (\cos 0^\circ + i \sin 0^\circ)^{1/n}$

Thus, n^{th} root of unity are $1, \alpha, \alpha^2, \dots, \alpha^{n-1}$

Where $\alpha = e^{\frac{2\pi i}{n}} = \cos \frac{2\pi}{n} + i \sin \frac{2\pi}{n}$

If $z = x + iy$ then $|z| = \sqrt{\{\text{Re}(z)\}^2 + \{\text{Im}(z)\}^2}$

Now, $x^n - 1 = (x - \alpha_0)(x - \alpha_1)(x - \alpha_2) \dots (x - \alpha_{n-1})$

$$\frac{x^n - 1}{x - 1} = (x - \alpha_1)(x - \alpha_2) \dots (x - \alpha_{n-1})$$

$[\because \alpha_0 = 1 \text{ if } k = 0]$

$$\Rightarrow 1 + x + x^2 + \dots + x^{n-1} = (x - \alpha_1)(x - \alpha_2) \dots (x - \alpha_{n-1})$$

Putting $x = 1$

$$n = (1 - \alpha_1)(1 - \alpha_2) \dots (1 - \alpha_{n-1})$$

Taking modulus on both sides, we have

$$|1 - \alpha_1| |1 - \alpha_2| \dots (1 - \alpha_{n-1}) = n$$

Now $|1 - \alpha_k| = \left| 1 - \cos \frac{2k\pi}{n} - i \sin \frac{2k\pi}{n} \right|$

$$= \left| 2 \sin^2 \frac{k\pi}{n} - i 2 \frac{\sin k\pi}{n} \frac{\cos k\pi}{n} \right|$$

$$= \left| 2 \sin \left(\frac{k\pi}{n} \right) \left(\frac{\sin k\pi}{n} - i \frac{\cos k\pi}{n} \right) \right|$$

$$= 2 \frac{\sin k\pi}{n}$$


$$\therefore |1 - \alpha_1| |1 - \alpha_2| \dots |1 - \alpha_{n-1}| = n$$

$$\Rightarrow \left(2 \sin \frac{\pi}{n} \right) \left(2 \sin \frac{2\pi}{n} \right) \dots \left(2 \sin \frac{(n-1)\pi}{n} \right) = n$$

$$2^{n-1} \sin \pi/n \frac{\sin 2\pi}{n} \dots \frac{\sin (n-1)\pi}{n} = n$$

TEST Edge In JEE Advanced questions based on properties of modulus of complex numbers, arguments and properties of n^{th} roots of unity are asked. To solve such type of questions students are advised to learn properties of complex number such as if $|z_1| \leq 1, |z_2| \leq 1$, then

$$|z_1 - z_2|^2 \leq (|z_1| - |z_2|)^2 + [\arg(z_1) - \arg(z_2)]^2$$

47. (b)  **Idea** Apply basic geometrical concepts of parallel lines, triangles coplanarity and collinearity easily of lines with the application of permutation and combination *i.e.*, number of triangles formed by joining n points out of which m are collinear is ${}^n C_3 - {}^m C_3$.

The straight lines L_1, L_2 and L_3 are parallel and lie in the same plane.

Total number of points are $= m + n + k$

The total number of triangles formed by these points $= {}^{m+n+k} C_3$

\Rightarrow but out of these $(m + n + k)$ points, m points lie on L_1 , n points lie on L_2 and k point, lie on L_3 and by joining these points on the same line we do not get a triangle.

Hence the total number of

$$\Delta = {}^{m+n+k} C_3 - {}^m C_3 - {}^n C_3 - {}^k C_3$$

TEST Edge Question based on functional applications of permutation and combination are asked. To solve such type of questions students are advised to learn functional application such as the number of all arrangements of n different objects taken r at time.

(a) When particular object is always include in arrangement is ${}^{n-1} C_{r-1} \times r!$.

(b) When a particular object is never taken in each arrangement is ${}^{n-1} C_r \times r!$.

