IIT JEE

SOLVED PAPER 2012

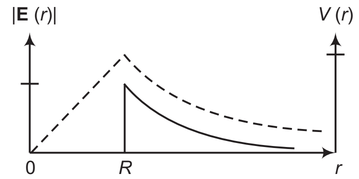
Physics

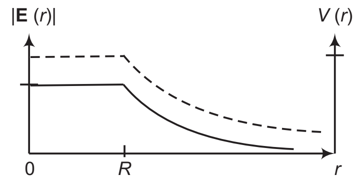
Paper 1

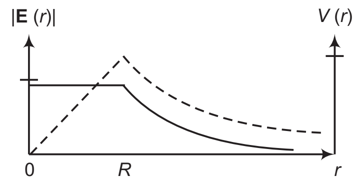
Section I

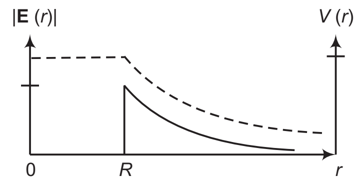
Directions (Q. Nos. 1-10) *This section contains* 10 *multiple choice questions. Each question has four choices (a), (b), (c) and (d) out of which only one is correct.*

1. Consider a thin spherical shell of radius R with its centre at the origin, carrying uniform positive surface charge density. The variation of the magnitude of the electric field $|\mathbf{E}(r)|$ and the electric potential $V(r)$ with the distance r from the centre, is best represented by which graph?

(a)


(b)


(c)


(d)


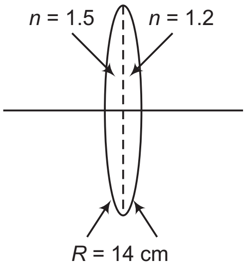
2. Young's double slit experiment is carried out by using green, red and blue light, one colour at a time. The fringe widths recorded are β_G, β_R and β_B respectively. Then,

(a) $\beta_G > \beta_B > \beta_R$
(b) $\beta_B > \beta_G > \beta_R$
(c) $\beta_R > \beta_B > \beta_G$
(d) $\beta_R > \beta_G > \beta_B$

3. Two large vertical and parallel metal plates having a separation of 1 cm are connected to a DC voltage source of potential difference X. A proton is released at rest midway between the two plates. It is found to move at 45° to the vertical just after release. Then X is nearly

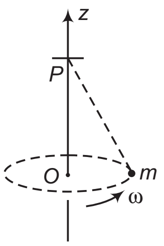
(a) 1×10^{-5} V
(b) 1×10^{-7} V
(c) 1×10^{-9} V
(d) 1×10^{-10} V

4. A bi-convex lens is formed with two thin plano-convex lenses as shown in the figure. Refractive index n of the first lens is 1.5 and that of the second lens is 1.2. Both the curved surfaces are of the same radius of curvature $R = 14$ cm. For this bi-convex lens, for an object distance of 40 cm, the image distance will be



(a) – 280.0 cm (b) 40.0 cm
(c) 21.5 cm (d) 13.3 cm

5. A small mass m is attached to a massless string whose other end is fixed at P as shown in the figure. The mass is undergoing circular motion in the x-y plane with centre at O and constant angular speed ω. If the angular momentum of the system, calculated about O and P are denoted by $\mathbf{L}_O$ and $\mathbf{L}_P$ respectively, then

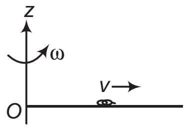


(a) $\mathbf{L}_O$ and $\mathbf{L}_P$ do not vary with time
(b) $\mathbf{L}_O$ varies with time while $\mathbf{L}_P$ remains constant
(c) $\mathbf{L}_O$ remains constant while $\mathbf{L}_P$ varies with time
(d) $\mathbf{L}_O$ and $\mathbf{L}_P$ both vary with time

