IIT JEE

SOLVED PAPER 2011

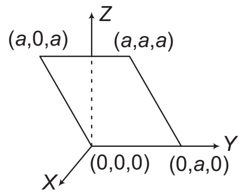
Physics

Paper I

SECTION I

Directions (Q. Nos. 1-7) *This section contains 7 multiple choice questions. Each question has four choices (a), (b), (c) and (d) out of which only one is correct.*

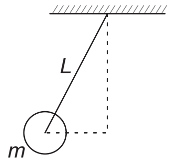
Consider an electric field $\mathbf{E} = E_0\hat{\mathbf{x}}$ where $\bar{E}_0$ is a constant. The flux through the shaded area (as shown in the figure) due to this field is



(a) $2E_0 a^2$ (b) $\sqrt{2}\, E_0 a^2$

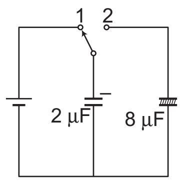
(c) $E_0 a^2$ (d) $\dfrac{E_0 a^2}{\sqrt{2}}$

A ball of mass (m) 0.5 kg is attached to the end of a string having length (L) 0.5 m. The ball is rotated on a horizontal circular path about vertical axis. The maximum tension that the string can bear is 324 N. The maximum possible value of angular velocity of ball (in rad/s) is



(a) 9 (b) 18 (c) 27 (d) 36

A 2μF capacitor is charged as shown in the figure. The percentage of its stored energy dissipated after the switch S is turned to position 2, is



(a) 0% (b) 20%

(c) 75% (d) 80%

5.6 L of helium gas at STP is adiabatically compressed to 0.7 L. Taking the initial temperature to be T_1, the work done in the process is

(a) $\dfrac{9}{8}RT_1$ (b) $\dfrac{3}{2}RT_1$ (c) $\dfrac{15}{8}RT_1$ (d) $\dfrac{9}{2}RT_1$

A police car with a siren of frequency 8 kHz is moving with uniform velocity 36 km/h towards a tall building which reflects the sound waves. The speed of sound in air is 320 m/s. The frequency of the siren heard by the car driver is

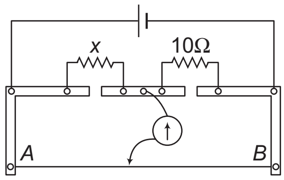
(a) 8.50 kHz (b) 8.25 kHz

(c) 7.75 kHz (d) 7.50 kHz

The wavelength of the first spectral line in the Balmer series of hydrogen atom is 6561 Å. The wavelength of the second spectral line in the Balmer series of singly ionized helium atom is

(a) 1215 Å (b) 1640 Å (c) 2430 Å (d) 4687 Å

A meter bridge is set up as shown in figure, to determine an unknown resistance X using a standard 10 Ω resistor. The galvanometer shows null point when tapping key is at 52 cm mark. The end-corrections are 1 cm and 2 cm respectively for the ends A and B. The determined value of X is



(a) 10.2 Ω (b) 10.6 Ω (c) 10.8 Ω (d) 11.1 Ω

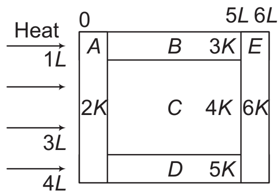
SECTION II

Directions (Q. Nos. 8-11) *This section contains 4 multiple choice questions. Each question has four choices (a), (b), (c) and (d) out of which one or more may be correct.*

An electron and a proton are moving on straight parallel paths with same velocity. They enter a semi-infinite region of uniform magnetic field perpendicular to the velocity. Which of the following statement(s) is/are true?

(a) They will never come out of the magnetic field region
(b) They will come out travelling along parallel paths
(c) They will come out at the same time
(d) They will come out at different times

A composite block is made of slabs A, B, C, D and E of different thermal conductivities (given in terms of a constant, K) and sizes (given in terms of length, L) as shown in the figure. All slabs are of same width. Heat Q flows only from left to right through the blocks. Then, in steady state

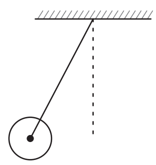


(a) heat flow through A and E slabs are same
(b) heat flow through slab E is maximum
(c) temperature difference across slab E is smallest
(d) heat flow through C = heat flow through B + heat flow through D

A spherical metal shell A of radius R_A and a solid metal sphere B of radius $R_B < (R_A)$ are kept far apart and each is given charge $+Q$. Now, they are connected by a thin metal wire. Then

(a) $E_A^{\text{inside}} = 0$ (b) $Q_A > Q_B$
(c) $\frac{\sigma_A}{\sigma_B} = \frac{R_B}{R_A}$ (d) $E_A^{\text{on surface}} < E_B^{\text{on surface}}$

A metal rod of length L and mass m is pivoted at one end. A thin disc of mass M and radius R (< L) is attached at its centre to the free end of the rod. Consider two ways the disc is attached. Case A—the disc is not free to rotate about its centre and case B—the disc is free to rotate about its centre. The rod-disc system performs SHM in vertical plane after being released from the same displaced position. Which of the following statement(s) is/are true?



(a) Restoring torque in case $A =$ Restoring torque in case B
(b) Restoring torque in case $A <$ Restoring torque in case B
(c) Angular frequency for case $A >$ Angular frequency for case B
(d) Angular frequency for case $A <$ Angular frequency for case B

SECTION III

Directions (Q. Nos. 12-16) *This section contains 2 paragraphs. Based upon one of the paragraph 2 multiple choice questions and based on the other paragraph 3 multiple choice questions have to be answered. Each of these questions has four choices (a), (b), (c) and (d) out of which only one is correct.*

Paragraph (Q. Nos. 12-13) *A dense collection of equal number of electrons and positive ions is called neutral plasma. Certain solids containing fixed positive ions surrounded by free electrons can be treated as neutral plasma. Let N be the number density of free electrons, each of mass m. When the electrons are subjected to an electric field, they are displaced relatively away from the heavy positive ions.If the electric field becomes zero, the electrons being to oscillate about the positive ions with a natural angular frequency ω_p, which is called the plasma frequency. To sustain the oscillations, a time varying electric field needs to be applied that has an angular frequency ω, where a part of the energy is absorbed and a part of it is reflected. As ω approaches ω_p, all the free electrons are set to resonance together and all the energy is reflected. This is the explanation of high reflectivity of metals.*

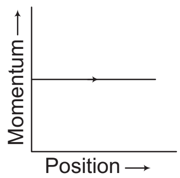
Taking the electronic charge as e and the permittivity as ε_0, use dimensional analysis to determine the correct expression for ω_p.

(a) $\sqrt{\dfrac{Ne}{m\varepsilon_0}}$ (b) $\sqrt{\dfrac{m\varepsilon_0}{Ne}}$ (c) $\sqrt{\dfrac{Ne^2}{m\varepsilon_0}}$ (d) $\sqrt{\dfrac{m\varepsilon_0}{Ne^2}}$

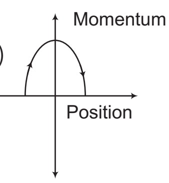
Estimate the wavelength at which plasma reflection will occur for a metal having the density of electrons $N \approx 4 \times 10^{27}\ \text{m}^{-1}$. Take $\varepsilon_0 = 10^{-11}$ and $m = 10^{-30}$, where these quantities are in proper SI units.

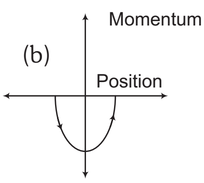
(a) 800 nm (b) 600 nm (c) 300 nm (d) 200 nm

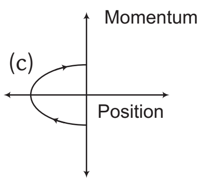
Paragraph (Q. Nos. 14-16) *Phase space diagrams are useful tools in analyzing all kinds of dynamical problems. They are especially useful in studying the changes in motion as initial position and momentum are changed. Here we consider some simple dynamical systems in one-dimension. For such systems, phase space is a plane in which position is plotted along horizontal axis and momentum is plotted along vertical axis. The phase space diagram is $x(t)$ vs $p(t)$ curve in this plane. The arrow on the curve indicates the time flow. For example, the phase space diagram for a particle moving with constant velocity is a straight line as shown in the figure. We use the sign convention in which position or momentum upwards (or to right) is positive and downwards (or to left) is negative.*

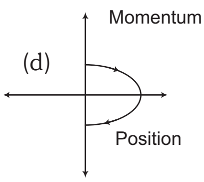


The phase space diagram for a ball thrown vertically up from ground is

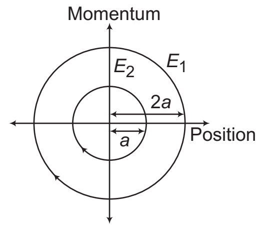
(a)


(b)


(c)


(d)


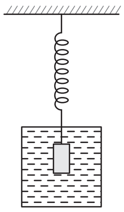
The phase space diagram for simple harmonic motion is a circle centered at the origin. In the figure, the two circles represent the same oscillator but for different initial conditions, and E_1 and E_2 are the total mechanical energies respectively. Then,

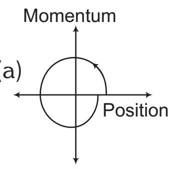


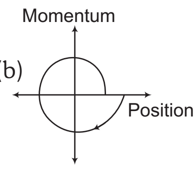
(a) $E_1 = \sqrt{2}E_2$ (b) $E_1 = 2\,E_2$

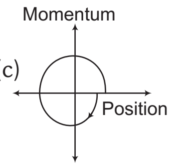
(c) $E_1 = 2E_2$ (d) $E_1 = 16\,E_2$

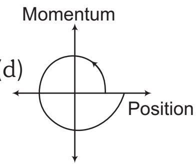
Consider the spring-mass system, with the mass submerged in water, as shown in the figure. The phase space diagram for one cycle of this system is



(a)


(b)


(c)


(d)


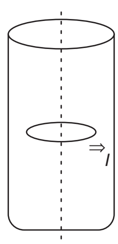
SECTION IV

Directions (Q. Nos. 17-23) *This section contains* 7 *questions. The answer to each question is a single digit integer, ranging from* 0 *to* 9.

Four point charges, each of $+q$, are rigidly fixed at the four corners of a square planar soap film of side a. The surface tension of the soap film is γ. The system of charges and planar film are in equilibrium, and $a = k\left[\dfrac{q^2}{\gamma}\right]^{1/N}$, where k is a constant. Then, N is

A block is moving on an inclined plane making an angle 45° with the horizontal and the coefficient of friction is μ. The force required to just push it up the inclined plane is 3 times the force required to just prevent it from sliding down. If we define $N = 10\,\mu$, then N is

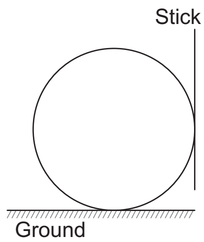
A long circular tube of length 10 m and radius 0.3 m carries a current I along its curved surface as shown. A wire loop of resistance 0.005 Ω and of radius 0.1 m is placed inside the tube with its axis coinciding with the axis of the tube. The current varies as $I = I_0 \cos 300\,t$, where I_0 is constant. If the magnetic moment of the loop is $N\mu_0 I_0 \sin 300\,t$, then N is



Steel wire of length L at 40° C is suspended from the ceiling and then a mass m is hung from its free end. The wire is cooled down from 40°C to 30°C to regain its original length L. The coefficient of linear thermal expansion of the steel is 10^{-5}/°C, Young's modulus of steel is 10^{11} N/m^2 and radius of the wire is 1 mm. Assume that $L >>$ diameter of the wire. Then, the value of m in kg is nearly

Four solid spheres each of diameter $\sqrt{5}$ cm and mass 0.5 kg ar placed with their centres at the corners of a square of side 4 cm. The moment of inertia of the system about the diagonal of the square is $N \times 10^{-4}$ kg-m^2, then N is

A boy is pushing a ring of mass 2 kg and radius 0.5 m with a stick as shown in the figure. The stick applies a force of 2 N on the ring and rolls it without slipping with an acceleration of 0.3 m/s^2. The coefficient of friction between the ground and the ring is large enough that rolling always occurs and the coefficient of friction between the stick and the ring is $\frac{P}{10}$. The value of P is

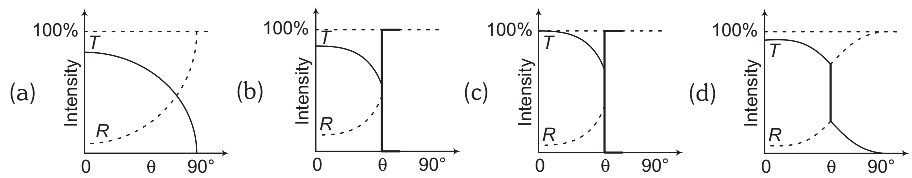


The activity of a freshly prepared radioactive sample is 10^{10} disintegrations per second, whose mean life is 10^{-9} s. The mass of an atom of this radioisotope is 10^{-25}kg. The mass (in mg) of the radioactive sample is

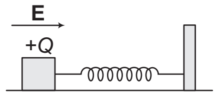
Paper II

Directions (Q. Nos. 1-8) *This section contains 8 multiple choice questions. Each question has four choices (a), (b), (c) and (d) out of which only one is correct.*

A light ray travelling in glass medium is incident on glass-air interface at an angle of incidence θ. The reflected (R) and transmitted (T) intensities, both as function of θ, are plotted. The correct sketch is



A wooden block performs SHM on a frictionless surface with frequency ν_0. The block carries a charge $+Q$ on its surface. If now a uniform electric field **E** is switched on as shown, then the SHM of the block will be



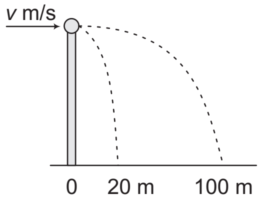
(a) of the dame frequency and with shifted mean position
(b) of the same frequency and with the same mean position
(c) of changed frequency and with shifted mean position

(d) of changed frequency and with the same mean position

The density of a solid ball is to be determined in an experiment. The diameter of the ball is measured with a screw gauge, whose pitch is 0.5 mm and there are 50 divisions on the circular scale. The reading on the main scale is 2.5 mm and that on the circular scale is 20 divisions. If the measured mass of the ball has a relative error of 2%, the relative percentage error in the density is

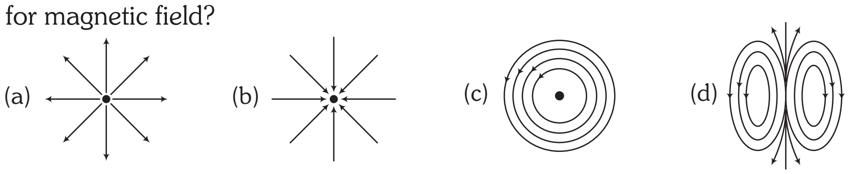
(a) 0.9% (b) 2.4% (c) 3.1% (d) 4.2%

A ball of mass 0.2 kg rests on a vertical post of height 5 m. A bullet of mass 0.01 kg, travelling with a velocity v m/s in a horizontal direction, hits the centre of the ball. After the collision, the ball and bullet travel independently. The ball hits the ground at a distance of 20 m and the bullet at a distance of 100 m from the foot of the post. The initial velocity v of the bullet is



(a) 250 m/s (b) $250\sqrt{2}$ m/s (c) 400 m/s (d) 500 m/s

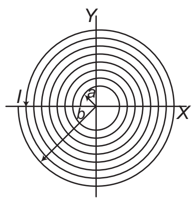
Which of the field patterns given in the figure is valid for electric field as well as for magnetic field?