48. (c)  **Idea** By the triangle law of inequality

$$|z_1 + z_2 + z_3| \leq |z_1| + |z_2| + |z_3|$$

$$\text{But } |z_1 + z_2 + z_3| = |z_1| + |z_2| + |z_3|$$

When all $z_1, z_2, z_3 \geq 0$ or ≤ 0 and trigonometric identities. $\cos^2 \theta = 1 - \sin^2 \theta$.

Based on property of modulus

Given equation is,

$$|\sin^2 x + 26 - x^2| = \cos^2 x + 2\sin^2 x + |25 - x^2|$$

$$\text{Now, } \cos^2 x + 2\sin^2 x + (25 - x^2)$$

$$\begin{aligned} &1 + \sin^2 x + 25 - x^2 \\ &= \sin^2 x + 26 - x^2 \end{aligned}$$

$$\text{i.e., } |x + y + z| = |x| + |y| + |z|$$


(where all $x, y, z \geq 0$ or ≤ 0)

$$\Rightarrow 25 - x^2 \geq 0$$

$$\Rightarrow x^2 - 25 \leq 0$$

$$x \in [-5, 5]$$

TEST Edge In JEE Advanced questions based on position of roots of a quadratic equation, maximum and minimum value of quadratic equation and arithmetic-geometric mean inequality are asked. To solve such type of questions students are advised to understand concept of arithmetic-geometric mean inequality such as if $a, b > 0$ and $a \neq b$, then $\frac{a+b}{2} > \sqrt{ab} > \frac{2}{\frac{1}{a} + \frac{1}{b}}$.

49. (b)  **Idea** We applied definition of probability so the general expression for the probability P of occurrence of an event used in the solution is

$$P = \frac{\text{Measure of the specified part of the region}}{\text{Measure of the whole region}}$$

Where, measure means length or area or volume of the region, when we are dealing with one, two or three dimensional space respectively.

$$\text{Total number of ways} = {}^{64}C_3$$


The number of ways of selecting squares consisting of 4 unit squares is $7 \times 7 = 49$

Also each squares with four unit squares form 4 L-shapes consisting of 3 squares

$$\begin{aligned} \text{Number of favourable outcomes} \\ &= 7 \times 7 \times 4 \\ &= 196 \end{aligned}$$

$$\text{Required probability} = \frac{196}{64} C_3$$

TEST Edge Questions based on Poisson distribution, binomial distribution, etc are also asked from advance probability. To solve such type of questions students are advised to understand the above mentioned concept such as mode of binomial distribution is $(n+1)P - 1 \leq r \leq (n+1)P$.

50. (c)  **Idea** Apply the concept of differentiation i.e., multiplication rule of differentiation, and solution of differential equation by the method of variable separable i.e.,

If general form of equation is $f(x)dx + g(y)dy = 0$ then solution is

$$\int f(x)dx = \int g(y)dy + c$$

Consider the given differential equation.

$$x(x+1) \frac{dy}{dx} + (x+1)y = \frac{f(xy)}{f'(xy)}$$

$$\Rightarrow \frac{x dy}{dx} + y = \frac{1}{(x+1)} \frac{f(xy)}{f'(xy)}$$

$$\Rightarrow \frac{x dy + y dx}{dx} = \frac{1}{x+1} \frac{f(xy)}{f'(xy)}$$

$$\Rightarrow d(xy) = \frac{dx}{x+1} \frac{f(xy)}{f'(xy)}$$

On separating the variables, we get

$$\frac{f'(xy)}{f(xy)} d(xy) = \frac{dx}{(x+1)}$$

Now on intergrating both sides, we get

$$\int \frac{f'(xy)}{f(xy)} d(xy) = \int \frac{dx}{(x+1)}$$

$$\Rightarrow \log(f(xy)) = \log(x+1) + \log c$$


$$\Rightarrow \log(f(xy)) = \log(c(x+1))$$

$$[\because \log m + \log n = \log(m.n)]$$

$\Rightarrow f(xy) = (x+1)c$, where c being an arbitrary constant.