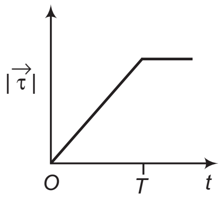
6. A mixture of 2 moles of helium gas (atomic mass = 4 amu) and 1 mole of argon gas (atomic mass = 40 amu) is kept at 300 K in a container. The ratio of the rms speeds $\left(\frac{v_{\text{rms}}\text{ (helium)}}{v_{\text{rms}}\text{ (argon)}}\right)$ is

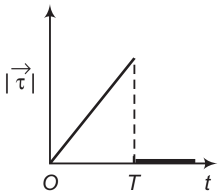
(a) 0.32
(b) 0.45
(c) 2.24
(d) 3.16

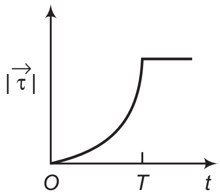
7. A thin uniform rod, pivoted at O, is rotating in the horizontal plane with constant angular speed ω, as shown in the figure.

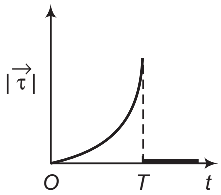


At time $t = 0$, a small insect starts from O and moves with constant speed v with respect to the rod towards the other end. It reaches the end of the rod at $t = T$ and stops. The angular speed of the system remains ω throughout. The magnitude of the torque $|\vec{\tau}|$ on the system about O, as a function of time is best represented by which plot?

(a) 

(b) 

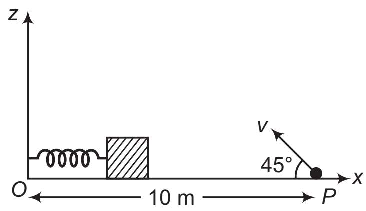
(c) 

(d) 

8. In the determination of Young's modulus $\left(Y = \frac{4MLg}{\pi l d^2}\right)$ by using Searle's method, a wire of length $L = 2$ m and diameter $d = 0.5$ mm is used. For a load $M = 2.5$ kg, an extension $l = 0.25$ mm in the length of the wire is observed. Quantities d and l are measured using a screw gauge and a micrometer, respectively. They have the same pitch of 0.5 mm. The number of divisions on their circular scale is 100. The contributions to the maximum probable error of the Y measurement is

(a) due to the errors in the measurements of d and l are the same
(b) due to the error in the measurement of d is twice that due to the error in the measurement of l.
(c) due to the error in the measurement of l is twice that due to the error in the measurement of d.
(d) due to the error in the measurement of d is four times that due to the error in the measurement of l.

9. A small block is connected to one end of a massless spring of unstretched length 4.9 m. The other end of the spring (see the figure) is fixed. The system lies on a horizontal frictionless surface. The block is stretched by 0.2 m and released from rest at $t = 0$. It then executes simple harmonic motion with angular frequency $\omega = \frac{\pi}{3}$ rad/s. Simultaneously at $t = 0$, a small pebble is projected with speed v from point P at an angle of 45° as shown in the figure. Point P is at a horizontal distance of 10 m from O. If the pebble hits the block at $t = 1$ s, the value of v is (take $g = 10$ m/s^2)



(a) $\sqrt{50}$ m/s
(b) $\sqrt{51}$ m/s
(c) $\sqrt{52}$ m/s
(d) $\sqrt{53}$ m/s

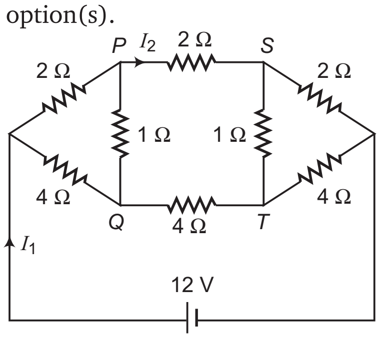
10. Three very large plates of same area are kept parallel and close to each other. They are considered as ideal black surfaces and have very high thermal conductivity. The first and third plates are maintained at temperatures $2T$ and $3T$ respectively. The temperature of the middle (*i.e.*, second) plate under steady state condition is