A point mass is subjected to two simultaneous sinusoidal displacements in x-direction, $x_1(t) = A \sin \omega t$ and $x_2(t) = A \sin\left(\omega t + \frac{2\pi}{3}\right)$. Adding a third sinusoidal displacement $x_3(t) = B \sin(\omega t + \phi)$ bring the mass to a complete rest. The values of B and ϕ are

(a) $\sqrt{2}A, \frac{3\pi}{4}$ (b) $A, \frac{4\pi}{3}$ (c) $\sqrt{3}A, \frac{5\pi}{6}$ (d) $A, \frac{\pi}{3}$

A long insulated copper wire is closely wound as a spiral of N turns. The spiral has inner radius a and outer radius b. The spiral lies in the XY-plane and a steady current I flows through the wire. The Z-component of the magnetic field at the centre of the spiral is



(a) $\frac{\mu_0 NI}{2(b-a)} \ln\left(\frac{b}{a}\right)$ (b) $\frac{\mu_0 NI}{2(b-a)} \ln\left(\frac{b+a}{b-a}\right)$

(c) $\frac{\mu_0 NI}{2b} \ln\left(\frac{b}{a}\right)$ (d) $\frac{\mu_0 NI}{2b} \ln\left(\frac{b+a}{b-a}\right)$

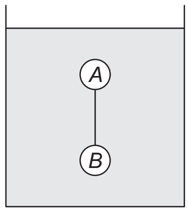
A satellite is moving with a constant speed v in a circular orbit about the earth. An object of mass m is ejected from the satellite such that it just escapes from the gravitational pull of the earth. At the time of its ejection, the kinetic energy of the object is

(a) $\frac{1}{2}mv^2$ (b) mv^2 (c) $\frac{3}{2}mv^2$ (d) $2\,mv^2$

SECTION II

Directions (Q. Nos. 9-12) *This section contains 4 multiple choice questions. Each question has four choices (a), (b), (c) and (d) out of which one or more may be correct.*

Two solid spheres A and B of equal volumes but of different densities d_A and d_B are connected by a string. They are fully immersed in a fluid of density d_F. They get arranged into an equilibrium state as shown in the figure with a tension in the string. The arrangement is possible only if

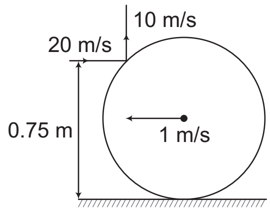


(a) $d_A < d_F$ (b) $d_B > d_F$
(c) $d_A > d_F$ (d) $d_A + d_B = 2d_F$

Which of the following statement(s) is/are correct?
(a) If the electric field due to a point charge varies as $r^{-2.5}$ instead of r^{-2}, then the Gauss's law will still be valid
(b) The Gauss's law can be used to calculate the field distribution around an electric dipole
(c) If the electric field between two point charges is zero somewhere, then the sign of the two charges is the same
(d) The work done by the external force in moving a unit positive charge from point A at potential V_A to point B at potential V_B is $(V_B - V_A)$

A series R-C circuit is connected to AC voltage source. Consider two cases; (A) when C is without a dielectric medium and (B) when C is filled with dielectric of constant 4. The current I_R through the resistor and voltage V_C across the capacitor are compared in the two cases. Which of the following is/are true?
(a) $I_R^A > I_R^B$ (b) $I_R^A < I_R^B$ (c) $V_C^A > V_C^B$ (d) $V_C^A < V_C^B$

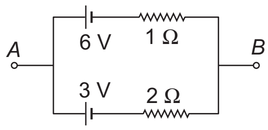
A thin ring of mass 2 kg and radius 0.5 m is rolling without slipping on a horizontal plane with velocity 1 m/s. A small ball of mass 0.1 kg, moving with velocity 20 m/s in the opposite directions, hits the ring at a height of 0.75 m and goes vertically up with velocity 10 m/s. Immediately after the collision



(a) the ring has pure rotation about its stationary CM
(b) the ring comes to a complete stop
(c) friction between the ring and the ground is to the left
(d) there is no friction between the ring and the ground

SECTION III

Directions (Q. Nos. 13-18) *This sections contain 6 questions. The answer to each of the question is a single-digit integer, ranging from* 0 *to* 9.

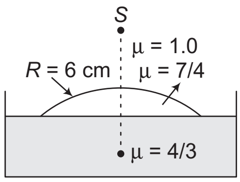


Two batteries of different emfs and different internal resistances are connected as shown. The voltage across AB in volt is

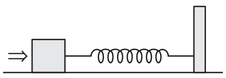
A series R-C combination is connected to an AC voltage of angular frequency $\omega = 500$ rad/s. If the impedance of the R-C circuit is $R\sqrt{1.25}$, the time constant (in millisecond) of the circuit is

A train is moving along a straight line with a constant acceleration a. A boy standing in the train throws a ball forward with a speed of 10 m/s, at an angle of 60° to the horizontal. The boy has to move forward by 1.15 m inside the train to catch the ball back at the initial height. The acceleration of the train in m/s^2, is

Water (with refractive index = $\frac{4}{3}$) in a tank is 18 cm deep. Oil of refractive index $\frac{7}{4}$ lies on water making a convex surface of radius of curvature $R = 6$ cm as shown. Consider oil to act as a thin lens. An object S is placed 24 cm above water surface. The location of its image is at x cm above the bottom of the tank. Then, x is



A block of mass 0.18 kg is attached to a spring of force constant 2 N/m. The coefficient of friction between the block and the floor is 0.1. Initially, the block is at rest and the spring is unstretched. An impulse is given to the block as shown in the figure. The block slides a distance of 0.06 m and comes to rest for the first time. The initial velocity of the block in m/s is $v = \frac{N}{10}$. Then, N is

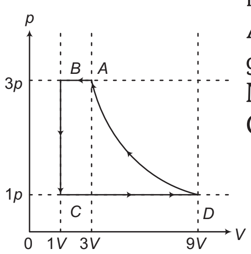


A silver sphere of radius 1 cm and work function 4.7 eV is suspended from an insulating thread in free-space. It is under continuous illumination of 200 nm wavelength light. As photoelectrons are emitted, the sphere gets charged and acquires a potential. The maximum number of photoelectrons emitted from the sphere is $A \times 10^Z$ (where $1 < A < 10$). The value of Z is

SECTION IV

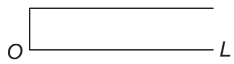
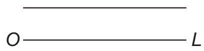
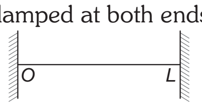
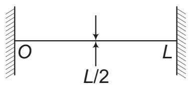
Directions (Q. Nos. 19-20) *This section contains 2 questions. Each question has four statements (A, B, C and D) given in Column* I *and five statements (p,q, r, s and t) in Column* II. *Any given statement in Column* I *can have correct matching with one or more statement(s) given in Column* II.

One mole of a monatomic ideal gas is taken through a cycle *ABCDA* as shown in the *p-V* diagram. Column II gives the characteristics involved in the cycle. Match them with each of the processes given in Column I.



Column I	Column II
(A) Process $A \to B$	(p) Internal energy decreases
(B) Process $B \to C$	(q) Internal energy increases
(C) Process $C \to D$	(r) Heat is lost
(D) Process $D \to A$	(s) Heat is gained
	(t) Work is done on the gas

Column I showns four systems, each of the same length L, for producing standing waves. The lowest possible natural frequency of a system is called its fundamental frequency, whose wavelength is denoted as λ_f . Match each system with statements given in Column II describing the nature and wave length of the standing waves.

Column I	Column II
(A) Pipe closed at one end 	(p) Longitudinal waves
(B) Pipe open at both ends 	(q) Transverse waves
(C) Stretched wire clamped at both ends 	(r) $\lambda_f = L$
(D) Stretched wire clamped at both ends and at mid-point 	(s) $\lambda_f = 2L$
	(t) $\lambda_f = 4L$

ANSWERS

Paper I

1. (c)	**2.** (d)	**3.** (d)	**4.** (a)	**5.** (a)	**6.** (a)
7. (b)	**8.** (b,d)	**9.** (a,c,d)	**10.** (a,b,c,d)	**11.** (a,d)	**12.** (c)
13. (b)	**14.** (d)	**15.** (c)	**16.** (b)	**17.** (3)	**18.** (5)
19. (6)	**20.** (3)	**21.** (9)	**22.** (4)	**23.** (1)	

Paper II

1. (c)	**2.** (a)	**3.** (c)	**4.** (d)	**5.** (c)	**6.** (b)
7. (a)	**8.** (b)	**9.** (a,b,d)	**10.** (c,d)	**11.** (b,c)	**12.** (a,c)
13. (5)	**14.** (4)	**15.** (5)	**16.** (2)	**17.** (4)	**18.** (7)

19. (A) → p,r,t; (B) → p,r; (C) → q, s; (D) → r **20.** (A) → p,t; (B) → p,s; (C) → q, s; (D) → q, r

Hints & Solutions

Paper I

1. Electric flux, $\mathbf{E} \cdot \mathbf{S}$, or $\phi = ES \cos\theta$

Here, θ is the angle between **E** and **S**. In this question $\theta = 45°$, because **S** is perpendicular to the surface.

$$E = E_0$$

$$\Rightarrow \quad S = (\sqrt{2}a)(a) = \sqrt{2}a^2$$

$\therefore$ $\phi = (E_0)(\sqrt{2}a^2)\cos 45° = E_0\, a^2$

$\therefore$ Correct option is (c).

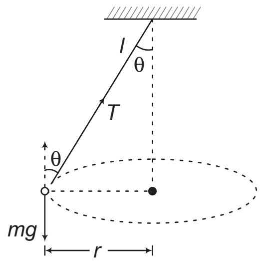
Analysis of Question

(i) Question is moderately tough.

(ii) The given shaded area is a rectangle not a square. One side of this rectangle is a and other side is $\sqrt{2}a$.

(iii) Electric field is uniform, whose magnitude is E_0 and direction is positive x. In uniform electric field we can use, $\phi = ES\cos\theta$

2.



$$r = l\sin\theta$$

$T\cos\theta$ component will cancel mg.

$T\sin\theta$ component will provide necessary centripetal force to the ball towards centre C.

$$\therefore \quad T\sin\theta = mr\omega^2 = m(l\sin\theta)\,\omega^2$$

or $$T = ml\omega^2$$

$$\therefore \quad \omega = \sqrt{\frac{T}{ml}}$$

or $$\omega_{max} = \sqrt{\frac{T_{max}}{ml}} = \sqrt{\frac{324}{0.5\times 0.5}}$$

$$= 36 \text{ rad/s}$$

$\therefore$ Correct option is (d).

Analysis of Question

(i) Question is simple.

(ii) This is called the conical pendulum.

(iii) The interesting fact in this problem is that ω or T is independent of θ.

$$\omega \propto \sqrt{T}$$

If ω is increased, T will also increase.

3. $q_i = C_i V = 2V = q$ (say)

This charge will remain constant after switch is shifted from position 1 to position 2.

$$U_i = \frac{1}{2}\frac{q^2}{C_i} = \frac{q^2}{2\times 2} = \frac{q^2}{4}$$

$$U_f = \frac{1}{2}\frac{q^2}{C_f} = \frac{q^2}{2\times 10} = \frac{q^2}{20}$$

$\therefore$ Energy dissipated $= U_i - U_f = \dfrac{q^2}{5}$

This energy dissipated $\left(= \dfrac{q^2}{5}\right)$ is 80%

of the initial stored energy $\left(= \dfrac{q^2}{4}\right)$.

Analysis of Question

(i) This question is moderately tough.

(ii) In a capacitor circuit, redistribution of charge takes place under following three conditions.

(a) *A* switch is closed.

(b) *A* closed switch is opened.

(c) *A* switch is shifted from one position to another position.

In the redistribution of charge, energy is dissipated.

4. At STP, 22.4 L of any gas is 1 mole.

$$\therefore \quad 5.6 \text{ L} = \frac{5.6}{22.4} = \frac{1}{4} \text{ moles} = n$$

In adiabatic process,

$$TV^{\gamma-1} = \text{constant}$$

$$\therefore \quad T_2 V_2^{\gamma-1} = T_1 V_1^{\gamma-1}$$

or $$T_2 = T_1\left(\frac{V_1}{V_2}\right)^{\gamma-1}$$

$$\gamma = \frac{C_p}{C_V} = \frac{5}{3} \text{ for monoatomic He gas.}$$

$$\therefore \quad T_2 = T_1 \left(\frac{5.6}{0.7}\right)^{\frac{5}{3}-1} = 4T_1$$

$\because$ Further in adiabatic process,

$$Q = 0$$

$$\therefore \quad W + \Delta U = 0$$

or

$$W = -\Delta U$$

$$= -nC_V\Delta T$$

$$= -n\left(\frac{R}{\gamma - 1}\right)(T_2 - T_1)$$

$$= -\frac{1}{4}\left(\frac{R}{\frac{5}{3}-1}\right)(4T_1 - T_1)$$

$$= -\frac{9}{8}RT_1$$

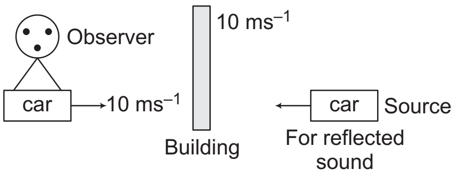
$\therefore$ Correct option is (a).

Analysis of Question

(i) From calculation point of view question is moderately tough.

(ii) Volume of gas is decreasing. Therefore, work done by the gas should be negative.

5. $36 \text{ km/h}^{-1} = 36 \times \frac{5}{18} = 10 \text{ ms}^{-1}$



Apparent frequency of sound heard by car driver (observer) reflected from the building will be

$$f' = f\left(\frac{v + v_0}{v - v_s}\right)$$

$$= 8\left(\frac{320 + 10}{320 - 10}\right)$$

$$= 8.5 \text{ kHz}$$

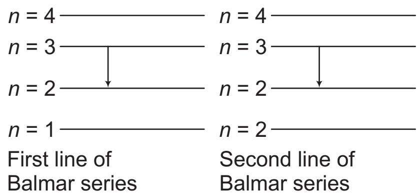
$\therefore$ Correct option is (a).

Analysis of Question

(i) Question is simple.

(ii) Driver will listen two sounds, direct and reflected. Direct sound will be of 8 kHz as driver has no relative motion with the car. But reflected sound is of increased frequency because driver and image of car both are approaching towards each other.

6.



For hydrogen or hydrogen type atoms

$$\frac{1}{\lambda} = RZ^2\left(\frac{1}{n_f^2} - \frac{1}{n_i^2}\right)$$

In the transition from $n_i \longrightarrow n_f$

$$\therefore \quad \lambda \propto \frac{1}{Z^2\left(\frac{1}{n_f^2} - \frac{1}{n_i^2}\right)}$$

$$\therefore \quad \frac{\lambda_2}{\lambda_1} = \frac{Z_1^2\left(\frac{1}{n_f^2} - \frac{1}{n_i^2}\right)_1}{Z_2^2\left(\frac{1}{n_f^2} - \frac{1}{n_i^2}\right)_2}$$

$$\lambda_2 = \frac{\lambda_1 Z_1^2\left(\frac{1}{n_f^2} - \frac{1}{n_i^2}\right)_1}{Z_2^2\left(\frac{1}{n_f^2} - \frac{1}{n_i^2}\right)_2}$$

Substituting the values, we have

$$= \frac{(6561 \text{ Å})(1)^2\left(\frac{1}{2^2} - \frac{1}{3^2}\right)}{(2)^2\left(\frac{1}{2^2} - \frac{1}{4^2}\right)} = 1215 \text{ Å}$$

$\therefore$ Correct option is (a).

Analysis of Question

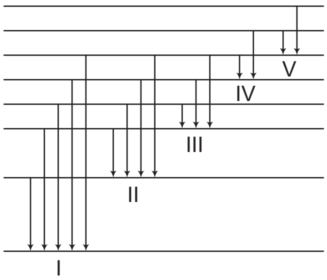
(i) Question is simple.

(ii) In modern physics, mostly questions are asked on the emission of photon by the transition of electron from some higher energy state to some lower energy state.

(iii) For hydrogen and hydrogen like atoms which have only single electron, we can use the formula

$$\frac{1}{\lambda} = RZ^2 \left(\frac{1}{n_f^2} - \frac{1}{n_i^2} \right)$$

(iv) Further, the student should also remember different series, as shown in figure below.



I → Lymen series
II → Balmer series
III → Paschen series
IV → Brackett series
V → Pfund series

7. Using the concept of balanced Wheatstone bridge, we have

$$\frac{P}{Q} = \frac{R}{S}$$

$$\therefore \quad \frac{X}{(52+1)} = \frac{10}{(48+2)}$$

$$\therefore \quad X = \frac{10 \times 53}{50} = 10.6\ \Omega$$

∴ Correct option is (b).

Analysis of Question

Question is moderately tough, because normally end corrections are not taught in the coaching/schools in this type of problem.

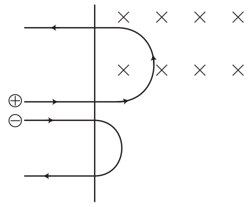
8. $r = \frac{mv}{Bq}$

or $r \propto m$

∴ $r_e < r_p$ as $m_e < m_p$

Further, $T = \frac{2\pi m}{Bq}$

or $T \propto m$



∴ $T_e < T_p$

$t_e = \frac{T_e}{2}$

and $t_p = \frac{T_p}{2}$

or $t_e < t_p$

∴ Correct options are (b) and (d).

Note In JEE 2011, official answer key, correct options were given bc, bd, bcd, here student should realise that both the electron and proton enter the magnetic field at the same time or at different times.

Analysis of Question

(i) Question is simple.

(ii) In the situation given above particle will never complete the circle unless magnetic field is all around.

9. Thermal resistance

$$R = \frac{l}{KA}$$

$$\therefore \quad R_A = \frac{L}{(2K)(4Lw)} = \frac{1}{8Kw}$$

(Here, w = width)

$$R_B = \frac{4L}{3K\,(Lw)} = \frac{4}{3Kw}$$

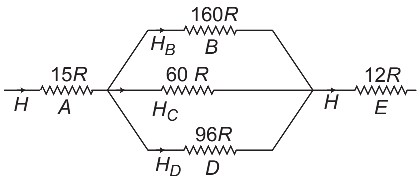
$$R_C = \frac{4L}{(4K)(2Lw)} = \frac{1}{2Kw}$$

$$R_D = \frac{4L}{(5K)(Lw)} = \frac{4}{5Kw}$$

$$R_E = \frac{L}{(6K)(Lw)} = \frac{1}{6Kw}$$

$$R_A : R_B : R_C : R_D : R_E = 15 : 160 : 60 : 96 : 12$$

So, let us write, $R_A = 15\,R, R_B = 160\,R$ etc and draw a simple electrical circuit as shown in figure.