TEST Edge In JEE Advance question based on solution of homogeneous differential equations and linear differential equation are also asked. To solve such type of questions students are advised to learn the method of solving mentioned differential equation such as linear equation of the form $\frac{dx}{dy} + Px = Q$ then it solution is $X = \int Q \cdot \text{IF} dy + c$

$$\text{where IF} = e^{\int P dy}$$

51. (b,c,d)  **Idea** Dot product of two non-zero vectors inclined at an angle θ is given as $\mathbf{a} \cdot \mathbf{b} = ab \cos \theta, 0 \leq \theta \leq \pi$

Scalar triple product of three vectors \mathbf{a}, \mathbf{b} and \mathbf{c} is $(\mathbf{a} \times \mathbf{b}) \cdot \mathbf{c} = |\mathbf{a}| |\mathbf{b}| |\mathbf{c}| \sin \theta \cos \phi$ where θ is the angle between \mathbf{a} and \mathbf{b} ϕ is the angle between $\mathbf{a} \times \mathbf{b}$ and \mathbf{c} . It is also represented as $[\mathbf{a} \mathbf{b} \mathbf{c}]$.

$$\text{Now, } \mathbf{u} \cdot \mathbf{v} = |\mathbf{u}| |\mathbf{v}| \cos \theta = \cos \theta$$

$$\text{Given, } \mathbf{w} + \mathbf{w} \times \mathbf{u} = \mathbf{v}$$

$$\Rightarrow \mathbf{u} \cdot (\mathbf{w} + \mathbf{w} \times \mathbf{u}) = \mathbf{u} \cdot \mathbf{v}$$

$$\Rightarrow \mathbf{u} \cdot \mathbf{w} = \mathbf{u} \cdot \mathbf{v} = \cos \theta \quad [\mathbf{u} \cdot \mathbf{w} \times \mathbf{u}] = 0$$

$$\text{also } \mathbf{v} \cdot (\mathbf{w} + \mathbf{w} \times \mathbf{u}) = \mathbf{v} \cdot \mathbf{v}$$

$$\mathbf{v} \cdot \mathbf{w} + [\mathbf{v} \mathbf{w} \mathbf{u}] = 1$$

$$\begin{aligned} \mathbf{v} \cdot \mathbf{w} + \beta &= 1 & (\because \beta &= [\mathbf{u} \mathbf{v} \mathbf{w}]) \\ \mathbf{v} \cdot \mathbf{w} &= 1 - \beta \end{aligned}$$

Similarly, $\mathbf{w} \cdot (\mathbf{w} + \mathbf{w} \times \mathbf{u}) = \mathbf{w} \cdot \mathbf{v}$
 $\mathbf{w} \cdot \mathbf{w} = \mathbf{w} \cdot \mathbf{v} = 1 - \beta$

$$\text{also } \beta^2 = [\mathbf{u} \mathbf{v} \mathbf{w}]^2 = \begin{vmatrix} \mathbf{u} \cdot \mathbf{u} & \mathbf{u} \cdot \mathbf{v} & \mathbf{u} \cdot \mathbf{w} \\ \mathbf{v} \cdot \mathbf{u} & \mathbf{v} \cdot \mathbf{v} & \mathbf{v} \cdot \mathbf{w} \\ \mathbf{w} \cdot \mathbf{u} & \mathbf{w} \cdot \mathbf{v} & \mathbf{w} \cdot \mathbf{w} \end{vmatrix}$$


$$\beta^2 = \begin{vmatrix} 1 & \cos \theta & \cos \theta \\ \cos \theta & 1 & 1 - \beta \\ \cos \theta & 1 - \beta & 1 - \beta \end{vmatrix} = \beta(1 - \beta - \cos^2 \theta)$$

$$(\because \mathbf{u} \cdot \mathbf{u} = |\mathbf{u}|^2 = 1, \mathbf{u} \cdot \mathbf{v} = \cos \theta, \mathbf{w} \cdot \mathbf{w} = \mathbf{w} \cdot \mathbf{v} = 1 - \beta)$$

$$\begin{aligned} \beta^2 &= \beta(\sin^2 \theta - \beta) \Rightarrow \beta = \sin^2 \theta - \beta \\ \Rightarrow \beta &= \frac{1}{2} \sin^2 \theta \end{aligned}$$