(a) $\left(\frac{65}{2}\right)^{\frac{1}{4}} T$
(b) $\left(\frac{97}{4}\right)^{\frac{1}{4}} T$
(c) $\left(\frac{97}{2}\right)^{\frac{1}{4}} T$
(d) $(97)^{\frac{1}{4}} T$

Section II

Directions (Q. Nos. 11-15) *This section contains 5 multiple choice questions. Each question has four choices (a), (b), (c) and (d) out of which one or more are correct.*

11. For the resistance network shown in the figure, choose the correct option(s).



(a) The current through PQ is zero
(b) $I_1 = 3$ A
(c) The potential at S is less than that at Q
(d) $I_2 = 2$ A

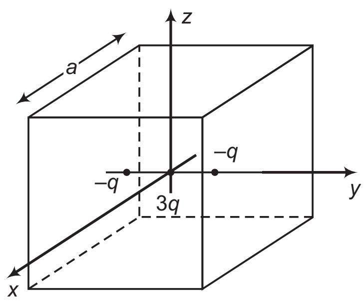
12. A person blows into open-end of a long pipe. As a result, a high-pressure pulse of air travels down the pipe. When this pulse reaches the other end of the pipe,

(a) a high-pressure pulse starts travelling up the pipe, if the other end of the pipe is open
(b) a low-pressure pulse starts travelling up the pipe, if the other end of the pipe is open
(c) a low-pressure pulse starts travelling up the pipe, if the other end of the pipe is closed
(d) a high-pressure pulse starts travelling up the pipe, if the other end of the pipe is closed

13. Consider the motion of a positive point charge in a region where there are simultaneous uniform electric and magnetic fields $\mathbf{E} = E_0\,\hat{\mathbf{j}}$ and $\mathbf{B} = B_0\,\hat{\mathbf{j}}$. At time $t = 0$, this charge has velocity $\mathbf{v}$ in the x-y plane, making an angle θ with the x-axis. Which of the following option(s) is(are) correct for time $t > 0$?

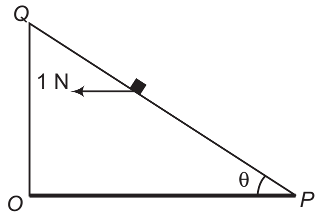
(a) If $\theta = 0°$, the charge moves in a circular path in the x-z plane.
(b) If $\theta = 0°$, the charge undergoes helical motion with constant pitch along the y-axis
(c) If $\theta = 10°$, the charge undergoes helical motion with its pitch increasing with time, along the y-axis.
(d) If $\theta = 90°$, the charge undergoes linear but accelerated motion along the y-axis.

14. A cubical region of side a has its centre at the origin. It encloses three fixed point charges, $-q$ at $(0, -a/4, 0)$, $+3q$ at $(0, 0, 0)$ and $-q$ at $(0, +a/4, 0)$. Choose the correct option(s).



(a) The net electric flux crossing the plane $x = +a/2$ is equal to the net electric flux crossing the plane $x = -a/2$
(b) The net electric flux crossing the plane $y = +a/2$ is more than the net electric flux crossing the plane $y = -a/2$
(c) The net electric flux crossing the entire region is q/ε_0
(d) The net electric flux crossing the plane $z = +a/2$ is equal to the net electric flux crossing the plane $x = +a/2$

15. A small block of mass of 0.1 kg lies on a fixed inclined plane PQ which makes an angle θ with the horizontal. A horizontal force of 1 N acts on the block through its centre of mass as shown in the figure. The block remains stationary if (take $g = 10 \text{ m/s}^2$)