H = Heat current = Rate of heat flow.

$H_A = H_E = H$ [let]

$\therefore$ Option (a) is correct.

In parallel, current distributes in inverse ratio of resistance.

$$\therefore \; H_B : H_C : H_D = \frac{1}{R_B} : \frac{1}{R_C} : \frac{1}{R_D}$$

$$= \frac{1}{160} : \frac{1}{60} : \frac{1}{96}$$

$$= 9 : 24 : 15$$

$$\therefore \quad H_B = \left(\frac{9}{9+24+15}\right) H = \frac{3}{16} H$$

$$H_C = \left(\frac{24}{9+24+15}\right) H = \frac{1}{2} H$$

and $$H_D = \left(\frac{15}{9+24+15}\right) H = \frac{5}{16} H$$

$$H_C = H_B + H_D$$

$\therefore$ Option (d) is correct.

Temperature difference (let us call it T)

= (Heat current) × (Thermal resistance)

$T_A = H_A R_A = (H)(15R) = 15\,HR$

$T_B = H_B R_B = \left(\frac{3}{16} H\right)(160\,R) = 30\,HR$

$T_C = H_C R_C = \left(\frac{1}{2} H\right)(60\,R) = 30\,HR$

$T_D = H_D R_D = \left(\frac{5}{16} H\right)(96\,R) = 30\,HR$

$T_E = H_E R_E = (H)(12\,R) = 12\,HR$

Here, T_E is minimum. Therefore, option (c) is also correct.

$\therefore$ Correct options are (a), (c) and (d).

Analysis of Question

(i) From calculation point of view, question is difficult otherwise question is simple.

(ii) In heat transfer, questions are mainly asked from conduction and radiation topic.

10. Inside a conducting shell, electric field is always zero. Therefore, option (a) is correct. When the two are connected, their potentials become the same.

$\therefore \quad V_A = V_B$

or $$\frac{Q_A}{R_A} = \frac{Q_B}{R_B}$$

$\left(\because V = \frac{1}{4\pi\varepsilon_0} \frac{Q}{R}\right)$

Since, $R_A > R_B$

$\therefore \quad Q_A > Q_B$

$\therefore$ Option (b) is correct.

Potential is also equal to,

$$V = \frac{\sigma R}{\varepsilon_0}$$

$$V_A = V_B$$

$\therefore \quad \sigma_A R_A = \sigma_B R_B$

or $$\frac{\sigma_A}{\sigma_B} = \frac{R_B}{R_A}$$

$\therefore$ Option (c) is correct.

Electric field on surface,

$$E = \frac{\sigma}{E_0} \text{ or } E \propto \sigma$$

or $\sigma_A < \sigma_B$

Since, $\sigma_A < \sigma_B$

$\therefore \quad E_A < E_B$

$\therefore$ Option (d) is also correct.

$\therefore$ Correct options are (a), (b), (c) and (d).

Analysis of Question

(i) Question is simple.

(ii) $E = \frac{1}{4\pi\varepsilon_0} \frac{Q}{R^2} = \frac{\sigma}{\varepsilon_0}$ (On surface)

As, σ = surface charge density

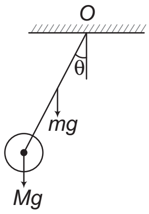
$= \frac{Q}{4\pi R^2}$

(iii) $V = \frac{1}{4\pi\varepsilon_0} \frac{Q}{R} = \frac{\sigma R}{\varepsilon_0}$

11. $\tau_A = \tau_B = mg \frac{L}{2} \sin\theta + MgL \sin\theta$

= Restoring torque about point O.

In case A, moment of inertia will be more. Hence, angular acceleration $(\alpha = \tau / I)$ will be less. Therefore, angular frequency will be less.



$\therefore$ Correct options are (a) and (d).

Analysis of Question

Question is difficult from my point of view. Because this type of SHM is rarely taught in the class and questions of this type are not given in standard books.

12. N = Number of electrons per unit volume

$$\therefore \quad [N] = [L^{-3}], [e] = [q] = [It] = [AT]$$

$$[\varepsilon_0] = [M^{-1}L^{-3}T^4A^2]$$

Substituting the dimensions, we can see that,

$$\left[\sqrt{\frac{Ne^2}{m\varepsilon_0}}\right] = [T^{-1}]$$

Angular frequency has also the dimension $[T^{-1}]$.

$\therefore$ Correct option is (c).

Analysis of Question

(i) From calculation point of view, question is moderately difficult. Otherwise it is simple.

(ii) Students may commit a mistake in the dimensions of N. It is not dimensionless.

13. $$\omega = 2\pi f = \frac{2\pi c}{\lambda}$$

$$\therefore \quad \lambda = \frac{2\pi c}{\omega} = \frac{2\pi c}{\sqrt{Ne^2 / m\varepsilon_0}}$$

Substituting the values, we get

$$\lambda = 600 \text{ nm}$$

$\therefore$ Correct option is (b).

Analysis of Question

(i) Paragraph does not make the solution very clear.

(ii) Solution is simple but based on error and trial method.

14. Momentum is first positive but decreasing. Displacement (or say position). is initially zero.

It will first increase. At highest point, momentum is zero and displacement is maximum. After that momentum is downwards (negative) and increasing but displacement is decreasing. Only (d) option satisfies these conditions.

Analysis of Question

(i) Question is simple.

(ii) Since all the graphs given in question are parabolic type, we need not to check the mathematical part of the question.

15. $$E = \frac{1}{2} m\omega^2 A^2$$

or $$E \propto A^2$$

$$\frac{E_2}{E_1} = \left(\frac{A_2}{A_1}\right)^2 = \left(\frac{a}{2a}\right)^2$$

or $$E_1 = 4E_2$$

$\therefore$ Correct option is (c).

Analysis of Question

(i) Question is simple.

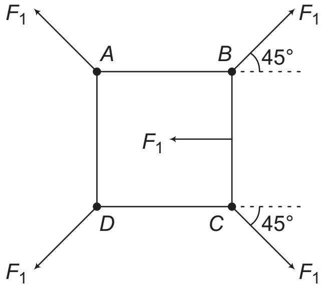
(ii) Since, oscillator is same, $\omega_1 = \omega_2$

16. In all the given four figures, at mean position the position coordinate is zero. At the same time mass is starting from the extreme position in all fours cases. In figures (c) and (d), extreme position is more than the initial extreme position. But due to viscosity opposite should be the case. Hence, the answer should be either option (a) or option (b).

Correct answer is (b), because mass starts from positive extreme position (from uppermost position). Then, it will move downwards or momentum should be negative.

Analysis of Question

Question is simple but (*a*) and (*b*) options are confusing.

17. 

F_1 = Net electrostatic force on anyone charge due to rest of three charges

$$= \frac{1}{4\pi\varepsilon_0}\frac{q^2}{a^2}\left(\sqrt{2}+\frac{1}{2}\right)$$

$$F_2 = \text{Surface tension force} = \gamma a$$

If we see the equilibrium of line BC,

then, $\quad 2F_1 \cos 45° = F_2$

or $\quad \sqrt{2}\, F_1 = F_2$

or $\quad \dfrac{1}{4\pi\varepsilon_0}\dfrac{q^2}{a^2}\left(2+\dfrac{1}{\sqrt{2}}\right) = \gamma a$

$\therefore \quad a^3 = \dfrac{1}{4\pi\varepsilon_0}\left(2+\dfrac{1}{\sqrt{2}}\right)\dfrac{q^2}{\gamma}$

or $\quad a = \left\{\dfrac{1}{4\pi\varepsilon_0}\left(2+\dfrac{1}{\sqrt{2}}\right)\right\}^{1/3}\left[\dfrac{q^2}{\gamma}\right]^{1/3}$

$$= k\left[\frac{q^2}{\gamma}\right]^{1/3}$$

where, $\quad k = \left\{\dfrac{1}{4\pi\varepsilon_0}\left(2+\dfrac{1}{\sqrt{2}}\right)\right\}^{1/3}$

Therefore, $N = 3$

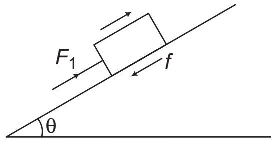
Answer is 3.

Analysis of Question

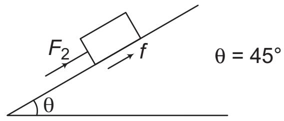
(i) Question is moderately difficult.

(ii) In my opinion problems of surface tension or viscosity are always not very simple questions.

18.



Moving upwords



Just remains stationary

$$F_1 = mg\sin\theta + \mu mg\cos\theta$$
$$F_2 = mg\sin\theta - \mu mg\cos\theta$$

Given that, $F_1 = 3F_2$

or $\quad (\sin 45° + \mu\cos 45°)$
$$= 3(\sin 45° - \mu\cos 45°)$$

On solving, we get $\mu = 0.5$

$\therefore \quad N = 10\,\mu = 5$

$\therefore$ Answer is 5.

Analysis of Question

Question is simple. Only one has to take care of direction and magnitude of friction.

19. Take the circular tube as a long solenoid. The wires are closely wound. Magnetic field inside the solenoid is

$$B = \mu_0 ni$$

Here, n = number of turns per unit length

$\therefore ni$ = current per unit length

In the given problem,

$$ni = \frac{I}{L}$$

$\therefore \quad B = \dfrac{\mu_0 I}{L}$

Flux passing through the circular coil is

$$\phi = BS = \left(\frac{\mu_0 I}{L}\right)(\pi r^2)$$

Induced emf, $e = -\dfrac{d\phi}{dt} = -\left(\dfrac{\mu_0\pi r^2}{LR}\right)\cdot\dfrac{dI}{dt}$

Induced current,

$$i = \frac{e}{R} = -\left(\frac{\mu_0\pi r^2}{LR}\right)\frac{dI}{dt}$$

Magnetic moment $iA = i\pi r^2$

or $M = -\left(\frac{\mu_0 \pi^2 r^4}{LR}\right) \cdot \frac{dI}{dt}$ ...(i)

Given, $I = I_0 \cos 300\, t$

$\therefore \frac{dI}{dt} = -300 I_0 \sin(300\, t)$

Substituting in Eq. (i), we get

$$M = \left(\frac{300\,\pi^2 r^4}{LR}\right) \mu_0 I_0 \sin 300\, t$$

$$\therefore \quad N = \frac{300\,\pi^2 r^4}{LR}$$

Substituting the values, we get

$$N = \frac{300\,(22/7)^2\,(0.1)^4}{(10)(0.005)}$$

$$= 5.926$$

or $N \simeq 6$

Analysis of Question

Question is difficult to understand inside the examination hall. But 5 to 10% question in IIT JEE are always difficult. Students should not panic. Because topper of IIT JEE scores approximately 80-90%.

20. $\Delta l_1 = \frac{FL}{AY} = \frac{mgL}{\pi r^2 Y}$ = Increase in length

$\Delta l_2 = L\,\alpha\,\Delta\theta$ = Decrease in length

To regain its original length,

$$\Delta l_1 = \Delta l_2$$

$$\therefore \quad \frac{mgL}{\pi r^2 Y} = L\,\alpha\,\Delta\theta$$

$$\therefore \quad m = \left(\frac{r^2 Y \alpha\, \Delta\theta}{g}\right)$$

Substituting the values, we get

$$m \simeq 3 \text{ kg}$$

$\therefore$ Answer is 3.

Analysis of Question

Question is very simple. In my opinion any student who have gone through the syllabus once can solve this problem easily.

21. $r = \frac{d}{2} = \frac{\sqrt{5}}{2}$ cm

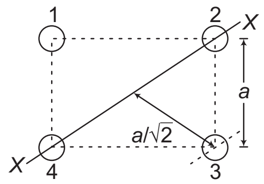
$$= \frac{\sqrt{5}}{2} \times 10^{-2} \text{ m}$$

$m = 0.5$ kg

$a = 4$ cm

$= 4 \times 10^{-2}$ m

$$I_{XX} = I_1 + I_2 + I_3 + I_4$$



$$= \left[\frac{2}{5} mr^2 + m\left(\frac{a}{\sqrt{2}}\right)^2\right] + \frac{2}{5} mr^2$$

$$+ \left[\frac{2}{5} mr^2 + m\left(\frac{a}{\sqrt{2}}\right)^2\right] + \frac{2}{5} mr^2$$

Substituting the values, we get

$$I_{XX} = 9 \times 10^{-4} \text{ Kgm}^{-2}$$

$\therefore \quad N = 9$

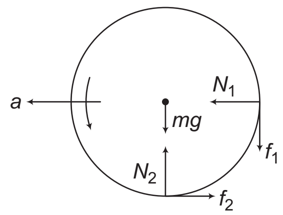
Answer is 9.

Analysis of Question

(i) Question is simple.

(ii) Only theorem of parallel axes is to be used properly.

22.



There is no slipping between ring and ground. Hence, f_2 is not maximum. But there is slipping between ring and stick. Therefore, f_1 is maximum. Now, let us write the equations.

$$I = mR^2 = (2)\,(0.5)^2$$

$$= \frac{1}{2} \text{ kgm}^{-2}$$

$$N_1 - F_2 = ma$$

or $N_1 - F_2 = (2)\,(0.3) = 0.6$ N ...(i)

$$a = R\,\alpha = \frac{R\tau}{I}$$

$$= \frac{R(f_2 - f_1)R}{I} = \frac{R^2(f_2 - f_1)}{I}$$

$$\therefore \quad 0.3 = \frac{(0.5)^2\,(f_2 - f_1)}{(1/2)}$$

or $\quad f_2 - f_1 = 0.6\text{ N} \quad \ldots\text{(ii)}$

$$N_1^2 + f_1^2 = (2)^2 = 4 \quad \ldots\text{(iii)}$$

Further $\quad F_1 = \mu N_1 = \left(\frac{P}{10}\right)N_1 \quad \ldots\text{(iv)}$

Solving above four equation, we get

$$P \simeq 3.6$$

Therefore, the correct answer should be 4.

Analysis of Question

(i) Question is moderately tough from concept point of view. But calculations are lengthy.

(ii) One has to think about the two components of the force applied by the stick.

(iii) Answer comes an integer when you consider only N_1.

23. Activity $\left(-\frac{dN}{dt}\right) = \lambda N = \left(\frac{1}{t_{\text{mean}}}\right) \times N$

$$\therefore \quad N = \left(-\frac{dN}{dt}\right) \times t_{\text{mean}}$$

= Total number of atoms

Mass of one atom is 10^{-25} kg $= m$ (say)

$\therefore$ Total mass of radioactive substance

= (number of atoms) × (mass of one atom)

$$= \left(-\frac{dN}{dt}\right)(t_{\text{mean}})(m)$$

Substituting the values, we get

Total mass of radioactive substance

= 1 mg

$\therefore$ Answer is 1.

Analysis of Question

Question is very simple. Again in my opinion one can solve it.

Paper II

1. After critical angle reflection will be 100% and transmission is 0%. Options (b) and (c) satisfy this condition. But option (c) is the correct option. Because in option (b) transmission is given 100% at $\theta = 0°$, which is not true.

Analysis of Question

From examination point of view question is moderately difficult. Because it takes little time to understand the given graphs clearly.

2. Frequency or time period of SHM depends on variable forces. It does not depend on constant external force. Constant external force can only change the mean position. For example, in the given question mean position is at natural length of spring in the absence of electric field. Whereas in the presence of electric field mean position will be obtained after a compression of x_0. Where x_0 is given by

$$kx_0 = QE$$

or $$x_0 = \frac{QE}{k}$$

$\therefore$ Correct answer is (a).

Analysis of Question

(i) Question is simple.

(ii) I think almost all the serious aspirants of JEE can solve this problem easily.

3. Least count of screw gauge $= \frac{0.5}{50}$

$= 0.01 \text{ mm} = \Delta r$

Diameter, $r = 2.5 \text{ mm} + 20 \times \frac{0.5}{50}$

$= 2.70 \text{ mm}$

$$\frac{\Delta r}{r} = \frac{0.01}{2.70}$$

or $\frac{\Delta r}{r} \times 100 = \frac{1}{2.7}$

Now, density $d = \frac{m}{V} = \frac{m}{\frac{4}{3}\pi\left(\frac{r}{2}\right)^3}$

Here, r is the diameter.

$$\therefore \quad \frac{\Delta d}{d} \times 100 = \left\{\frac{\Delta m}{m} + 3\left(\frac{\Delta r}{r}\right)\right\} \times 100$$

$$= \frac{\Delta m}{m} \times 100 + 3 \times \left(\frac{\Delta r}{r}\right) \times 100$$

$$= 2\% + 3 \times \frac{1}{2.7}$$

$$= 3.11\%$$

Analysis of Question

(i) Question is moderately difficult.

(ii) In practical part, questions in JEE are asked from vernier callipers and screw gauge.

4. Time taken by the bullet and ball to strike the ground is

$$t = \sqrt{\frac{2h}{g}} = \sqrt{\frac{2 \times 5}{10}} = 1 \text{ s}$$

Let v_1 and v_2 are the velocities of ball and bullet after collision.