TEST Edge Generally, in JEE Advanced cross product, vector triple product and addition of vectors related questions are asked. To solve such type of questions students are advised to understand the mentioned concept and also acquainted yourself with the properties of cross product such as

$$\mathbf{a} \times \mathbf{b} \neq \mathbf{b} \times \mathbf{a} \text{ and } (\mathbf{a} \times \mathbf{b})^2 = |\mathbf{a}|^2 |\mathbf{b}|^2 - (\mathbf{a} \cdot \mathbf{b})^2$$

52. (b,d)  **Idea** Apply intersecting chords theorem concept of intersecting chord inside the circle and the concept of arithmetic-geometric mean inequality i.e., $AM \geq GM$

\therefore Chord PT and QR intersect each other then

$$PS \times ST = QS \times SR \quad \dots(i)$$

Now, $AM > GM$

$$\frac{1}{PS} + \frac{1}{ST} > \sqrt{\frac{1}{PS} \times \frac{1}{ST}}$$

$$\Rightarrow \frac{1}{PS} + \frac{1}{ST} > \frac{2}{\sqrt{PS \times ST}}$$

$$\Rightarrow \frac{1}{PS} + \frac{1}{ST} > \frac{2}{\sqrt{QR \times SR}} \quad [\text{Using Eq. (i)}] \dots(ii)$$

Again $\frac{QS + SR}{2} > \sqrt{QS \times SR}$

$$\Rightarrow \frac{1}{\sqrt{QS \times SR}} > \frac{2}{QS + SR}$$

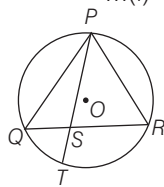
$$\Rightarrow \frac{1}{\sqrt{QS \times SR}} > \frac{2}{QR} \quad (\because QS + SR = QR)$$

$$\Rightarrow \frac{2}{\sqrt{QS \times SR}} > \frac{4}{QR} \quad \dots(iii)$$


From Eqs. (ii) and (iii), we have

$$\frac{1}{PS} + \frac{1}{ST} > \frac{4}{QR}$$

Hence, answer option (b) and (d) are correct.



TEST Edge Questions based on solution set of inequality equation, properties of inequality, tangent of a circle, sector of a circle are asked in JEE Advanced. To solve such type of questions students are advised to understand the concept of inequalities and also acquainted yourself with theorem of circles such as alternate segment theorem and equation of normal of slope m to the circle $x^2 + y^2 = r^2$ is $my = -x \pm r\sqrt{1 + m^2}$.

53. (a,b,c,d)  **Idea** To solve this problem, use the concept of algebra of functions, periodic function i.e., $f: X \rightarrow Y$ is a periodic function if $f(x + T) = f(x) \forall x \in X$ and properties of periodic functions.

$$f(x + 1) = \frac{f(x) - 5}{f(x) - 3} \quad \dots(i)$$

$$\Rightarrow f(x)f(x + 1) - 3f(x - 1) = f(x) - 5$$

$$\Rightarrow f(x)[f(x + 1) - 1] = 3f(x + 1) - 5$$

$$\Rightarrow f(x) = \frac{3f(x + 1) - 5}{f(x + 1) - 1}$$

Replacing x by $x - 1$, we get

$$f(x - 1) = \frac{3f(x) - 5}{f(x) - 1} \quad \dots(ii)$$

Using Eq. (i), $f(x + 2) = \frac{f(x + 1) - 5}{f(x + 1) - 3}$

$$\begin{aligned} &= \frac{\frac{f(x) - 5}{f(x) - 3} - 5}{\frac{f(x) - 5}{f(x) - 3} - 3} \\ &= \frac{f(x) - 5}{f(x) - 3} - 3 \end{aligned}$$