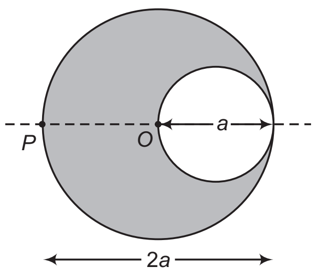


(a) $\theta = 45°$.
(b) $\theta > 45°$ and a frictional force acts on the block towards P
(c) $\theta > 45°$ and a frictional force acts on the block towards Q
(d) $\theta < 45°$ and a frictional force acts on the block towards Q

Section III

Directions (Q. Nos. 16-20) *This section contains 5 questions. The answer to each question is a single digit integer, ranging from 0 to 9 (both inclusive).*

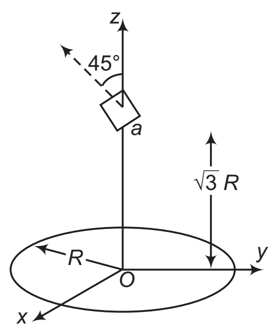
16. A cylindrical cavity of diameter a exists inside a cylinder of diameter $2a$ as shown in the figure. Both the cylinder and the cavity are infinitely long. A uniform current density J flows along the length. If the magnitude of the magnetic field at the point P is given by $\frac{N}{12}\mu_0 aJ$, then the value of N is



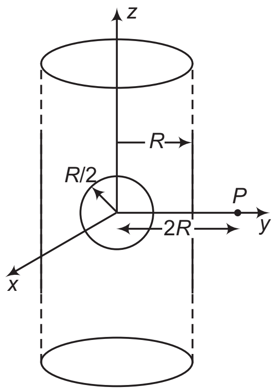
17. A proton is fired from very far away towards a nucleus with charge $Q = 120\,e$, where e is the electronic charge. It makes a closest approach of 10 fm to the nucleus. The de-Broglie wavelength (in units of fm) of the proton at its start is

[Take the proton mass, $m_p = (5/3) \times 10^{-27}$ kg;
$h/e = 4.2 \times 10^{-15}$ J-s/C;
$\frac{1}{4\pi\varepsilon_0} = 9 \times 10^9$ m/F; 1 fm $= 10^{-15}$ m]

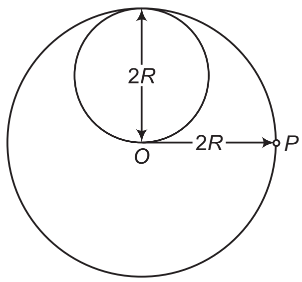
18. A circular wire loop of radius R is placed in the x-y plane centred at the origin O. A square loop of side $a\,(a << R)$ having two turns is placed with its centre at $z = \sqrt{3}\,R$ along the axis of the circular wire loop, as shown in figure. The plane of the square loop makes an angle of 45° with respect to the z-axis. If the mutual inductance between the loops is given by $\frac{\mu_0 a^2}{2^{p/2} R}$, then the value of p is



19. An infinitely long solid cylinder of radius R has a uniform volume charge density ρ. It has a spherical cavity of radius $R/2$ with its centre on the axis of the cylinder, as shown in the figure. The magnitude of the electric field at the point P, which is at a distance $2R$ from the axis of the cylinder, is given by the expression $\frac{23\rho R}{16k\varepsilon_0}$. The value of k is



20. A lamina is made by removing a small disc of diameter $2R$ from a bigger disc of uniform mass density and radius $2R$, as shown in the figure. The moment of inertia of this lamina about axes passing through O and P is I_O and I_P, respectively. Both these axes are perpendicular to the plane of the lamina. The ratio $\frac{I_P}{I_O}$ to the nearest integer is



Paper 2

Section I

Directions (Q. Nos. 21-28) *This section contains 8 multiple choice questions. Each question has four choices (a), (b), (c) and (d) out of which only one is correct.*

21. A thin uniform cylindrical shell, closed at both ends, is partially filled with water. It is floating vertically in water in half-submerged state. If ρ_c is the relative density of the material of the shell with respect to water, then the correct statement is that the shell is