Then applying

$$x = vt$$

We have, $20 = v_1 \times 1$

or $v_1 = 20 \text{ ms}^{-1}$

$100 = v_2 \times 1$ or $v_2 = 100 \text{ m/s}^{-1}$

Now, from conservation of linear momentum before and after collision we have,

$$0.01\, v = (0.2 \times 20) + (0.01 \times 100)$$

On solving, we get

$$v = 500 \text{ ms}^{-1}$$

$\therefore$ Correct answer is (d).

Analysis of Question

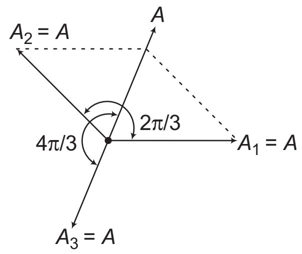
Question is moderately lengthy from calculation point of view, otherwise it is simple.

5. Correct answer is (c), because induced electric field lines (produced by change in magnetic field) and magnetic field lines form closed loops.

Analysis of Question

Question is simple, provided you have an idea of induced electric field lines.

6.



Resultant amplitude of x_1 and x_2 is A at angle $\left(\frac{\pi}{3}\right)$ from A_1. To make resultant of x_1, x_2 and x_3 to be zero. A_3 should be equal to A at angle $\phi = \frac{4\pi}{3}$ as shown in figure.

$\therefore$ Correct answer is (b).

Alternate Solution

If we substitute,

$$x_1 + x_2 + x_3 = 0$$

or $$A \sin \omega t + A \sin\left(\omega t + \frac{2\pi}{3}\right) + B \sin(\omega t + \phi) = 0$$

Then, by applying simple mathematics, we can prove that

$$B = A$$

and $$\phi = \frac{4\pi}{3}$$

Analysis of Question

(i) Question is simple.

(ii) Question can be solved by applying mathematics also.

(iii) Amplitudes of two or more sine or cosine functions of same frequency ω can be added by vector method.

7. If we take a small strip of dr at distance r from centre, then number of turns in this strip would be

$$dN = \frac{N}{b-a} dr$$

Magnetic field due to this element at the centre of the coil will be

$$dB = \frac{\mu_0 (dN) I}{2r} = \frac{\mu_0 \, NI}{2(b-a)} \frac{dr}{r}$$

$$\therefore \quad B = \int_{r=a}^{r=b} dB = \frac{\mu_0 NI}{2(b-a)} \ln \frac{b}{a}$$

∴ Correct answer is (a).

Analysis of Question

(i) If we see this problem independently, then I will rate this question moderately difficult. But the idea of this question is taken from question number 3.245 of IE Irodov.

(ii) Interestingly the same question was asked in IIT-JEE 2001 also.

8. In circular orbit of a satellite, potential energy

$$= -2 \times (\text{kinetic energy})$$

$$= -2 \times \frac{1}{2} mv^2 = -mv^2$$

Just to escape from the gravitational pull, its total mechanical energy should be zero. Therefore, its kinetic energy should be $+mv^2$.

∴ Correct answer is (b).

Analysis of Question

(i) Question is moderately difficult.

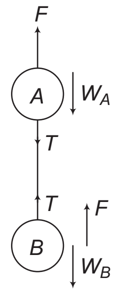
(ii) In circular orbit of satellite

$$U = -\frac{GMm}{r} \text{ and } K = \frac{GMm}{2r}$$

or $U = -2K$

Here, U is potential energy and K is kinetic energy.

9.



$$F = \text{Upthrust} = Vd_F g$$

Equilibrium of A

$$Vd_F g = T + W_A$$
$$= T + Vd_A g \quad \ldots\text{(i)}$$

Equilibrium of B,

$$T + Vd_F g = Vd_B g \quad \ldots\text{(ii)}$$

Adding Eqs. (i) and (ii), we get

$$2d_f = d_A + d_B$$

∴ Option (d) is correct.

From Eq. (i), we can see that

$$d_F > d_A \quad [\text{as } T > 0]$$

∴ Option (a) is correct.

From Eq. (ii) we can see that,

$$d_B > d_F$$

∴ Option (a) is correct.

∴Correct options are (a), (b) and (d).

Analysis of Question

Question is moderately difficult but conceptwise it is good.

10. If charges are of opposite signs, then the two fields are along the same direction. So, they cannot be zero. Hence, the charges should be of same sign.

Therefore, option (c) is correct.

Further, work done by external force = change in potential energy

$$\therefore \quad W_{A \to B} = q\,(\Delta V)$$
$$= (+1)(V_B - V_A)$$

or $\quad W_{A \to B} = V_B - V_A$

Therefore, option (d) is also correct.

∴Correct options are (c) and (d).

Analysis of Question

(i) Question is simple.

(ii) $W = \Delta U$ (by external force)

If charge is just moved, without changing the kinetic energy. If nothing is mentioned in the question regarding the kinetic energy then we take $W = \Delta U$.

11. $Z = \sqrt{R^2 + X_C^2} = \sqrt{R^2 + \left(\frac{1}{\omega C}\right)^2}$

In case (b) capacitance C will be more.

Therefore, impedance Z will be less.

Hence, current will be more.

$\therefore$ Option (b) is correct.

Further, $V_C = \sqrt{V^2 - V_R^2}$

$= \sqrt{V^2 - (IR)^2}$

In case (b), since current I is more.

Therefore, V_C will be less.

$\therefore$ Option (c) is correct.

$\therefore$ Correct options are (b) and (c).

Analysis of Question

(i) Question is moderately difficult.

(ii) In my opinion problems of alternating currents are not very difficult.

(iii) Topic of AC is small. One can feel comfortable in this topic by putting less efforts.

12. The data is incomplete. Let us assume that friction from ground on ring is not impulsive during impact.

From linear momentum conservation in horizontal direction, we have

$(-2\times 1) + (0.1\times 20)$

$= (0.1\times 0) + (2\times v) \quad \overset{-\text{ve}\quad +\text{ve}}{\longleftarrow\!\!\longrightarrow}$

Here, v is the velocity of CM of ring after impact.

Solving the above equation, we have

$v = 0$

Thus, CM becomes stationary.

$\therefore$ Correct answer is (a).

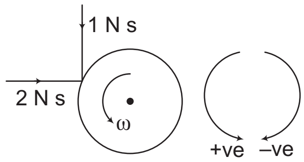
Linear impulse during impact

(i) In horizontal direction

$J_1 = \Delta P = 0.1 \times 20 = 2\,\text{Ns}$

(ii) In vertical direction

$J_2 = \Delta P = 0.1 \times 10 = 1\,\text{Ns}$



Writing the equation (about CM)

Angular impulse

= Change in angular momentum

$1 \times \left(\frac{\sqrt{3}}{2} \times \frac{1}{2}\right) - 2 \times 0.5 \times \frac{1}{2}$

$= 2 \times (0.5)^2 \left[\omega - \frac{1}{0.5}\right]$

Solving this equation ω comes out to be positive or ω anti-clockwise. So just after collision rightwards slipping is taking place.

Hence, friction is leftwards.

Therefore, option (c) is also correct.

$\therefore$ Correct options are (a) and (c).

Note In JEE 2011 official answer key, correct option were given a, ac.

Analysis of Question

(i) Question is moderately difficult.

(ii) In such type of problems impulse due to friction during collision is ignored.

13. V_{AB} = Equivalent emf of two batteries in parallel.

$= \frac{E_1/r_1 + E_1/r_2}{1/r_1 + 1/r_2}$

$= \frac{(6/1) + (3/2)}{(1/1) + (1/2)} = 5\,\text{V}$

$\therefore$ Answer is 5.

Analysis of Question

(i) In my opinion this is the simplest question of JEE 2011.

(ii) When no current is drawn from this equivalent battery, then

$V_{AB} = V = E$

Otherwise, $V = E \pm ir$

(iii) Similar type of question was asked in JEE 1981 and cancelled paper of JEE 1997.

14. $Z = \sqrt{R^2 + X_C^2} = R\sqrt{1.25}$

$\therefore \quad R^2 + X_C^2 = 1.25\, R^2$

or $\quad X_C = \dfrac{R}{2}$

or $\quad \dfrac{1}{\omega C} = \dfrac{R}{2}$

$\therefore$ Time constant $= CR = \dfrac{2}{\omega}$

$$= \frac{2}{500}\ \text{s} = 4\ \text{ms}$$

$\therefore$ Answer is 4.

Analysis of Question

(i) Question is very simple.

(ii) I think this is one of the simplest formula based question of this paper.

15. $t = T = \dfrac{2u \sin \theta}{g}$

$$= \frac{2 \times 10 \times \sin 60^\circ}{10} = \sqrt{3}\ \text{s}$$

Displacement of train in time $t = \dfrac{1}{2} at^2$

Displacement of boy with respect to train

$$= 1.15\ \text{m}$$

$\therefore$ Displacement of boy with respect to ground $= \left(1.15 + \dfrac{1}{2} at^2\right)$

Displacement of ball with respect to ground $= (u \cos 60^\circ) t$

To catch the ball back at initial height,

$$1.15 + \frac{1}{2} at^2 = (u \cos 60^\circ)\, t$$

$$\therefore \quad 1.15 + \frac{1}{2} a (\sqrt{3})^2 = 10 \times \frac{1}{2} \times \sqrt{3}$$

Solving this equation, we get

$$a = 5\ \text{ms}^{-2}$$

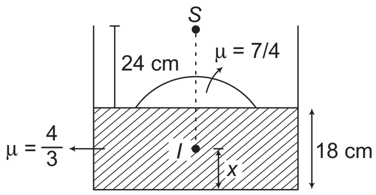
$\therefore$ Answer is 5.

Analysis of Question

(i) Question is moderately tough.

(ii) Velocity of ball given in the question is with respect to ground.

16.



Two refractions will take place, first from spherical surface and the other from the plane surface.

So, applying

$$\frac{\mu_2}{v} - \frac{\mu_1}{u} = \frac{\mu_2 - \mu_1}{R}$$

two times with proper sign convention. Ray of light is travelling downwards. Therefore, downward direction is taken as positive direction.

$$\frac{7/4}{v} - \frac{1.0}{-24} = \frac{7/4 - 1.0}{+6} \qquad \ldots \text{(i)}$$

$$\frac{4/3}{(18 - x)} - \frac{7/4}{v} = \frac{4/3 - 7/4}{\infty} \qquad \ldots \text{(ii)}$$

Solving these equations, we get

$$x = 2\,\text{cm}$$

$\therefore$ Answer is 2.

Analysis of Question

(i) Question is moderately difficult from calculation point of view, otherwise it is simple.

(ii) $\dfrac{\mu_2}{v} - \dfrac{\mu_1}{u} = \dfrac{\mu_2 - \mu_1}{R}$ can be applied for

plane surface also with $R = \infty$

17. Decrease in mechanical energy

= Work done against friction

$$\therefore \quad \frac{1}{2} mv^2 - \frac{1}{2} kx^2 = \mu\, mgx$$

or $\quad v = \sqrt{\dfrac{2\mu\, mgx + kx^2}{m}}$

Substituting the values, we get

$$v = 0.4\ \text{ms}^{-1} = \left(\frac{4}{10}\right) \text{ms}^{-1}$$

$\therefore$ Answer is 4.

Analysis of Question

(i) Question is simple.

(ii) If μ_s and μ_k two values of coefficient of friction are given, then we will take μ_k.

18. Photoemission will stop when potential on silver sphere becomes equal to the stopping potential.

$$\therefore \quad \frac{hc}{\lambda} - W = eV_0$$

Here, $V_0 = \frac{1}{4\pi\varepsilon_0} \frac{ne}{r}$

$$\therefore \quad \left(\frac{1240}{1200} eV\right) - (4.7\ eV)$$

$$= \frac{9 \times 10^9 \times n \times 1.6 \times 10^{-19}}{10^{-2}}$$

$$(6.2 - 4.7) = \frac{9 \times 10^9 \times n \times 1.6 \times 10^{-19}}{10^{-2}}$$

or $n = \frac{1.5 \times 10^{-2}}{9 \times 1.6 \times 10^{-10}} = 1.04 \times 10^7$

$\therefore$ Answer is 7.

Analysis of Question

(i) Question is moderately difficult.

(ii) In eV_0 if we substitute the value of $e = 1.6 \times 10^{-19}$ C, then answer is in J. If we leave it then, answer is in eV.

19. Internal energy $\propto T \propto pV$

This is because

$$U = \frac{nf}{2} RT = \frac{f}{2} pV$$

Here, n = number of moles

f = degree of freedom

If the product pV increases, then internal energy will increase and if product decreases, the internal energy will decrease.

Further, work is done on the gas, if volume of gas decreases. For heat exchange.

$$Q = W + \Delta U$$

Work done is area under p-V graph. If volume increases work done by gas is positive and if volume decreases work done by gas is negative. Further ΔU is positive if product of pV is increasing and ΔU is negative, if product of pV is decreasing. If heat is taken by the gas Q is positive and if heat is lost by the gas Q is negative.

Keeping the above points in mind the answer to this question is as under.

(A) → (p, r, t)
(B) → (p, r)
(C) → (q, s)
(D) → (r, t)

Analysis of Question

(i) Calculation wise, question is slightly lengthy. Otherwise question is theory based and simple.

(ii) In process DA,

$$p_A V_A = p_D V_D$$

$\therefore \quad T_A = T_D$

or $\quad \Delta U = 0$

Further, volume of gas is decreasing. Therefore, work is done on the gas or work done by gas is negative. Therefore, Q is negative or heat is lost.

(iii) This question covers almost all the concepts of first law of thermodynamics.

20. In organ pipes, longitudinal waves are formed. In string, transverse waves are formed. Open end of pipe is displacement antinode and closed end is displacement node. In case of string fixed end of a string is node.

Further, least distance between a node and an antinode is $\frac{\lambda}{4}$ and between two nodes is $\frac{\lambda}{2}$. Keeping these points in mind answer to this question is as under;

(A) → (p, t)
(B) → (p, s)
(C) → (q, s)

IIT JEE

UNSOLVED PAPER 2011

Mathematics

Paper I

SECTION I

Directions (Q. Nos. 1-7) *This section contains 7 multiple choice questions. Each question has four choices (a), (b), (c) and (d) out of which only one is correct.*

Let $\mathbf{a} = \hat{\mathbf{i}} + \hat{\mathbf{j}} + \hat{\mathbf{k}}$, $\mathbf{b} = \hat{\mathbf{i}} - \hat{\mathbf{j}} + \hat{\mathbf{k}}$ and $\mathbf{c} = \hat{\mathbf{i}} - \hat{\mathbf{j}} - \hat{\mathbf{k}}$ be three vectors. A vector $\mathbf{v}$ in the plane of $\mathbf{a}$ and $\mathbf{b}$ whose projection of $\mathbf{c}$ is $\frac{1}{\sqrt{3}}$, is given by

(a) $\hat{\mathbf{i}} - 3\hat{\mathbf{j}} + 3\hat{\mathbf{k}}$
(b) $-3\hat{\mathbf{i}} - 3\hat{\mathbf{j}} - \hat{\mathbf{k}}$
(c) $3\hat{\mathbf{i}} - \hat{\mathbf{j}} + 3\hat{\mathbf{k}}$
(d) $\hat{\mathbf{i}} + 3\hat{\mathbf{k}} - 3\hat{\mathbf{k}}$

Let the straight line $x = b$ divide the area enclosed by $y = (1 - x)^2$, $y = 0$ and $x = 0$ into two parts $R_1 (0 \le x \le b)$ and $R_2 (b \le x \le 1)$ such that $R_1 - R_2 = \frac{1}{4}$. Then, b equals to

(a) $\frac{3}{4}$
(b) $\frac{1}{2}$
(c) $\frac{1}{3}$
(d) $\frac{1}{4}$

A straight line L through the point $(3, -2)$ is inclined at an angle $60°$ to the line $\sqrt{3}x + y = 1$. If L also intersects the X-axis, then the equation of L is

(a) $y + \sqrt{3}x + 2 - 3\sqrt{3} = 0$
(b) $y - \sqrt{3}x + 2 + 3\sqrt{3} = 0$
(c) $\sqrt{3}y - x + 3 + 2\sqrt{3} = 0$
(d) $\sqrt{3}y + x - 3 + 2\sqrt{3} = 0$

The value of $\int_{\sqrt{\log 2}}^{\sqrt{\log 3}} \frac{x \sin x^2}{\sin x^2 + \sin(\log 6 - x^2)} dx$ is

(a) $\frac{1}{4} \log \frac{3}{2}$
(b) $\frac{1}{2} \log \frac{3}{2}$
(c) $\log \frac{3}{2}$
(d) $\frac{1}{6} \log \frac{3}{2}$

Let (x_0, y_0) be the solution of the following equations $(2x)^{\log 2} = (3y)^{\log 3}$, $3^{\log x} = 2^{\log y}$, then x_0 is equal to

(a) $\frac{1}{6}$
(b) $\frac{1}{2}$
(c) $\frac{1}{2}$
(d) 6

Let $P = \{\theta : \sin\theta - \cos\theta = \sqrt{2}\cos\theta\}$ and $Q = \{\theta : \sin\theta + \cos\theta = \sqrt{2}\sin\theta\}$ be two sets. Then,