$$= \frac{2f(x) - 5}{f(x) - 2} \quad \dots(iii)$$

Using Eq. (ii),

$$f(x - 2) = \frac{3f(x - 1) - 5}{f(x - 1) - 1} = \frac{3\left[\frac{3f(x) - 5}{f(x) - 1}\right] - 5}{\frac{3f(x) - 5}{f(x) - 1} - 1}$$

$$= \frac{2f(x) - 5}{f(x) - 2} \quad \dots(iv)$$


From Eqs. (iii) and (iv), we have $f(x + 2) = f(x - 2)$

$\Rightarrow f(x + 4) = f(x) \Rightarrow f(x)$ is periodic with period 4

TEST Edge In JEE Advanced domain and range of real function, equality of functions, inverse function and types of functions related questions are asked. To solve such type of questions students are advised to understand above topics and also acquainted yourself with the properties of composition of functions such as if f, g and n are functions from R to R

$$\text{then } (f + g)oh = foh + goh$$

$$(f \cdot g)oh = (foh) \cdot (goh)$$

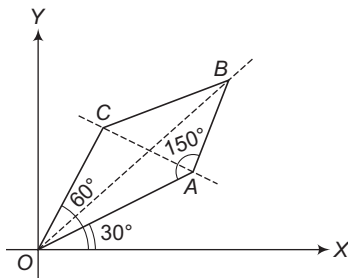
54. (a, b)  **Idea** To solve this problem, use the concept of representation of line equation in cartesian coordinate system, properties and area of rhombus and cosine rule *i.e.*, in any triangle ABC . Cosing of an angle can express in terms of sides

$$\cos A = \frac{b^2 + c^2 - a^2}{2bc},$$

$$\cos B = \frac{c^2 + a^2 - b^2}{2ac}$$

$$\text{and } \cos C = \frac{a^2 + b^2 - c^2}{2ab}$$

Let the length of the sides of the rhombus be l units.



$$\Rightarrow \text{Area} = l^2 \sin 30^\circ$$

(since, the angle between consecutive sides is 30°)

$$\Rightarrow 2 = l^2 \sin 30^\circ$$

$$\Rightarrow l = 2 \text{ units}$$

Now, applying cosine rule in ΔOAB , we get

$$OB^2 = OA^2 + AB^2 - 2OA \cdot AB \cdot \cos 150^\circ$$

$$= 4 + 4 - 2(4) \cdot \left(-\frac{\sqrt{3}}{2}\right)$$

$$= 4(2 + \sqrt{3})$$

$$\Rightarrow OB = 2\sqrt{2 + \sqrt{3}}$$

Since, the angle between consecutive sides is 30° , hence the slope of OB is equal to 1.

\therefore Coordinates of the point B are given by

$$B \left(\frac{2\sqrt{2 + \sqrt{3}}}{\sqrt{2}}, \frac{2\sqrt{2 + \sqrt{3}}}{\sqrt{2}} \right)$$

(using polar coordinates $(r \cos \theta, r \sin \theta)$)

Here $\theta = 45^\circ$ (as $\angle BOX = 45^\circ$)

$$\Rightarrow B = (\sqrt{4 + 2\sqrt{3}}, \sqrt{4 + 2\sqrt{3}})$$

$$\sqrt{4 + 2\sqrt{3}} \equiv (1 + \sqrt{3}, 1 + \sqrt{3})$$


also since the rhombus can lie entirely in I or III quadrants, hence coordinates of B can be

$$(1 + \sqrt{3}, 1 + \sqrt{3})$$

$$\text{or } (-1 - \sqrt{3}, -1 - \sqrt{3})$$

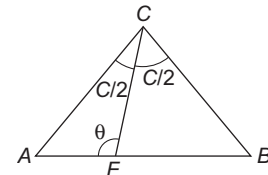
TEST Edge In JEE Advanced questions related to distance formula, section formula and different points related to a triangle such as centroid, incentre, etc are asked. To solve such type of questions students are advised to understand fundamentals of cartesian coordinate system such as quadrants, condition of collinearity and also acquainted yourself with the properties of triangles such as Napier's analogy *i.e.*, In any ΔABC ,