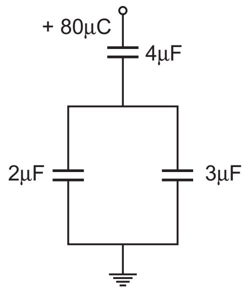
(a) more than half-filled if ρ_c is less than 0.5

(b) more than half-filled if ρ_c is more than 1.0

(c) half-filled if ρ_c is more than 0.5

(d) less than half-filled if ρ_c is less than 0.5

22. In the given circuit, a charge of +80 μC is given to the upper plate of the 4 μF capacitor. Then in the steady state, the charge on the upper plate of the 3 μF capacitor is



(a) + 32 μC

(b) + 40 μC

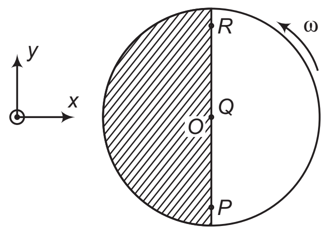
(c) + 48 μC

(d) + 80 μC

23. Two moles of ideal helium gas are in a rubber balloon at 30°C. The balloon is fully expandable and can be assumed to require no energy in its expansion. The temperature of the gas in the balloon is slowly changed to 35°C. The amount of heat required in raising the temperature is nearly (take $R = 8.31$ J/mol-K)

(a) 62 J (b) 104 J
(c) 124 J (d) 208 J

24. Consider a disc rotating in the horizontal plane with a constant angular speed ω about its centre O. The disc has a shaded region on one side of the diameter and an unshaded region on the other side as shown in the figure. When the disc is in the orientation as shown, two pebbles P and Q are simultaneously projected at an angle towards R. The velocity of projection is in the y-z plane and is same for both pebbles with respect to the disc. Assume that (i) they land back on the disc before the disc has completed $\frac{1}{8}$ rotation, (ii) their range is less than half the disc radius, and (iii) ω remains constant throughout. Then

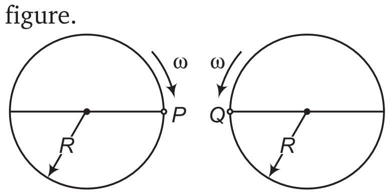


(a) P lands in the shaded region and Q in the unshaded region
(b) P lands in the unshaded region and Q in the shaded region
(c) both P and Q land in the unshaded region
(d) both P and Q land in the shaded region

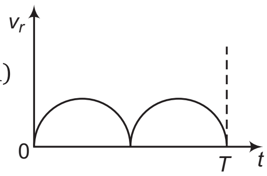
25. A student is performing the experiment of resonance column. The diameter of the column tube is 4 cm. The frequency of the tuning fork is 512 Hz. The air temperature is 38° C in which the speed of sound is 336 m/s. The zero of the meter scale coincides with the top end of the resonance column tube. When the first resonance occurs, the reading of the water level in the column is

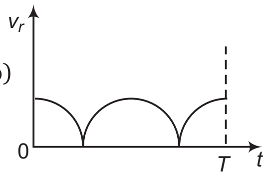
(a) 14.0 cm (b) 15.2 cm
(c) 16.4 cm (d) 17.6 cm

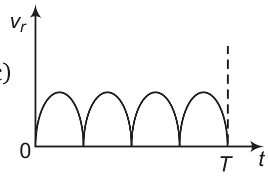
26. Two identical discs of same radius R are rotating about their axes in opposite directions with the same constant angular speed ω. The discs are in the same horizontal plane. At time $t = 0$, the points P and Q are facing each other as shown in the figure.