(a) $P \subset Q$ and $Q - P \neq \Phi$
(b) $Q \not\subset P$
(c) $P \not\subset Q$
(d) $P = Q$

Let α and β be the roots of $x^2 - 6x - 2 = 0$ with $\alpha > \beta$. If $a_n = \alpha^n - \beta^n$ for $n \geq 1$, then the value of $\frac{a_{10} - 2a_8}{2a_9}$ is

(a) 1 (b) 2 (c) 3 (d) 4

SECTION II

Directions (Q. Nos. 8-11) *This section contains 4 multiple choice questions. Each question has four choices (a), (b), (c) and (d) out of which one ore more may be correct.*

Let M and N be two 3×3 non-singular skew-symmetric matrices such that $MN = NM$. If P^T denotes the transpose of P, then $M^2N^2(M^TN)^{-1}(MN^{-1})^T$ is equal to

(a) M^2 (b) $-N^2$ (c) $-M^2$ (d) MN

. Let the eccentricity of the hyperbola $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ be reciprocal to that of the ellipse $x^2 + 4y^2 = 4$. If the hyperbola passes through a focus of the ellipse, then

(a) the equation of the hyperbola is $\frac{x^2}{3^2} - \frac{y^2}{2^2} = 1$

(b) a focus of the hyperbola is (2, 0)

(c) the eccentricity of the hyperbola is $\sqrt{\frac{5}{3}}$

(d) the equation of the hyperbola is $x^2 - 3y^2 = 3$

The vector(s) which is/are coplanar with vectors $\hat{\mathbf{i}} + \hat{\mathbf{j}} + 2\hat{\mathbf{k}}$ and $\hat{\mathbf{i}} + 2\hat{\mathbf{j}} + \hat{\mathbf{k}}$, are perpendicular to the vector $\hat{\mathbf{i}} + \hat{\mathbf{j}} + \hat{\mathbf{k}}$ is/are

(a) $\hat{\mathbf{j}} - \hat{\mathbf{k}}$ (b) $-\hat{\mathbf{i}} + \hat{\mathbf{j}}$ (c) $\hat{\mathbf{i}} - \hat{\mathbf{j}}$ (d) $-\hat{\mathbf{j}} + \hat{\mathbf{k}}$

Let $f : R \to R$ be a function such that $f(x + y) = f(x) + f(y), \forall x, y \in R$. If $f(x)$ is differentiable at $x = 0$, then

(a) $f(x)$ is differentiable only in a finite interval containing zero
(b) $f(x)$ is continuous, $\forall x \in R$
(c) $f'(x)$ is constant, $\forall x \in R$
(d) $f(x)$ is differentiable except at finitely many points

SECTION III

Directions (Q. Nos. 12-16) *This section contains 2 paragraphs. Based upon one of the paragraphs 2 multiple choice questions and based on the other paragraph 3 multiple choice questions have the be answered. Each of these questions has four choices (a), (b), (c) and (d) out of which only one is correct.*

Paragraph (Q. Nos. 12-13)

Let U_1 and U_2 be two urns such that U_1 contains 3 white and 2 red balls and U_2 contains only 1 white ball. A fair coin is tossed. If head appears then 1 ball is drawn at random from U_1 and put into U_2. However, if tail appears then 2 balls are drawn at random from U_1 and put into U_2. Now, 1 ball is drawn at random from U_2.

The probability of the drawn ball from U_2 being white is

(a) $\frac{13}{30}$ (b) $\frac{23}{30}$ (c) $\frac{19}{30}$ (d) $\frac{11}{30}$

Given that the drawn ball from U_2 is white, the probability that head appeared on the coin is

(a) $\frac{17}{23}$ (b) $\frac{11}{23}$ (c) $\frac{15}{23}$ (d) $\frac{12}{23}$

Paragraph (Q. Nos. 14-16)

Let a, b and c be three real numbers satisfying $[a \ \ b \ \ c]\begin{bmatrix}1 & 9 & 7\\8 & 2 & 7\\7 & 3 & 7\end{bmatrix}=[0 \ \ 0 \ \ 0]$ ...(E)

If the point $P(a, b, c)$, with reference to Eq. (E), lies on the plane $2x + y + z = 1$, then the value of $7a + b + c$ is

(a) 0 (b) 12 (c) 7 (d) 6

Let ω be a solution of $x^3 - 1 = 0$ with Im (ω) > 0. If $a = 2$, with b and c satisfying Eq. (E), then the value of $\frac{3}{\omega^a}+\frac{1}{\omega^b}+\frac{3}{\omega^c}$

(a) – 2 (b) 2 (c) 3 (d) – 3

Let $b = 6$, with a and c satisfying Eq. (E). If α and β are the roots of the quadratic equation $ax^2 + bx + c = 0$, then $\sum_{n=0}^{\infty}\left(\frac{1}{\alpha}+\frac{1}{\beta}\right)^n$ is

(a) 6 (b) 7 (c) $\frac{6}{7}$ (d) ∞

SECTION IV

Directions (Q. Nos. 17-23) *This section contains 7 questions. The answer to each of the questions is a single digit integer, ranging from 0 to 9.*

The positive integer value of $n > 3$ satisfying the equation

$$\frac{1}{\sin\left(\frac{\pi}{n}\right)}=\frac{1}{\sin\left(\frac{2\pi}{n}\right)}+\frac{1}{\sin\left(\frac{3\pi}{n}\right)} \text{ is}$$

Let $a_1, a_2, a_3, \ldots, a_{100}$ be an arithmetic progression with $a_1 = 3$ and $S_p = \sum_{i=1}^{p} a_i$, $1 \le p \le 100$. For any integer n with $1 \le n \le 20$, let $m = 5n$. If $\frac{S_m}{S_n}$ does not depend on n, then a_2 is

The minimum value of the sum of real numbers a^{-5}, a^{-4}, $3a^{-3}$, 1, a^8 and a^{10} with $a > 0$ is

Let $f : [1, \infty) \to [2, \infty)$ be a differentiable function such that $f(1) = 2$. If $6\int_1^x f(t)dt = 3x\ f(x) - x^3$ for all $x \ge 1$, then the value of $f(2)$ is

Let $f(\theta) = \sin\left[\tan^{-1}\left(\frac{\sin\theta}{\sqrt{\cos 2\theta}}\right)\right]$, where $-\frac{\pi}{4} < \theta < \frac{\pi}{4}$. Then, the value of $\frac{d}{d(\tan\theta)}(f(\theta))$ is

Consider the parabola $y^2 = 8x$. Let Δ_1 be the area of the triangle formed by the end points of its latusrectum and the point $P\left(\frac{1}{2}, 2\right)$ on the parabola and Δ_2 be the area of the triangle formed by drawing tangents at P and at the end points of the latusrectum. Then, $\frac{\Delta_1}{\Delta_2}$ is

If z is any complex number satisfying $|z - 3 - 2i| \le 2$, then the maximum value of $|2z - 6 + 5i|$ is

Paper II

SECTION I

Directions (Q. Nos. 1-8) *This section contains 8 multiple choice questions. Each question has four choices (a), (b), (c) and (d) out of which only one is correct.*

Let $P(6, 3)$ be a point on the hyperbola $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$. If the normal at the point P intersects the X-axis at (9, 0), then the eccentricity of the hyperbola is

(a) $\sqrt{\frac{5}{2}}$ (b) $\sqrt{\frac{3}{2}}$ (c) $\sqrt{2}$ (d) $\sqrt{3}$

Let (x, y) be any point on the parabola $y^2 = 4x$. Let P be the point that divides the line segment from (0, 0) to (x, y) in the ratio 1 : 3. Then, the locus of P is

(a) $x^2 = y$ (b) $y^2 = 2x$ (c) $y^2 = x$ (d) $x^2 = 2y$

Let $f(x) = x^2$ and $g(x) = \sin x$ for all $x \in R$. Then, the set of all x satisfying $(fogogof)(x) = (gogof)(x)$, where $(fog)(x) = f(g(x))$ is

(a) $\pm\sqrt{n\pi}, n \in \{0, 1, 2, \dots\}$ (b) $\pm\sqrt{n\pi}, n \in \{1, 2, \dots.\}$

(c) $\frac{\pi}{2} + 2n\pi, n \in \{\dots, -2, -1, 0, 1, 2, \dots\}$ (d) $2n\pi, n \in \{\dots, -2, -1, 0, 1, 2, \dots\}$

Let $f : [-1, 2] \to [0, \infty)$ be a continuous function such that $f(x) = f(1 - x)$ for all $x \in [-1, 2]$. Let $R_1 = \int_{-1}^{2} x f(x) dx$ and R_2 be the area of the region bounded by $y = f(x)$, $x = -1$, $x = 2$ and the X-axis. Then,

(a) $R_1 = 2R_2$ (b) $R_1 = 3R_2$ (c) $2R_1 = R_2$ (d) $3R_1 = R_2$

If $\lim_{x \to 0} [1 + x \log(1 + b^2)]^{1/x} = 2b\sin^2\theta$ $b > 0$ and $\theta \in (-\pi, \pi]$, then the value of θ is

(a) $\pm\frac{\pi}{4}$ (b) $\pm\frac{\pi}{3}$ (c) $\pm\frac{\pi}{6}$ (d) $\pm\frac{\pi}{2}$

The circle passing through the point $(-1, 0)$ and touching the Y-axis at $(0, 2)$, also passes through the point

(a) $\left(-\frac{3}{2}, 0\right)$ (b) $\left(-\frac{5}{2}, 2\right)$ (c) $\left(-\frac{3}{2}, \frac{5}{2}\right)$ (d) $(-1, -4)$

Let $\omega \neq 1$ be a cube root of unity and S be the set of all non-singular matrices of the form $\begin{bmatrix} 1 & a & b \\ \omega & 1 & c \\ \omega^2 & \omega & 1 \end{bmatrix}$, where each of a, b and c is either ω or ω^2. Then, the number of distinct matrices in the set S is

(a) 2 (b) 6 (c) 4 (d) 8

The value of b for which the equations $x^2 + bx - 1 = 0$, $x^2 + x + b = 0$ have one root in common is

(a) $-\sqrt{2}$ (b) $-i\sqrt{3}$ (c) $i\sqrt{5}$ (d) $\sqrt{2}$

SECTION II

Directions (Q. Nos. 9-12) *This section contains 4 multiple choice questions. Each question has four choices (a), (b), (c) and (d) out of which one or more may be correct.*

If $f(x) = \begin{cases} -x - \frac{\pi}{2}, & x \leq -\frac{\pi}{2} \\ -\cos x, & -\frac{\pi}{2} < x \leq 0 \\ x - 1, & 0 < x \leq 1 \\ \log x, & x > 1 \end{cases}$, then

(a) $f(x)$ is continuous at $x = -\frac{\pi}{2}$ (b) $f(x)$ is not differentiable at $x = 0$

(c) $f(x)$ is differentiable at $x = 1$ (d) $f(x)$ is differentiable at $x = -\frac{3}{2}$

Let L be a normal to the parabola $y^2 = 4x$. If L passes through the point $(9, 6)$, then L is given by

(a) $y - x + 3 = 0$ (b) $y + 3x - 33 = 0$ (c) $y + x - 15 = 0$ (d) $y - 2x + 12 = 0$

Let E and F be two independent events. The probability that exactly one of them occurs is 11/25 and the probability of none of them occurring is 2/25. If $P(T)$ denotes the probability of occurrence of the event T, then

(a) $P(E)=\frac{4}{5}, P(F)=\frac{3}{5}$ (b) $P(E)=\frac{1}{5}, P(F)=\frac{2}{5}$

(c) $P(E)=\frac{2}{5}, P(F)=\frac{1}{5}$ (d) $P(E)=\frac{3}{5}, P(F)=\frac{4}{5}$

Let $f:(0,1)\to R$ be defined by $f(x)=\frac{b-x}{1-bx}$, where b is a constant such that $0<b<1$. Then,

(a) f if not invertible on (0, 1)

(b) $f\neq f^{-1}$ on (0, 1) and $f'(b)=\frac{1}{f'(0)}$

(c) $f=f^{-1}$ on (0, 1) and $f'(b)=\frac{1}{f'(0)}$

(d) f^{-1} is differentiable on (0, 1)

SECTION III

Directions (Q. Nos. 13-18) *This section contains 6 questions. The answer to each of the questions is a single digit integer, ranging from 0 to 9.*

Let $y'(x)+y(x)g'(x)=g(x)g'(x)\,y(0)=0, x\in R$, where $f'(x)$ denotes $\frac{d\,f(x)}{dx}$ and $g(x)$ is a given non-constant differentiable function on R with $g(0)=g(2)=0$. Then, the value of $y(2)$ is...

Let $\mathbf{a}=-\hat{\mathbf{i}}-\hat{\mathbf{k}}$, $\mathbf{b}=-\hat{\mathbf{i}}+\hat{\mathbf{j}}$ and $\mathbf{c}=\hat{\mathbf{i}}+2\hat{\mathbf{j}}+3\hat{\mathbf{k}}$ be three given vectors. If $\mathbf{r}$ is a vector such that $\mathbf{r}\times\mathbf{b}=\mathbf{c}\times\mathbf{b}$ and $\mathbf{r}\cdot\mathbf{a}=0$, then the value of $\mathbf{r}\cdot\mathbf{b}$ is

Let $\omega=e^{i\pi/3}$ and a, b, c, x, y, z be non-zero complex numbers such that
$a+b+c=x$, $a+b\omega+c\omega^2=y$, $a+b\omega^2+c\omega=z$.

Then, the value of $\frac{|x|^2+|y|^2+|z|^2}{|a|^2+|b|^2+|c|^2}$ is

Let M be a 3×3 matrix satisfying

$$M\begin{bmatrix}0\\1\\0\end{bmatrix}=\begin{bmatrix}-1\\2\\3\end{bmatrix}, M\begin{bmatrix}1\\-1\\0\end{bmatrix}=\begin{bmatrix}1\\1\\-1\end{bmatrix} \text{ and } M\begin{bmatrix}1\\1\\1\end{bmatrix}=\begin{bmatrix}0\\0\\12\end{bmatrix}$$

Then, the sum of the diagonal entries of M is

The number of distinct real roots of $x^4-4x^3+12x^2+x-1=0$ is

The straight line $2x-3y=1$ divides the circular region $x^2+y^2\le 6$ into two parts. If $S=\left\{\left(2,\frac{3}{4}\right),\left(\frac{5}{2},\frac{3}{4}\right),\left(\frac{1}{4},-\frac{1}{4}\right),\left(\frac{1}{8},\frac{1}{4}\right)\right\}$, then the number of point(s) in S lying inside the smaller part is

SECTION IV

Directions (Q. Nos. 19-20) *This section contains 2 questions. Each question has four statements (A, B, C and D) given in Column* I *and five statements (p, q, r, s and t) in Column* II. *Any given statement in Column* I *can have correct matching with one or more statement(s) given in Column* II.