$$\tan \frac{C-A}{2} = \frac{c-a}{c+a} \cot \frac{B}{2}$$

55. (a, c)  **Idea** When two sides and angle between them is known, then area of triangle is given by

$$\Delta = \frac{1}{2} ab \sin C, \Delta = \frac{1}{2} bc \sin A$$

$$\Delta = \frac{1}{2} ca \sin B$$



$$\Delta = \Delta_1 + \Delta_2$$

$$\frac{1}{2} ab \sin C = \frac{1}{2} b(CF) \sin \left(\frac{C}{2}\right)$$

$$+ \frac{1}{2} (a)(CF) \sin (C/2)$$

$$\Rightarrow ab \sin C = CF \sin (C/2)(a + b)$$

$$\Rightarrow CF = \frac{ab \sin C}{\sin C/2 (a + b)}$$

$$= \frac{2 ab \sin C/2 \cos C/2}{\sin C/2 (a + b)}$$

$$CF = \frac{2 ab \cos C/2}{a + b}$$

also in ΔCFB

$$\frac{CF}{\sin B} = \frac{a}{\sin (\pi - \theta)} \quad (\text{using sine rule})$$

$$\frac{CF}{\sin B} = \frac{a}{\sin \theta}$$

$$\frac{CF}{\sin B} = \frac{a}{\sin \left(B + \frac{C}{2}\right)} \quad (\theta = B + C/2)$$

$$\Rightarrow CF = \frac{a \sin B}{\sin (B + C/2)}$$

$$= \frac{b \sin A}{\sin \left(B + \frac{C}{2}\right)}$$

$$\left(\text{using sine rule } \frac{a}{\sin A} = \frac{b}{\sin B} \right)$$

TEST Edge In JEE Advanced trigonometry *i.e.*, ratios of half angle of triangles, different type of circle connected with triangle and regular polygon related questions are asked. To solve such type of questions students are advised to understand the application of trigonometry and properties of triangle such as radius of in circle of a triangle is given by

$$r = \frac{a \sin \frac{B}{2} \sin \frac{C}{2}}{\cos \frac{A}{2}}$$

$$= \frac{b \sin \frac{A}{2} \sin \frac{C}{2}}{\cos \frac{B}{2}}$$

$$= \frac{C \sin \frac{B}{2} \sin \frac{A}{2}}{\cos \frac{C}{2}}$$

56. (7) **Idea** For greatest integer function $[x]$, if $n \leq x < n+1$, then $[x] = n$ and use the concept of property of definite integral *i.e.*,
 $\int_a^b f(x) dx = \int_a^c f(x) dx + \int_c^b f(x) dx$ ($a < c < b$)
 $f(x) = x^2 + 1$ and $\alpha(x) = [x]$

From property

$$\int_0^b f(x) d\alpha(x) + \int_a^b \alpha(x) df(x) = \alpha(b)f(b) - \alpha(a)f(a)$$

$$\Rightarrow \int_0^2 (x^2 + 1) d[x] + \int_0^2 [x] 2x dx = [2](4 + 1) - 0$$

$$= 10$$

$$\int_0^2 (x^2 + 1) d[x] = 10 - 2 \int_0^2 x [x] dx$$

$$= 10 - 2 \left\{ \int_0^1 0 dx + \int_1^2 x dx \right\}$$

$$= 10 - 2 \left\{ 0 + \left[\frac{x^2}{2} \right]_1^2 \right\}$$

$$= 10 - 2 \left\{ \frac{4}{2} - \frac{1}{2} \right\} = 10 - 2 \cdot \frac{3}{2}$$

$$10 - 3 = 7$$

TEST Edge Questions based on definite integration by substitution, definite integral as a limit of sum and types of functions *i.e.*, one-one function, onto function, etc are asked in JEE Advanced. To solve such type of questions students are advised to understand above concept and also acquainted yourself with approximation in definite integration such as

$$\left| \int_a^b f(x) g(x) dx \right| \leq \sqrt{\left\{ \int_a^b f^2(x) dx \right\} \left\{ \int_a^b g^2(x) dx \right\}}$$