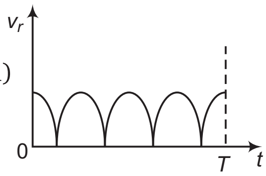


The relative speed between the two points P and Q is v_r. In one time period (T) of rotation of the discs, v_r as a function of time is best represented by

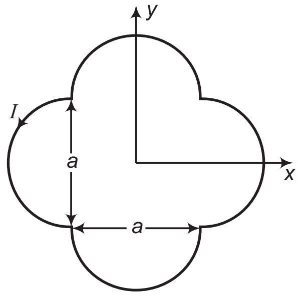
(a)


(b)


(c)


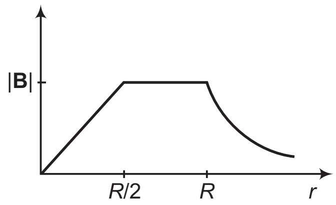
(d)


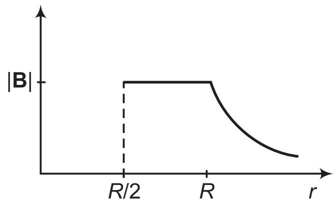
27. A loop carrying current I lies in the x-y plane as shown in the figure. The unit vector $\mathbf{k}$ is coming out of the plane of the paper. The magnetic moment of the current loop is

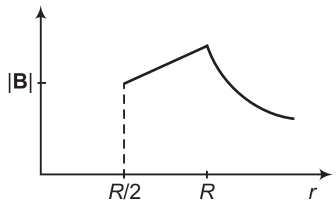


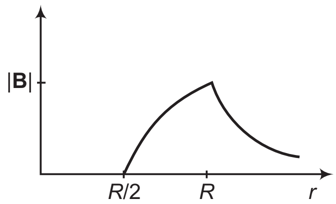
(a) $a^2 I \mathbf{k}$
(b) $\left(\frac{\pi}{2}+1\right) a^2 I \mathbf{k}$
(c) $-\left(\frac{\pi}{2}+1\right) a^2 I \mathbf{k}$
(d) $(2\pi+1) a^2 I \mathbf{k}$

28. An infinitely long hollow conducting cylinder with inner radius $R/2$ and outer radius R carries a uniform current density along its length. The magnitude of the magnetic field, $|\mathbf{B}|$ as a function of the radial distance r from the axis is best represented by

(a) 

(b) 

(c) 

(d) 

Section II

Directions (Q. Nos. 29-34) *This section contains 6 multiple choice questions relating to three paragraphs with two questions on each paragraph. Each question has four choices (a), (b), (c) and (d) out of which only one is correct.*

Paragraph

Most materials have the refractive index, $n > 1$. So, when a light ray from air enters a naturally occurring material, then by Snell's law, $\frac{\sin\theta_1}{\sin\theta_2}=\frac{n_2}{n_1}$, it is understood that the refracted ray bends towards the normal. But it never emerges on the same side of the normal as the incident ray. According to electromagnetism, the refractive index of the medium is given by the relation, $n=\left(\frac{c}{v}\right)=\pm\sqrt{\varepsilon_r \mu_r}$, where c is the speed of electromagnetic waves in vacuum, v its speed in the medium, ε_r and μ_r are the relative permittivity and permeability of the medium respectively.

In normal materials, both ε_r and μ_r are positive, implying positive n for the medium. When both ε_r and μ_r are negative, one must choose the negative root of n. Such negative refractive index materials can now be artificially prepared and are called meta-materials. They exhibit significantly different optical behaviour, without violating any physical laws. Since n is negative, it results in a change in the direction of propagation of the refracted light. However, similar to normal materials, the frequency of light remains unchanged upon refraction even in meta-materials.