19. Match the statements given in Column I with the values given in Column II.

Column I	Column II
(A) If $\mathbf{a} = \hat{\mathbf{j}} + \sqrt{3}\hat{\mathbf{k}}$, $\mathbf{b} = -\hat{\mathbf{j}} + \sqrt{3}\hat{\mathbf{k}}$ and $\mathbf{c} = 2\sqrt{3}\hat{\mathbf{k}}$ form a triangle, then the internal angle of the triangle between **a** and **b** is	(p) $\frac{\pi}{6}$
(B) If $\int_a^b [f(x) - 3x]dx = a^2 - b^2$, then the value of $f\left(\frac{\pi}{6}\right)$ is	(q) $\frac{2\pi}{3}$
(C) The value of $\frac{\pi^2}{\log 3}\int_{7/6}^{5/6} \sec(\pi x)dx$ is	(r) $\pi/3$
(D) The maximum value of $\left\lvert \arg\left(\frac{1}{1-z}\right)\right\rvert$ for $\lvert z\rvert = 1, z \neq 1$ is given by	(s) π
	(t) $\pi/2$

20. Match the statements given in Column I with the intervals/union of intervals given in Column II.

Column I	Column II
(A) The set $\left\{\text{Re}\left(\frac{2iz}{1-z^2}\right) : z \text{ is a complex number}, \lvert z\rvert = 1, z \neq \pm 1\right\}$	(p) $(-\infty, -1) \cup (1, \infty)$
(B) The domain of the function $f(x) = \sin^{-1}\left[\frac{8(3)^{x-2}}{1-3^{2(x-1)}}\right]$ is	(q) $(-\infty, 0) \cup (0, \infty)$
(C) If $f(\theta) = \begin{vmatrix} 1 & \tan\theta & 1 \\ -\tan\theta & 1 & \tan\theta \\ -1 & -\tan\theta & 1 \end{vmatrix}$, then the set $\{f(\theta) : 0 \leq \theta < \frac{\pi}{2}\}$ is	(r) $[2, \infty)$
(D) If $f(x) = x^{3/2}(3x - 10)$, $x \geq 0$, then $f(x)$ is increasing in	(s) $(-\infty, -1] \cup [1, \infty)$
	(t) $(-\infty, 0] \cup [2, \infty)$

ANSWERS

Paper I

1. (c)	**2.** (b)	**3.** (b)	**4.** (a)	**5.** (c)	**6.** (d)
7. (c)	**8.** (c)	**9.** (b, d)	**10.** (a, d)	**11.** (b, c)	**12.** (b)
13. (d)	**14.** (d)	**15.** (a)	**16.** (b)	**17.** (7)	**18.** (3 or 9)
19. (8)	**20.** (8/3)	**21.** (1)	**22.** (2)	**23.** (5)	

Paper II

1. (b)	**2.** (c)	**3.** (b)	**4.** (c)	**5.** (d)	**6.** (d)
7. (a)	**8.** (b)	**9.** (a, b, c, d)	**10.** (a, b, d)	**11.** (a, d)	**12.** (a)
13. (0)	**14.** (9)	**15.** (3)	**16.** (9)	**17.** (2)	**18.** (2)

19. (A) → q; (B) → p; (C) → s; (D) → s   **20.** (A) → s; (B) → t; (C) → r; (D) → r

Hints & Solutions

Paper I

1. Let $\mathbf{v} = \mathbf{a} + \lambda\, \mathbf{b}$

$$\mathbf{v} = (1+\lambda)\,\hat{\mathbf{i}} + (1-\lambda)\,\hat{\mathbf{j}} + (1+\lambda)\hat{\mathbf{k}}$$

Projection of $\mathbf{v}$ on $\mathbf{c} = \dfrac{1}{\sqrt{3}}$

$$\Rightarrow \quad \frac{\mathbf{v}\cdot\mathbf{c}}{|\mathbf{c}|} = \frac{1}{\sqrt{3}}$$

$$\Rightarrow \quad \frac{(1+\lambda)-(1-\lambda)-(1+\lambda)}{\sqrt{3}} = \frac{1}{\sqrt{3}}$$

$$\Rightarrow \quad 1+\lambda-1+\lambda-1-\lambda = 1$$

$$\Rightarrow \quad \lambda - 1 = 1 \Rightarrow \lambda = 2$$

$$\therefore \quad \mathbf{v} = 3\hat{\mathbf{i}} - \hat{\mathbf{j}} + 3\hat{\mathbf{k}}$$

2. Here, area between 0 to b is R_1 and b to 1 is R_2.

$$\therefore \int_0^b (1-x)^2\,dx - \int_b^1 (1-x)^2\,dx = \frac{1}{4}$$

$$\Rightarrow \left(\frac{(1-x)^3}{-3}\right)_0^b - \left(\frac{(1-x)^3}{-3}\right)_b^1 = \frac{1}{4}$$

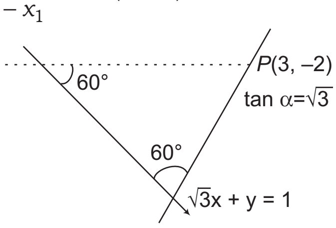
$$\Rightarrow -\frac{1}{3}\{(1-b)^3 - 1\} + \frac{1}{3}\{0-(1-b)^3\} = \frac{1}{4}$$

$$\Rightarrow -\frac{2}{3}(1-b)^3 = -\frac{1}{3} + \frac{1}{4} = -\frac{1}{12}$$

$$\Rightarrow (1-b)^3 = \frac{1}{8}$$

$$\Rightarrow (1-b) = \frac{1}{2} \Rightarrow b = \frac{1}{2}$$

3. A straight line passing through P and making an angle of $\alpha = 60°$, is given by $\dfrac{y-y_1}{x-x_1} = \tan(\theta \pm \alpha)$



where, $\sqrt{3}x + y = 1$

$$\Rightarrow \quad y = -\sqrt{3}x + 1$$

Then, $\tan\theta = -\sqrt{3}$

$$\Rightarrow \frac{y+2}{x-3} = \frac{\tan\theta \pm \tan\alpha}{1 \mp \tan\theta\tan\alpha}$$

$$\Rightarrow \frac{y+2}{x-3} = \frac{-\sqrt{3}+\sqrt{3}}{1-(-\sqrt{3})(\sqrt{3})}$$

and $$\frac{y+2}{x-3} = \frac{-\sqrt{3}-\sqrt{3}}{1+(-\sqrt{3})(\sqrt{3})}$$

$$\Rightarrow y + 2 = 0$$

and $$\frac{y+2}{x-3} = \frac{-2\sqrt{3}}{1-3} = \sqrt{3}$$

$$\Rightarrow y + 2 = \sqrt{3}x - 3\sqrt{3}$$

Neglecting, $y + 2 = 0$ as it does not intersect Y-axis.

4. Put $x^2 = t \Rightarrow x\,dx = \dfrac{dt}{2}$

$$\therefore \quad I = \int_{\log 2}^{\log 3} \frac{\sin t \cdot \frac{dt}{2}}{\sin t + \sin(\log 6 - t)} \quad \ldots\text{(i)}$$

Using, $\int_a^b f(x)dx = \int_a^b f(a+b-x)dx$

$$I = \frac{1}{2}\int_{\log 2}^{\log 3} \frac{\sin(\log 2 + \log 3 - t)}{\sin(\log 2 + \log 3 - t) + \sin(\log 6 - (\log 2 + \log 3 - t))}\,dt$$

$$I = \frac{1}{2}\int_{\log 2}^{\log 3} \frac{\sin(\log 6 - t)}{\sin(\log 6 - t) + \sin(t)}\,dt$$

$$\therefore I = \int_{\log 2}^{\log 3} \frac{\sin(\log 6 - t)}{\sin(\log 6 - t) + \sin t}\,dt \quad \ldots\text{(ii)}$$

On adding Eqs. (i) and (ii), we get

$$2I = \frac{1}{2}\int_{\log 2}^{\log 3} \frac{\sin t + \sin(\log 6 - t)}{\sin(\log 6 - t) + \sin t}\,dt$$

$$\therefore \quad 2I = \frac{1}{2}(t)_{\log 2}^{\log 3} = \frac{1}{2}(\log 3 - \log 2)$$

$$\Rightarrow I = \frac{1}{4}\log\left(\frac{3}{2}\right)$$

5. Taking log on both sides,

$\log 2 \cdot \log(2x) = \log 3\,(\log 3y)$

$\Rightarrow \quad \log 2\{\log 2+\log x\}$
$= \log 3\{\log 3+\log y\} \quad \ldots(\text{i})$

and $\quad \log x \cdot \log 3=\log y \log 2$

$$\log y=\frac{\log x \cdot \log 3}{\log 2} \quad \ldots(\text{ii})$$

From Eqs. (i) and (ii), we get

$$\log 2\{\log 2+\log x\} = \log 3 \cdot\left\{\log 3+\frac{\log x \cdot \log 3}{\log 2}\right\}$$

$$\Rightarrow (\log 2)^2+\log 2 \cdot \log x = (\log 3)^2+\frac{(\log 3)^2}{(\log 2)} \cdot \log x$$

$$\Rightarrow \log x\left\{\frac{(\log 3)^2}{\log 2}-\log 2\right\} = (\log 2)^2-(\log 3)^2$$

$$\Rightarrow \log x\left\{\frac{(\log 3)^2-(\log 2)^2}{\log 2}\right\} = (\log 2)^2-(\log 3)^2$$

$$\Rightarrow \log x=-\log 2=\log 2^{-1}$$

$$\therefore \quad x=\frac{1}{2}$$

6. $P=\{\theta: \sin \theta-\cos \theta=\sqrt{2} \cos \theta\}$

$\Rightarrow \quad \cos \theta(\sqrt{2}+1)=\sin \theta$

$\Rightarrow \quad \tan \theta=\sqrt{2}+1 \quad \ldots(\text{i})$

$Q=\{\theta: \sin \theta+\cos \theta=\sqrt{2} \sin \theta\}$

$\Rightarrow \quad \sin \theta(\sqrt{2}-1)=\cos \theta$

$$\Rightarrow \quad \tan \theta=\frac{1}{\sqrt{2}-1} \times \frac{\sqrt{2}+1}{\sqrt{2}+1}$$

$= (\sqrt{2}+1) \quad \ldots(\text{ii})$

$\therefore \quad P=Q$

7. $$\frac{a_{10}-2 a_8}{2 a_9}=\frac{(\alpha^{10}-\beta^{10})-2(\alpha^8-\beta^8)}{2(\alpha^9-\beta^9)}$$

$$=\frac{\alpha^8(\alpha^2-2)-\beta^8(\beta^2-2)}{2(\alpha^9-\beta^9)}$$

$[\because \alpha$ is root of $x^2-6x-2=0$
$\alpha^2-2=6\alpha]$

Also, β is root of $x^2-6x-2=0$

$\Rightarrow \quad \beta^2-2=6\beta$

$$=\frac{\alpha^8(6\alpha)-\beta^8(6\beta)}{2(\alpha^9-\beta^9)}=\frac{6(\alpha^9-\beta^9)}{2(\alpha^9-\beta^9)}=3$$

8. Given, $\quad M^T=-M, N^T=-N$

and $\quad MN=NM \quad \ldots(\text{i})$

$\therefore \quad M^2N^2(M^TN)^{-1}(MN^{-1})^T$

$= M^2N^2N^{-1}(M^T)^{-1}(N^{-1})^T \cdot M^T$

$= M^2N(NN^{-1})(-M)^{-1}(N^T)^{-1}(-M)$

$= M^2NI(-M^{-1})(-N)^{-1}(-M)$

$= -M^2NM^{-1}N^{-1}M$

$= -M \cdot (MN)M^{-1}N^{-1}M$

$= -M(NM)M^{-1}N^{-1}M$

$= -MN(NM^{-1})N^{-1}M$

$= -M(NN^{-1})M=-M^2$

Note Here, non-singular word should not be used, since there is no non-singular 3×3 skew-symmetric matrix.

9. Here, equation of ellipse

$$\frac{x^2}{4}+\frac{y^2}{1}=1$$

$$\Rightarrow \quad e^2=1-\frac{b^2}{a^2}=1-\frac{1}{4}=\frac{3}{4}$$

$\therefore \quad e=\dfrac{\sqrt{3}}{2}$ and focus $(\pm ae, 0)$

$=(\pm\sqrt{3}, 0)$

For hyperbola $\dfrac{x^2}{a^2}-\dfrac{y^2}{b^2}=1$,

$e_1^2=1+\dfrac{b^2}{a^2}$ where, $e_1^2=\dfrac{1}{e^2}=\dfrac{4}{3}$

$$\Rightarrow \quad 1+\frac{b^2}{a^2}=\frac{4}{3} \Rightarrow \frac{b^2}{a^2}=\frac{1}{3} \quad \ldots(\text{i})$$

and hyperbola passes through $(\pm\sqrt{3}, 0)$.

Now, $\quad \dfrac{3}{a^2}=1 \Rightarrow a^2=3 \quad \ldots(\text{ii})$

From Eqs. (i) and (ii), we get

$b^2=1 \quad \ldots(\text{iii})$

$\therefore$ Equation of hyperbola is

$$\frac{x^2}{3}-\frac{y^2}{1}=1$$

Focus is $(\pm ae, 0)$.

Now, $\quad \left(\pm\sqrt{3} \cdot \dfrac{2}{\sqrt{3}}, 0\right) \Rightarrow (\pm 2, 0)$

Hence, both options (b) and (d) are correct.

10. Let $\mathbf{a}=\hat{\mathbf{i}}+\hat{\mathbf{j}}+2\hat{\mathbf{k}}, \mathbf{b}=\hat{\mathbf{i}}+2\hat{\mathbf{j}}+\hat{\mathbf{k}}$

and $\quad \mathbf{c}=\hat{\mathbf{i}}+\hat{\mathbf{j}}+\hat{\mathbf{k}}$

$\therefore$ A vector coplanar to **a** and **b**, and perpendicular to **c**.

Now, $\lambda\,(\mathbf{a}\times\mathbf{b})\times\mathbf{c}$

$\Rightarrow\quad \lambda\,\{(\mathbf{a}\cdot\mathbf{c})\,\mathbf{b}-(\mathbf{b}\cdot\mathbf{c})\,\mathbf{a}\}$

$\Rightarrow\quad \lambda\,\{(1+1+4)\,(\hat{\mathbf{i}}+2\hat{\mathbf{j}}+\hat{\mathbf{k}})$
$\qquad -(1+2+1)\,(\hat{\mathbf{i}}+\hat{\mathbf{j}}+2\hat{\mathbf{k}})\}$

$\Rightarrow\quad \lambda\,\{6\hat{\mathbf{i}}+12\hat{\mathbf{j}}+6\hat{\mathbf{k}}-6\hat{\mathbf{i}}-6\hat{\mathbf{j}}-12\hat{\mathbf{k}}\}$

$\Rightarrow\quad \lambda\,\{6\hat{\mathbf{j}}-6\hat{\mathbf{k}}\}\Rightarrow 6\lambda\,(\hat{\mathbf{j}}-\hat{\mathbf{k}})$

For $\lambda=\dfrac{1}{6}\Rightarrow$ Option (a) is correct.

For $\lambda=-\dfrac{1}{6}\Rightarrow$ Option (d) is correct.

11. $f(x+y)=f(x)+f(y)$, as $f(x)$ is differentiable at $x=0$.

$\Rightarrow\quad f'(0)=k \quad \ldots\text{(i)}$

Now, $f'(x)=\lim\limits_{h\to 0}\dfrac{f(x+h)-f(x)}{h}$

$=\lim\limits_{h\to 0}\dfrac{f(x)+f(h)-f(x)}{h}$

$=\lim\limits_{h\to 0}\dfrac{f(h)}{h} \qquad \left[\dfrac{0}{0}\text{ from}\right]$

Given, $f(x+y)=f(x)+f(y),\ \forall\ x,y$

$\therefore\quad f(0)=f(0)+f(0),$

when $\quad x=y=0\Rightarrow f(0)=0$

Using L'Hospital's rule,

$\lim\limits_{h\to 0}\dfrac{f'(h)}{1}=f'(0)=k \quad \ldots\text{(ii)}$

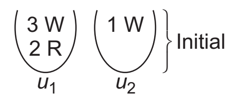
$\Rightarrow\quad f'(x)=k$, on integrating both sides,
$f(x)=kx+C$, as $f(0)=0\Rightarrow C=0$

So, $\quad f(x)=kx$

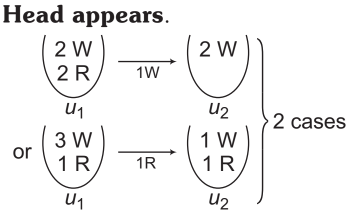
$\therefore\quad f(x)$ is continuous for all $x\in R$ and $f'(x)=k$, *i.e.* constant for all $x\in R$.

Hence, both (b) and (c) are correct.

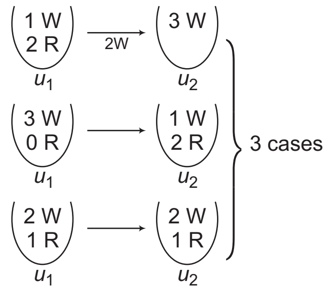
Solutions (Q. Nos. 12-13)



Head appears.



Tail appears.