57. (2) **Idea** Apply the concept of area of triangle when co-ordinates of three points is known *i.e.*,

$$\text{Area of triangle} = \frac{1}{2} \begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix} \text{ and projection}$$

of points in different quadrants

The coordinates of vertices of projected triangle will be

$$A'(-1, 1, 0), B'(1, -1, 0), C'(1, 1, 0)$$

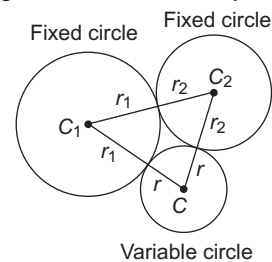
$$\Rightarrow \text{Area of triangle} = \frac{1}{2} \begin{vmatrix} -1 & 1 & 1 \\ 1 & -1 & 1 \\ 1 & 1 & 1 \end{vmatrix}$$

$$= 2 \text{ square units}$$

TEST Edge Question based on direction cosines, direction ratios, angle between two lines, orthocentre of triangle, excentre of triangle, etc related to three dimensional geometry are asked in JEE Advanced. To solve such type of questions students are advised to learn the above mentioned concepts and also acquaint yourself with different equation of plane such as intercept form *i.e.*,
 $\frac{x}{a} + \frac{y}{b} + \frac{z}{c} = 1$

58. (6) **Idea** Apply the concept of circle touching each other externally then distance between the centres equals the sum of their radii *i.e.*,
 $|C_1C_2| = r_1 + r_2$ and the definition of hyperbola to calculate the eccentricity.

Let us assume that C_1 and C_2 are fixed circles touching each other externally.



Since C_1 touches C_2 externally

$$\therefore C_1C_2 = r_1 + r_2$$

Let us also assume that C is a variable circle which touches both of C_1 and C_2 externally.

Since C touches C_1 externally,

$$\therefore CC_1 = r + r_1$$

Since, C touches C_2 externally,

$$\therefore CC_2 = r + r_2$$

$$\therefore |CC_1 - CC_2| = |r_1 - r_2| = \text{Constant}$$

∴ Locus of C is a hyperbola, whose eccentricity is

$$= \frac{|C_1 + C_2|}{|C_1 - C_2|} = \frac{|r_1 + r_2|}{|r_1 - r_2|}$$

Now, $\frac{r_1}{r_2} = \frac{7}{5}$


$$\Rightarrow \frac{|r_1 + r_2|}{|r_1 - r_2|} = \frac{7 + 5}{7 - 5} = 6$$

So, the curve is hyperbola and $e = \frac{r_1 + r_2}{r_1 - r_2} = 6$

∴ $k = 6$

TEST Edge In JEE Advanced question based on parabola and its related terms such as equation of directrix, latus rectum, vertex, etc and ellipse are asked. To solve such type of questions students are advised to understand various terms related to parabola and ellipse such as pairs of tangent of ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2}$ is $SS_1 = T^2$

where, $S = \frac{x^2}{a^2} + \frac{y^2}{b^2} - 1$, $S_1 = \frac{x_1^2}{a^2} + \frac{y_1^2}{b^2} - 1$ and $T = \frac{xx_1}{a^2} + \frac{yy_1}{b^2} - 1$

59. (1)  **Idea** Apply the concept of dot product $\mathbf{a} \cdot \mathbf{b} = ab \cos \theta$ and cross product *i.e.*, $\mathbf{a} \times \mathbf{b} = ab \sin \theta \hat{n}$ and also basic algebraic formula to solve this type of questions.