29. Choose the correct statement.

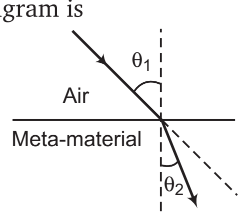
(a) The speed of light in the meta-material is $v = c\,|n|$

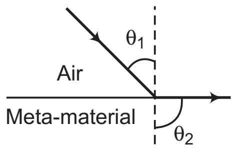
(b) The speed of light in the meta-material is $v = \dfrac{c}{|n|}$

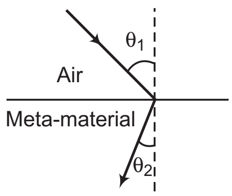
(c) The speed of light in the meta-material is $v = c$

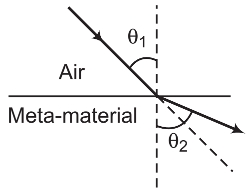
(d) The wavelength of the light in the meta-material (λ_m) is given by $\lambda_m = \lambda_{\text{air}}\,|n|$, where λ_{air} is the wavelength of the light in air.

30. For light incident from air on a meta-material, the appropriate ray diagram is

(a)


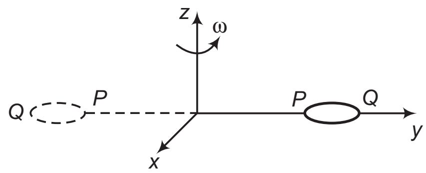
(b)


(c)


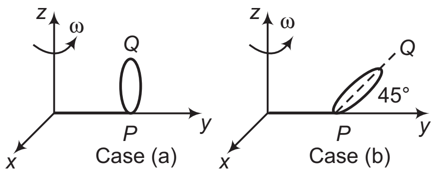
(d)


Paragraph

The general motion of a rigid body can be considered to be a combination of (i) a motion of its centre of mass about an axis, and (ii) its motion about an instantaneous axis passing through the centre of mass. These axes need not be stationary. Consider, for example, a thin uniform disc welded (rigidly fixed) horizontally at its rim to a massless stick, as shown in the figure. When the disc-stick system is rotated about the origin on a horizontal frictionless plane with angular speed ω, the motion at any instant can be taken as a combination of (i) a rotation of the centre of mass of the disc about the z-axis, and (ii) a rotation of the disc through an instantaneous vertical axis passing through its centre of mass (as is seen from the changed orientation of points P and Q). Both these motions have the same angular speed ω in this case.



Now consider two similar systems as shown in the figure : Case (a) the disc with its face vertical and parallel to x-z plane; Case (b) the disc with its face making an angle of 45° with x-y plane and its horizontal diameter parallel to x-axis. In both the cases, the disc is welded at point P, and the systems are rotated with constant angular speed ω about the z-axis.



31. Which of the following statements regarding the angular speed about the instantaneous axis (passing through the centre of mass) is correct?

(a) It is $\sqrt{2}\omega$ for both the cases

(b) It is ω for case (a); and $\frac{\omega}{\sqrt{2}}$ for case (b)

(c) It is ω for case (a); and $\sqrt{2}\omega$ for case (b)

(d) It is ω for both the cases

32. Which of the following statements about the instantaneous axis (passing through the centre of mass) is correct?

(a) It is vertical for both the cases (a) and (b)

(b) It is vertical for case (a); and is at 45° to the x-z plane and lies in the plane of the disc for case (b)

(c) It is horizontal for case (a); and is at 45° to the x-z plane and is normal to the plane of the disc for case (b)

(d) It is vertical for case (a); and is at 45° to the x-z plane and is normal to the plane of the disc for case (b)

Paragraph

The β-decay process, discovered around 1900, is basically the decay of a neutron (n). In the laboratory, a proton (p) and an electron (e^-) are observed as the decay products of the neutron. Therefore, considering the decay of a neutron as a two-body decay process, it was predicted theoretically that the kinetic energy of the electron should be a constant. But experimentally, it was observed that the electron kinetic energy has a continuous spectrum. Considering a three-body decay process, *i.e.*, $n \rightarrow p + e^- + \bar{\nu}_e$, around 1930, Pauli explained the observed electron energy spectrum. Assuming the anti-neutrino $(\bar{\nu}_e)$ to be massless and possessing negligible energy, and the neutron to be at rest, momentum and energy conservation principles are applied. From this calculation, the maximum kinetic energy of the electron is 0.8×10^6 eV. The kinetic energy carried by the proton is only the recoil energy.