12. Now, probability of the drawn ball from V_2 being white is

$\Rightarrow P\ (\text{white}/V_2)$

$$=P(H)\cdot\left\{\frac{{}^3C_1}{{}^5C_1}\times\frac{{}^2C_1}{{}^2C_1}+\frac{{}^2C_1}{{}^5C_1}\times\frac{{}^1C_1}{{}^2C_1}\right\}$$
$$+P(T)\left\{\frac{{}^3C_2}{{}^5C_2}\times\frac{{}^3C_2}{{}^3C_2}+\frac{{}^2C_2}{{}^5C_2}\times\frac{{}^1C_1}{{}^3C_2}+\frac{{}^3C_1\cdot{}^2C_1}{{}^5C_2}\times\frac{{}^2C_1}{{}^3C_2}\right\}$$

Now, $P\ (\text{white}/V_2)=\dfrac{1}{2}\left\{\dfrac{3}{5}\times 1+\dfrac{2}{5}\times\dfrac{1}{2}\right\}$

$$+\frac{1}{2}\left\{\frac{3}{10}\times 1+\frac{1}{10}\times\frac{1}{3}+\frac{6}{10}\times\frac{2}{3}\right\}=\frac{23}{30}$$

13. P (head appeared/white from V_2)

$$=P(H)\cdot\frac{\left\{\frac{{}^3C_1}{{}^5C_1}\times\frac{{}^2C_1}{{}^2C_1}+\frac{{}^2C_1}{{}^5C_1}\times\frac{{}^1C_1}{{}^2C_1}\right\}}{\frac{23}{30}}$$

$$=\frac{1}{2}\,\frac{\left\{\frac{3}{5}\times 1+\frac{2}{5}\times\frac{1}{2}\right\}}{\frac{23}{30}}=\frac{12}{23}$$

Solutions (Q. Nos. 14-16)

Given, $[a\ b\ c]_{1\times 3}\begin{bmatrix}1&9&7\\8&2&7\\7&3&7\end{bmatrix}_{3\times 3}=[0\,0\,0]$

$$\Rightarrow\quad \begin{bmatrix}a+8b+7c\\9a+2b+3c\\7a+7b+7c\end{bmatrix}=\begin{bmatrix}0\\0\\0\end{bmatrix}$$

$\Rightarrow\quad a+8b+7c=0 \quad \ldots\text{(i)}$

$\Rightarrow\quad 9a+2b+3c=0 \quad \ldots\text{(ii)}$

$\Rightarrow\quad a+b+c=0 \quad \ldots\text{(iii)}$

On multiplying Eq. (iii) by 2, then subtract from Eq. (ii), we get

$$7a + c = 0 \quad \ldots\text{(iv)}$$

Again multiplying Eq. (iii) by 3, then subtract from Eq. (ii), we get

$$6a - b = 0 \quad \ldots\text{(v)}$$

$\therefore \quad b = 6a$ and $c = -7a$

14. As (a, b, c) lies on $2x + y + z = 1$

$\Rightarrow \quad 2a + b + c = 1$

$\Rightarrow \quad 2a + 6a - 7a = 1$

$\Rightarrow \quad a = 1, b = 6$ and $c = -7$

$\therefore \quad 7a + b + c = 7 + 6 - 7 = 6$

15. If $a = 2$, $b = 12$ and $c = -14$

$\therefore \quad \dfrac{3}{\omega^a} + \dfrac{1}{\omega^b} + \dfrac{3}{\omega^c}$

$\Rightarrow \quad \dfrac{3}{\omega^2} + \dfrac{1}{\omega^{12}} + \dfrac{3}{\omega^{-14}} = \dfrac{3}{\omega^2} + 1 + 3\omega^2$

$= 3\omega + 1 + 3\omega^2$

$= 1 + 3(\omega + \omega^2)$

$= 1 - 3 = -2$

16. If $b = 6$, $a = 1$ and $c = -7$

$\therefore \quad ax^2 + bx + c = 0$

$\Rightarrow \quad x^2 + 6x - 7 = 0$

$\Rightarrow \quad (x + 7)(x - 1) = 0$

$\therefore \quad x = 1, -7$

$\Rightarrow \quad \sum_{n=0}^{\infty} \left(\dfrac{1}{1} - \dfrac{1}{7}\right)^n \Rightarrow \sum_{n=0}^{\infty} \left(\dfrac{6}{7}\right)^n$

$\Rightarrow \quad 1 + \dfrac{6}{7} + \left(\dfrac{6}{7}\right)^n + \ldots \infty$

$= \dfrac{1}{1 - \dfrac{6}{7}} = \dfrac{1}{1/7} = 7$

17. Given, $n > 3 \in$ Integer

and $\dfrac{1}{\sin\left(\dfrac{\pi}{n}\right)} = \dfrac{1}{\sin\left(\dfrac{2\pi}{n}\right)} + \dfrac{1}{\sin\left(\dfrac{3\pi}{n}\right)}$

$\Rightarrow \quad \dfrac{1}{\sin\dfrac{\pi}{n}} - \dfrac{1}{\sin\dfrac{3\pi}{n}} = \dfrac{1}{\sin\dfrac{2\pi}{n}}$

$\Rightarrow \quad \dfrac{\sin\dfrac{3\pi}{n} - \sin\dfrac{\pi}{n}}{\sin\dfrac{\pi}{n}\cdot\sin\dfrac{3\pi}{n}} = \dfrac{1}{\sin\dfrac{2\pi}{n}}$

$\Rightarrow \quad 2\cos\left(\dfrac{2\pi}{n}\right)\cdot\sin\dfrac{\pi}{n} = \dfrac{\sin\dfrac{\pi}{n}\cdot\sin\dfrac{3\pi}{n}}{\sin\dfrac{2\pi}{n}}$

$\Rightarrow \quad 2\sin\dfrac{2\pi}{n}\cdot\cos\dfrac{2\pi}{n} = \sin\dfrac{3\pi}{n}$

$\Rightarrow \quad \sin\dfrac{4\pi}{n} = \sin\dfrac{3\pi}{n}$

$\Rightarrow \quad \dfrac{4\pi}{n} = \pi - \dfrac{3\pi}{n} \Rightarrow \dfrac{7\pi}{n} = \pi \Rightarrow n = 7$

18. Given, $a_1 = 3, m = 5n$

and $\quad a_1, a_2, \ldots$ are in AP.

$\therefore \quad \dfrac{S_m}{S_n} = \dfrac{S_{5n}}{S_n}$ is independent of n.

Now, $\quad \dfrac{\dfrac{5n}{2}[2 \times 3 + (5n - 1)d]}{\dfrac{n}{2}[2 \times 3 + (n - 1)d]}$

$\Rightarrow \quad \dfrac{f\{(6 - d) + 5n\}}{(6 - d) + n}$

independent of n, if

$6 - d = 0 \Rightarrow d = 6$

$\therefore \quad a_2 = a_1 + d = 3 + 6 = 9$

Or

If $\quad d = 0$

$\dfrac{S_m}{S_n}$ is independent of n.

$\therefore \quad a_2 = 3$

19. Using AM $\geq$ GM,

$\dfrac{a^{-5} + a^{-4} + a^{-3} + a^{-3} + a^{-3} + 1 + a^8 + a^{10}}{8}$

$\geq (a^{-5} \cdot a^{-4} \cdot a^{-3} \cdot a^{-3} \cdot a^{-3} \cdot 1 \cdot a^8 \cdot a^{10})^{\frac{1}{8}}$

$\Rightarrow a^{-5} + a^{-4} + 3a^{-3} + 1 + a^8 + a^{10} \geq 8 \cdot 1$

Hence, minimum value is 8.

20. Given, $f(1) = \dfrac{1}{3}$ and $6\int_1^x f(t)\,dt$

$= 3x\,f(x) - x^3$, for all $x \geq 1$

Using (Newton-Leibnitz formula),

On Differentiating both sides,

$6f(x)\cdot 1 - 0 - 3f(x) + 3x\,f'(x) - 3x^2$

$\Rightarrow \quad 3x\,f'(x) - 3f(x) = 3x^2$

$\Rightarrow \quad f'(x) - \dfrac{1}{x} f(x) = x$

$\Rightarrow \frac{x f'(x) - f(x)}{x^2} = 1 \Rightarrow \frac{d}{dx}\left\{\frac{f(x)}{x}\right\} = 1$

On integrating both sides,

$\frac{f(x)}{x} = x + C \qquad [\because f(1) = \frac{1}{3}]$

$\Rightarrow \quad \frac{1}{3} = 1 + C \Rightarrow C = -\frac{2}{3}$

Now, $\quad f(x) = x^2 - \frac{2}{3}x$

$\Rightarrow \quad f(2) = 4 - \frac{4}{3} = \frac{8}{3}$

Note Here, $f(1) = 2$ does not satisfy given function.

$\therefore \quad f(1) = \frac{1}{3}$

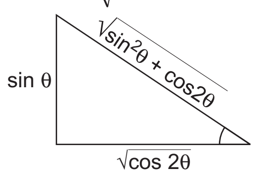
For that, $f(x) = x^2 - \frac{2}{3}x$

and $\quad f(2) = 4 - \frac{4}{3} = \frac{8}{3}$

21. $f(\theta) = \sin\left(\tan^{-1}\frac{\sin\theta}{\sqrt{\cos 2\theta}}\right), -\frac{\pi}{4} < \theta < \frac{\pi}{4}$

Let $\quad \tan^{-1}\frac{\sin\theta}{\sqrt{\cos 2\theta}} = \phi$

$\Rightarrow \quad \tan\phi = \frac{\sin\theta}{\sqrt{\cos 2\theta}}$



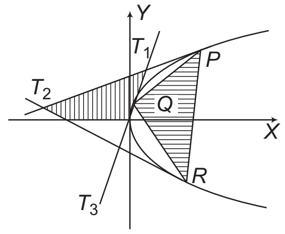
$\therefore \quad \sin\phi = \frac{\sin\theta}{\sqrt{\sin^2\theta + \cos 2\theta}}$

$= \frac{\sin\theta}{\sqrt{1 - \sin^2\theta}} = \frac{\sin\theta}{\cos\theta} = \tan\theta$

$\therefore \quad f(\theta) = \sin\phi = \tan\theta$

$\Rightarrow \quad \frac{d\,f(\theta)}{d(\tan\theta)} = 1$

22. As, we know area of triangle formed by three points on parabola is twice the area of triangle formed by corresponding tangents, i.e. area of $\Delta PQR = 2$ area of $\Delta T_1T_2T_3$.



$\therefore \quad \Delta_1 = 2\Delta_2$ or $\frac{\Delta_1}{\Delta_2} = 2$

23. Given, $|z - 3 - 2i| \le 2 \qquad \ldots$(i)

To find minimum of $|2z - 6 + 5i|$

or $\quad 2\left|z - 3 + \frac{5}{2}i\right|$

Using triangle inequality,

i.e. $\quad ||z_1| - |z_2|| \le |z_1 + z_2|$

$\therefore \quad \left|z - 3 + \frac{5}{2}i\right|$

$= \left|z - 3 - 2i + 2i + \frac{5}{2}i\right|$

$= \left|(z - 3 - 2i) + \frac{9}{2}i\right| \ge \left||z - 3 - 2i| - \frac{9}{2}\right|$

$\ge \left|2 - \frac{9}{2}\right| \ge \frac{5}{2} \Rightarrow \left|z - 3 + \frac{5}{2}i\right| \ge \frac{5}{2}$

or $\quad |2z - 6 + 5i| \ge 5$

Paper II

1. Equation of normal to hyperbola at (x_1, y_1) is

$$\frac{a^2x}{x_1} + \frac{b^2y}{y_1} = a^2 + b^2$$

$\therefore$ At (6, 3), $\quad \frac{a^2x}{6} + \frac{b^2y}{3} = a^2 + b^2$

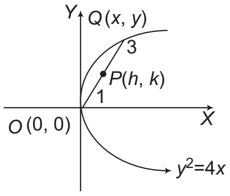
It passes throught (9, 0).

Now, $\quad \frac{a^2 \cdot 9}{6} = a^2 + b^2$

$\Rightarrow \quad \frac{3a^2}{2} - a^2 = b^2 \Rightarrow \frac{a^2}{b^2} = 2$

$\therefore \quad e^2 = 1 + \frac{b^2}{a^2} = 1 + \frac{1}{2} \Rightarrow e = \sqrt{\frac{3}{2}}$

2. By section formula,



$$h = \frac{x + 0}{4}, \; k = \frac{y + 0}{4}$$

$\therefore \quad x = 4h$, and $y = 4k$

Substituting in $y^2 = 4x$,

$(4k)^2 = 4(4h) \Rightarrow k^2 = h$

or $\quad y^2 = x$ is required locus.

3. $f(x) = x^2, g(x) = \sin x$

$(gof)(x) = \sin x^2$

$go(gof)(x) = \sin(\sin x^2)$

$(fogogof)(x) = (\sin(\sin x^2))^2 \quad \ldots(\text{i})$

Again, $(gof)(x) = \sin x^2$

$(gogof)(x) = \sin(\sin x^2) \quad \ldots(\text{ii})$

Given, $(fogogof)(x) = (gogof)(x)$

$\Rightarrow \quad (\sin(\sin x^2))^2 = \sin(\sin x^2)$

$\Rightarrow \quad \sin(\sin x^2)\{\sin(\sin x^2) - 1\} = 0$

$\Rightarrow \quad \sin(\sin x^2) = 0$ or $\sin(\sin x^2) = 1$

$\Rightarrow \quad \sin x^2 = 0$ or $\sin x^2 = \frac{\pi}{2}$

$\therefore \quad x^2 = n\pi$

(i.e. not possible as $-1 \le \sin\theta \le 1$)

$x = \pm\sqrt{n\pi}$

4. Here, $\quad R_1 = \int_{-1}^{2} x\, f(x)\, dx \quad \ldots(\text{i})$

Using, $\quad \int_a^b f(x)\, dx = \int_a^b f(a + b - x)\, dx$

$R_1 = \int_{-1}^{2} (1 - x)\, f(1 - x) dx,$

$[\because f(x) = f(1 - x)]$

$\therefore \quad R_1 = \int_{-1}^{2} (1 - x)\, f(x)\, dx \quad \ldots(\text{ii})$

Given, R_2 is area bounded by

$f(x), x - 1$ and $x = 2$

$\therefore \quad R_2 = \int_{-1}^{2} f(x)\, dx \quad \ldots(\text{iii})$

Adding Eqs. (i) and (ii), we get

$2R_1 = \int_{-1}^{2} f(x)\, dx \quad \ldots(\text{iv})$

$\therefore$ From Eqs. (iii) and (iv), we get

$2R_1 = R_2$

5. Here, $\lim_{x \to 0} \{1 + x \log(1 + b^2)\}^{1/x}$

$[1^\infty \text{ from}]$

$\Rightarrow \quad e^{\lim_{x \to 0} \{x \log(1 + b^2)\} \cdot \frac{1}{x}}$

$\Rightarrow \quad e^{\log(1 + b^2)} = (1 + b)^2 \quad \ldots(\text{i})$

Given,

$\lim_{x \to 0} \{1 + x \log(1 + b^2)\}^{1/x} = 2b \sin^2\theta$

$\Rightarrow \quad (1 + b^2) = 2b \sin^2\theta$

$\therefore \quad \sin^2\theta = \frac{1 + b^2}{2b} \quad \ldots(\text{ii})$

By AM $\ge$ GM,

$\frac{b + \frac{1}{b}}{2} \ge \left(b \cdot \frac{1}{b}\right)^{1/2} \Rightarrow \frac{b^2 + 1}{2b} \ge 1 \quad \ldots(\text{iii})$

From Eqs. (ii) and (iii), we get $\sin^2\theta = 1$

$\Rightarrow \quad \theta = \pm\frac{\pi}{2}$ as $\theta \in (-\pi, \pi]$

6. Equation of circle passing through a point (x_1, y_1) and touching the straight line L, is given by

$(x - x_1)^2 + (y - y_1)^2 = \lambda L = 0$

$\therefore$ Equation of circle passing through $(0, 2)$ and touching $x = 0$.

Now, $\quad (x - 0)^2 + (y - 2)^2 + \lambda x = 0 \ldots(\text{i})$

Also, it passes through $(-1, 0)$.

So, $1 + 4 - \lambda = 0 \Rightarrow \lambda = 5$

Eq. (i) becomes,

$x^2 + y^2 - 4y + 4 + 5x = 0$

$\Rightarrow \quad x^2 + y^2 + 5x - 4y + 4 = 0$

For x-intercept, put $y = 0$,

$x^2 + 5x + 4 = 0$

$\Rightarrow \quad (x + 1)(x + 4) = 0$

$\therefore \quad x = -1, -4$

7. $|A| \ne 0$, as non-singular.

$$\therefore \quad \begin{vmatrix} 1 & a & b \\ \omega & 1 & c \\ \omega^2 & \omega & 1 \end{vmatrix} \ne 0$$

$\Rightarrow \quad 1(1 - c\omega) - a(\omega - c\omega^2)$

$$+ b(\omega^2 - \omega^2) \neq 0$$

$\Rightarrow \quad 1 - c\omega - a\omega + ac\omega^2 \neq 0$

$\Rightarrow \quad (1 - c\omega)(1 - a\omega) \neq 0$

$\Rightarrow \quad a \neq \frac{1}{\omega}, c \neq \frac{1}{\omega} \Rightarrow a = \omega, c = \omega$

and $\quad b \in \{\omega, \omega^2\} \Rightarrow$ 2 solutions

8. If $\quad a_1x^2 + b_1x + c_1 = 0$

and $\quad a_2x^2 + b_2x + c_2 = 0$

have a common real root, then

$\Rightarrow \quad (a_1c_2 - a_2c_1)^2 = (b_1c_2 - b_2c_1)(a_1b_2 - a_2b_1)$

$\therefore \left.\begin{matrix} x^2 + bx - 1 = 0 \\ x^2 + x + b = 0 \end{matrix}\right\}$ have a common root.