Now, $\mathbf{a} \cdot \mathbf{b} = 0$, $\mathbf{a} \cdot \mathbf{c} = 0$, $\mathbf{b} \cdot \mathbf{c} = |\mathbf{b}| |\mathbf{c}| \cos \pi/3$
 $\mathbf{b} \cdot \mathbf{c} = 1/2$ $[\because \mathbf{a} \perp \mathbf{b}, \mathbf{a} \perp \mathbf{c}]$


$$|\mathbf{a} \times \mathbf{b}| = |\mathbf{a}| |\mathbf{b}| \sin 90^\circ = 1$$

$$|\mathbf{a} \times \mathbf{c}| = |\mathbf{a}| |\mathbf{c}| \sin 90^\circ = 1$$

$$\begin{aligned} \text{Now } |\mathbf{a} \times \mathbf{b} - \mathbf{a} \times \mathbf{c}|^2 &= |\mathbf{a} \times \mathbf{b}|^2 + |\mathbf{a} \times \mathbf{c}|^2 - 2(\mathbf{a} \times \mathbf{b}) \cdot (\mathbf{a} \times \mathbf{c}) \\ &= 1 + 1 - 2|\mathbf{a} \times \mathbf{b}| |\mathbf{a} \times \mathbf{c}| \cos 60^\circ \\ &= 2 - 2(1)(1) \frac{1}{2} \\ &= 1 \end{aligned}$$

(∴ the angle between $\mathbf{a} \times \mathbf{b}$ and $\mathbf{a} \times \mathbf{c}$ = angle between \mathbf{b} and \mathbf{c})

TEST Edge In JEE Advanced representation of vector, types of vector such as collinear vector, coplanar vector, etc, addition of vectors related questions are asked. To solved such type of questions students are advised to learn the basic facts of vectors and also acquainted yourself with the concept of product of three vectors such as scalar triple product *i.e.*, $(\mathbf{a} \times \mathbf{b}) \cdot \mathbf{c} = |\mathbf{a}| |\mathbf{b}| |\mathbf{c}| \sin \theta \cos \phi$ and vector triple product *i.e.*, $\mathbf{a} \times (\mathbf{b} \times \mathbf{c}) = (\mathbf{a} \cdot \mathbf{c}) \mathbf{b} - (\mathbf{a} \cdot \mathbf{b}) \mathbf{c}$.

60. (0)  **Idea** Apply the concept of transformation of one trigonometric ratio to another trigonometric ratio *i.e.*, $\tan x = \frac{\sin x}{\cos x}$, etc and concept of inequality *i.e.*, if $x > 0$, $x + \frac{1}{x} \geq 2$.

$$|\sin x \cos x| + \sqrt{(\tan x + \cot x)^2} = \sqrt{3}$$

$$|\sin x \cos x| + |\tan x + \cot x| = \sqrt{3}$$

$$|\sin x \cos x| + \left| \frac{1}{\sin x \cos x} \right| = \sqrt{3}$$

$$|\sin x \cos x| + \frac{1}{|\sin x \cos x|} = \sqrt{3}$$

We know if $x > 0$, $x + \frac{1}{x} \geq 2$

$$\therefore |\sin x \cos x| + \frac{1}{|\sin x \cos x|} \geq 2$$

Hence, $|\sin x \cos x| + \frac{1}{|\sin x \cos x|} = \sqrt{3}$ has no solution.

TEST Edge In JEE Advanced questions based on arithmetic-geometric mean inequality *i.e.*, AM and GM solution of inequalities and trigonometric identities are asked. To solve such type of questions students are advised to learn the concept of solution of trigonometric inequalities by graph and also acquainted yourself with the concept of arithmetic-geometric mean inequality such as

$$\frac{a_1 + a_2 + \dots + a_n}{n} \geq (a_1 \cdot a_2 \cdot \dots \cdot a_n)^{1/n} \geq \frac{n}{\frac{1}{a_1} + \frac{1}{a_2} + \dots + \frac{1}{a_n}}$$