$\Rightarrow \quad (1 + b)^2 = (b^2 + 1)(1 - b)$

$\Rightarrow \quad b^2 + 2b + 1 = b^2 - b^3 + 1 - b$

$\Rightarrow \quad b^3 + 3b = 0 \Rightarrow b(b^2 + 3) = 0$

$\Rightarrow \quad b = 0, \pm \sqrt{3}\, i$

9. $f(x) = \begin{cases} -x - \frac{\pi}{2}, & x \leq -\frac{\pi}{2} \\ -\cos x, & -\frac{\pi}{2} < x \leq 0 \\ x - 1, & 0 < x \leq 1 \\ \log x, & x > 1 \end{cases}$

Continuity at $x = -\frac{\pi}{2}$,

$f\left(-\frac{\pi}{2}\right) = -\left(-\frac{\pi}{2}\right) - \frac{\pi}{2} = 0$

RHL $\Rightarrow \lim_{h \to 0} -\cos\left(-\frac{\pi}{2} + h\right) = 0$

$\therefore$ Continuous at $x = 0$

Continuity at $x = 0 \Rightarrow f(0) = -1$

RHL $\Rightarrow \lim_{h \to 0}(0 + h) - 1 = -1$

$\therefore$ Continuous at $x = 0$

Continuity at $x = 1$; $f(1) = 0$

RHL $\Rightarrow \lim_{h \to 0} \log(1 + h) = 0$

$\therefore$ Continuous at $x = 1$

Here, $f'(x) = \begin{cases} -1, & x \leq -\frac{\pi}{2} \\ \sin x, & -\frac{\pi}{2} < x \leq 0 \\ 1, & 0 < x \leq 1 \\ \frac{1}{x}, & x > 1 \end{cases}$

Differentiable at $x = 0$,

LHD = 0, RHD = 1

$\therefore$ Not differentiable at $x = 0$

Differentiable at $x = 1$,

LHD = 1, RHD = 1

$\therefore$ Differentiable at $x = 1$

Also, for $x = -\frac{3}{2} \Rightarrow f(x) = -x - \frac{3}{2}$

$\therefore$ Differentiable at $x = -\frac{3}{2}$

10. Normal to $y^2 = 4x$, is

$y - mx - 2m - m^3$ which passes through (9, 6).

Now, $\quad 6 = 9m - 2m - m^3$

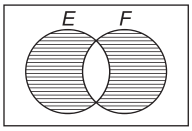
$\Rightarrow m^3 - 7m + 6 = 0 \Rightarrow m = 1, 2, -3$

$\therefore$ Equation of normals are

$y - x + 3 = 0,$

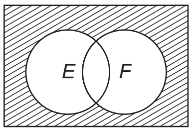
$y + 3x - 33 = 0$ and $y - 2x + 12 = 0$

11.



$P(E \cup F) - P(E \cap F) = \frac{11}{25}$ ...(i)

(i.e. only E or only F)



Neither of them occurs $= \frac{2}{25}$

$\Rightarrow \quad P(\bar{E} \cap \bar{F}) = \frac{2}{25}$ ...(ii)

From Eq. (i), we get

$P(E) + P(F) - 2P(E \cap F) = \frac{11}{25}$ ...(iii)

From Eq. (ii), we get

$(1 - P(E))(1 - P(F)) = \frac{2}{25}$

$\Rightarrow 1 - P(E) - P(F) + P(E) \cdot P(F) = \frac{2}{25}$ ...(iv)

From Eqs. (iii) and (iv), we get

$P(E) + P(F) = \frac{7}{5}$ and $P(E) \cdot P(F) = \frac{12}{25}$

$\therefore \quad P(E) \cdot \left\{ \frac{7}{5} - P(E) \right\} = \frac{12}{25}$

$\Rightarrow \quad (P(E))^2 - \frac{7}{5} P(E) + \frac{12}{25} = 0$

$\Rightarrow \quad \left(P(E) - \frac{3}{5} \right) \left(P(E) - \frac{4}{5} \right) = 0$

$\therefore \quad P(E) = \frac{3}{4} \text{ or } \frac{4}{5} \Rightarrow P(F) = \frac{4}{5} \text{ or } \frac{3}{5}$

12. Here, $f(x) = \frac{b - x}{1 - bx}$

where, $0 < b < 1, 0 < x < 1$

For function to be invertible it should be one-one onto.

$\therefore$ Check range :

Let $\quad f(x) = y \Rightarrow y = \frac{b - x}{1 - bx}$

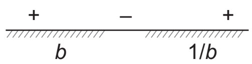
$\Rightarrow y - bxy = b - x \Rightarrow x(1 - by) = b - y$

$\Rightarrow \quad x = \frac{b - y}{1 - by}$

where, $0 < x < 1$

$\therefore \quad 0 < \frac{b - y}{1 - by} < 1$

$\frac{b - y}{1 - by} > 0$ and $\frac{b - y}{1 - by} < 1$



$\Rightarrow \quad y < b \text{ or } y > \frac{1}{b} \quad \ldots(\text{i})$

$\frac{(b - 1)(y + 1)}{1 - by} < -1 < y < \frac{1}{b} \quad \ldots(\text{ii})$

From Eqs. (i) and (ii), we get

$y \in \left(-1, \frac{1}{b} \right) \subset$ Codomain

Thus, $f(x)$ is not invertible.

13. $\frac{dy}{dx} + y \cdot g'(x) = g(x)\, g'(x)$

$\text{IF} = e^{\int g'(x)\, dx} = e^{g(x)}$

$\therefore$ Solution is

$y(e^{g(x)}) = \int g(x) \cdot g'(x) \cdot e^{g(x)} dx + C$

Put $\quad g(x) = t, g'(x) dx = dt$

$y(e^{g(x)}) = \int t \cdot e^t \, dt + C$

$= t \cdot e^t - \int 1 \cdot e^t \, dt + C = t \cdot e^t - e^t + C$

$y\, e^{g(x)} = (g(x) - 1)\, e^{g(x)} + C \quad \ldots(\text{i})$

Given, $y(0) = 0,\, g(0) = g(2) = 0$

$\therefore$ Eq. (i) becomes,

$y(0) \cdot e^{g(0)} = (g(0) - 1) \cdot e^{g(0)} + C$

$\Rightarrow \quad 0 = (-1) \cdot 1 + C \Rightarrow C = 1$

$\therefore \quad y(x) \cdot e^{g(x)} = (g(x) - 1)\, e^{g(x)} + 1$

$\Rightarrow y(2) \cdot e^{g(2)} = (g(2) - 1)\, e^{g(2)} + 1$

where, $\quad g(2) = 0$

$\Rightarrow \quad y(2) \cdot 1 = (-1) \cdot 1 + 1 \Rightarrow y(2) = 0$

14. $\mathbf{r} \times \mathbf{b} = \mathbf{c} \times \mathbf{b}$

$\Rightarrow \quad (\mathbf{r} - \mathbf{c}) \times \mathbf{b} = 0 \Rightarrow \mathbf{r} - \mathbf{c} + \lambda\, \mathbf{b}$

or $\quad \mathbf{r} = \mathbf{c} + \lambda\, \mathbf{b} \quad \ldots(\text{i})$

Given, $\mathbf{r} \cdot \mathbf{a} = 0$, taking dot product with $\mathbf{a}$ for Eq. (i).

Now, $\quad \mathbf{r} \cdot \mathbf{a} = \mathbf{a} \cdot \mathbf{c} + \lambda\, \mathbf{a} \cdot \mathbf{b}$

$\therefore \quad \lambda = \frac{-\vec{a} \cdot \vec{c}}{\vec{a} \cdot \vec{b}} \quad [\because \vec{r} \cdot \vec{a} = 0] \ldots(\text{i})$

From Eqs. (i) and (ii), we get

$$\mathbf{r} = \mathbf{c} - \frac{\mathbf{a} \cdot \mathbf{c}}{\mathbf{a} \cdot \mathbf{b}}\, \mathbf{b}$$

Taking dot with $\mathbf{b}$, we get

$$\mathbf{r} \cdot \mathbf{b} = \mathbf{c} \cdot \mathbf{b} - \frac{\mathbf{a} \cdot \mathbf{c}}{\mathbf{a} \cdot \mathbf{b}} (\mathbf{b} \cdot \mathbf{b})$$

where, $\begin{bmatrix} \mathbf{a} = -\hat{\mathbf{i}} - \hat{\mathbf{k}} \\ \mathbf{b} = -\hat{\mathbf{i}} + \hat{\mathbf{j}} \\ \mathbf{c} = \hat{\mathbf{i}} + 2\hat{\mathbf{j}} + 3\hat{\mathbf{k}} \end{bmatrix}$

$= (-1 + 2) - \frac{(-1 - 3)}{(1)} (1 + 1) = 1 + 8 = 9$

15. Here, $\omega = e^{i2\pi/3}$, then only integer solution exists.

Then, $\frac{|x^2| + |y^2| + |z^2|}{|a^2| + |b^2| + |c^2|} = 3$

16. Let $M = \begin{bmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{bmatrix}$

$\therefore \quad M \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} = \begin{bmatrix} -1 \\ 2 \\ 3 \end{bmatrix}, M \begin{bmatrix} 1 \\ -1 \\ 0 \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \\ -1 \end{bmatrix},$

$$M\begin{bmatrix}1\\1\\1\end{bmatrix}=\begin{bmatrix}0\\0\\12\end{bmatrix}$$

$$\Rightarrow \begin{bmatrix}a_2\\b_2\\c_2\end{bmatrix}=\begin{bmatrix}-1\\2\\3\end{bmatrix}, \begin{bmatrix}a_1-a_2\\b_1-b_2\\c_1-c_2\end{bmatrix}=\begin{bmatrix}1\\1\\-1\end{bmatrix},$$

$$\begin{bmatrix}a_1+a_2+a_3\\b_1+b_2+b_3\\c_1+c_2+c_3\end{bmatrix}=\begin{bmatrix}0\\0\\12\end{bmatrix}$$

$\Rightarrow \quad a_2=-1, b_2=2, c_2=3, a_1-a_2=1,$
$b_1-b_2=1, c_1-c_2=-1$

$\Rightarrow \quad a_1+a_2+a_3=0, b_1+b_2+b_3=0,$
$c_1+c_2+c_3=12$

$\therefore \quad a_1=0, b_2=2$ and $c_3=7$

Hence, sum of diagonal elements
$=0+2+7=9$

17. $f(x)=x^4-4x^3+12x^2+x-1$

$f'(x)=4x^3-12x^2+24x+1$

$f''(x)=12x^2-24x+24$
$=12(x^2-2x+2)$
$=12\{(x-1)^2+1\}>0$, for all x

$\Rightarrow f'(x)$ is increasing.

Since, $f'(x)$ is cubic and increasing.

$\Rightarrow f'(x)$ has only one real root and two imaginary roots.

$\therefore$ $f(x)$ cannot have all distinct root.

$\Rightarrow$ Atmost 2 real roots.

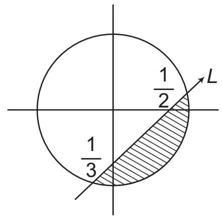
Now, $f(-1)=15$,
$f(0)=-1$ and $f(1)=9$

$\therefore$ $f(x)$ must have one root in $(-1, 0)$ and other in $(0, 1)$.

$\Rightarrow$ 2 real roots.

18. $x^2+y^2\le 6$ and $2x-3y=1$ is shown as,

For the point to lie in the shaded part, origin and the point lie on opposite side of straight line L.



$\therefore$ For any point in shaded part $L>0$ and for any point inside the circle $S<0$.

Now, for $\left(2, \frac{3}{4}\right)$ $L: 2x-3y-1$

$$L: 4-\frac{9}{4}-1=\frac{3}{4}>0$$

and $S: x^2+y^2-6,$

$$S: 4+\frac{9}{16}-6<0$$

$\Rightarrow \left(2, \frac{3}{4}\right)$ lies in shaded part.

For $\left(\frac{5}{2}, \frac{3}{4}\right) L: 5-9-1<0$ [neglect]

For $\left(\frac{1}{4}, -\frac{1}{4}\right) L: \frac{1}{2}+\frac{3}{4}-1>0\}$

$\therefore \left(\frac{1}{4}, -\frac{1}{4}\right)$ lies in the shaded part.

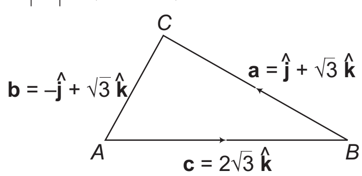
For $\left(\frac{1}{8}, \frac{1}{4}\right)$ $L: \frac{1}{4}-\frac{3}{4}-1<0$ [neglect]

$\Rightarrow$ Only 2 points lie in the shaded part.

19. (A) $\therefore |\mathbf{a}|=\sqrt{1+3}=2$

$|\mathbf{b}|=\sqrt{1+3}=2$

$|\mathbf{c}|=\sqrt{12}=2\sqrt{3}$



Using cosine law,

$$\cos C=\frac{|\mathbf{a}|^2+|\mathbf{b}|^2-|\mathbf{c}|^2}{2|\mathbf{a}||\mathbf{b}|}$$

$$=\frac{4+4-12}{2\times 2\times 2}=\frac{-4}{8}=\frac{-1}{2}$$

$$\Rightarrow \quad \angle C=120^\circ=\frac{2\pi}{3}$$

(B) $\int_a^b f(x)\,dx - 3\left(\frac{x^2}{2}\right)_a^b = (a^2 - b^2)$

$$\Rightarrow \int_a^b f(x)\,dx - \frac{3}{2}(b^2 - a^2) = (a^2 - b^2)$$

$$\Rightarrow \int_a^b f(x)dx = (a^2 - b^2) + \frac{3}{2}(b^2 - a^2)$$

$$= \frac{b^2 - a^2}{2}$$

$$\Rightarrow \int_a^b f(x)\,dx = \frac{b^2 - a^2}{2}$$

$$f(x) = x \Rightarrow f\left(\frac{\pi}{6}\right) = \frac{\pi}{6}$$

(C) $\frac{\pi^2}{\log_e 3}\int_{7/6}^{5/6} \sec(\pi x)\,dx$

$$\Rightarrow \frac{\pi^2}{\log_e 3}\left\{\frac{\log|\sec \pi x + \tan \pi x|}{\pi}\right\}_{7/6}^{5/6}$$

$$\Rightarrow \frac{\pi}{\log 3}\left\{\log\left|\sec\frac{5\pi}{6} + \tan\frac{5\pi}{6}\right| - \log\left|\sec\frac{7\pi}{6} + \tan\frac{7\pi}{6}\right|\right\}$$

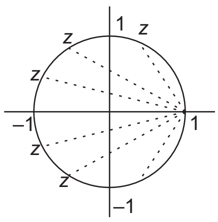
$$\Rightarrow \frac{\pi}{\log 3}\left\{\log|\sqrt{3}| - \log\left|\frac{1}{\sqrt{3}}\right|\right\}$$

$$\Rightarrow \frac{\pi}{\log 3}\{\log 3\} = \pi$$

(D) $\left|\arg\frac{1}{(1-z)}\right|$, for $|z| = 1$

$$\Rightarrow |\arg(1-z)^{-1}|$$

$$\Rightarrow |-\arg(1-z)| \Rightarrow |\arg(1-z)|$$



From figure, $\arg(z-1)$ is maximum $= \pi$.

20. (A) Given, $|z| = 1 \Rightarrow z\cdot\bar{z} = 1$

$$\therefore \frac{2iz}{1-z^2} = \frac{2iz}{z\cdot\bar{z} - z^2} = \frac{2i}{\bar{z} - z},$$

Let $z = x + iy$

$$\therefore z - \bar{z} = 2iy = \frac{2i}{-2iy} = -\frac{1}{y} \quad \ldots\text{(i)}$$

where, $y = \sqrt{1 - x^2}$

$\therefore \quad -1 \le y \le 1 \Rightarrow -1 \le y$

and $y \le 1 \Rightarrow -1 \ge \frac{1}{y}$ and $\frac{1}{y} \ge 1$

$$\Rightarrow \text{Re}\left(\frac{2iz}{1-z^2}\right) \in (-\infty, -1] \cup [1, \infty)$$

(B) $f(x) = \sin^{-1}\left(\frac{8(3^{x-2})}{1 - 3^{2(x-1)}}\right)$,

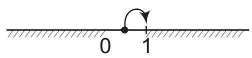
For domain, $-1 \le \frac{8(3^{x-2})}{1 - 3^{2(x-1)}} \le 1$

$$\Rightarrow -1 \le \frac{9\cdot(3^{x-2}) - (3^{x-2})}{1 - 3^{2(x-1)}} \le 1$$

$$\therefore -1 \le \frac{3^x - 3^{x-2}}{1 - 3^x\cdot 3^{(x-2)}} \le 1$$

$$\frac{3^x - 3^{x-2}}{1 - 3^x\cdot 3^{x-2}} \ge -1$$

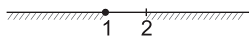
$$\Rightarrow \frac{(3^x - 1)(3^{x-2} - 1)}{(3^{x-1} + 1)(3^{x-1} - 1)} \ge 0$$



$$\Rightarrow x \in (-\infty, 0] \cup (1, \infty)$$

and $\frac{3^x - 3^{x-2}}{1 - 3^x\cdot 3^{x-2}} \le 1$

$$\Rightarrow \frac{(3^{x-2} - 1)(3^x + 1)}{(3^{x-1} + 1)(3^{x-1} - 1)} \ge 0$$



and $x \in (-\infty, 1) \cup [2, \infty)$

$\therefore \quad x \in (-\infty, 0] \cup [2, \infty)$

(C) $f(\theta) = \begin{vmatrix} 1 & \tan\theta & 1 \\ -\tan\theta & 1 & \tan\theta \\ -1 & -\tan\theta & 1 \end{vmatrix}$

$R_1 \to R_1 + R_3$,

$$f(\theta) = \begin{vmatrix} 0 & 0 & 2 \\ -\tan\theta & 1 & \tan\theta \\ -1 & -\tan\theta & 1 \end{vmatrix}$$

$$= 2(\tan^2\theta + 1) = 2\sec^2\theta \ge 2$$

$f(\theta) \in [2, \infty)$