33. If the anti-neutrino had a mass of 3 eV/c^2 (where c is the speed of light) instead of zero mass, what should be the range of the kinetic energy K, of the electron?

(a) $0 \le K \le 0.8 \times 10^6$ eV

(b) $3.0 \text{ eV} \le K \le 0.8 \times 10^6$ eV

(c) $3.0 \text{ eV} \le K < 0.8 \times 10^6$ eV

(d) $0 \le K < 0.8 \times 10^6$ eV

34. What is the maximum energy of the anti-neutrino?

(a) Zero

(b) Much less than 0.8×10^6 eV

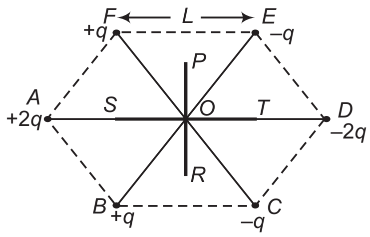
(c) Nearly 0.8×10^6 eV

(d) Much larger than 0.8×10^6 eV

Section III

Directions (Q. Nos. 35-40) *This section contains 6 multiple choice questions. Each question has four choices (a), (b), (c) and (d) out of which one or more are correct.*

35. Six point charges are kept at the vertices of a regular hexagon of side L and centre O as shown in the figure. Given that $K = \frac{1}{4\pi\varepsilon_0}\frac{q}{L^2}$, which of the following statements(s) is(are) correct.

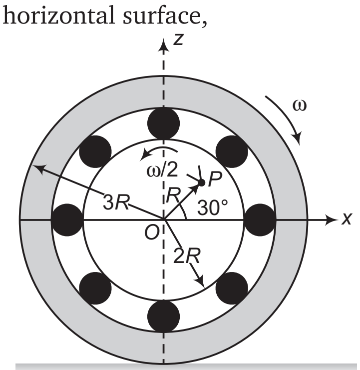


(a) The electric field at O is $6K$ along OD
(b) The potential at O is zero
(c) The potential at all points on the line PR is same
(d) The potential at all points on the line ST is same

36. Two solid cylinders P and Q of same mass and same radius start rolling down a fixed inclined plane from the same height at the same time. Cylinder P has most of its mass concentrated near its surface, while Q has most of its mass concentrated near the axis. Which statement(s) is(are) correct?
(a) Both cylinders P and Q reach the ground at the same time
(b) Cylinder P has larger linear acceleration than cylinder Q
(c) Both cylinders reach the ground with same translational kinetic energy
(d) Cylinder Q reaches the ground with larger angular speed

37. Two spherical planets P and Q have the same uniform density ρ, masses M_P and M_Q, and surface areas A and $4A$, respectively. A spherical planet R also has uniform density ρ and its mass is $(M_P + M_Q)$. The escape velocities from the planets P, Q and R, are V_P, V_Q and V_R, respectively. Then
(a) $V_Q > V_R > V_P$ (b) $V_R > V_Q > V_P$
(c) $V_R/V_P = 3$ (d) $V_P/V_Q = \frac{1}{2}$

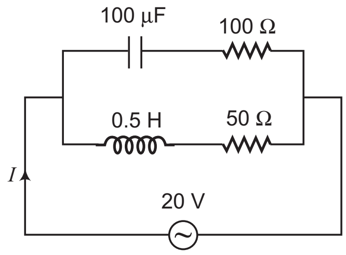
38. The figure shows a system consisting of (i) a ring of outer radius $3R$ rolling clockwise without slipping on a horizontal surface with angular speed ω and (ii) an inner disc of radius $2R$ rotating anti-clockwise with angular speed $\omega/2$. The ring and disc are separated by frictionless ball bearings. The system is in the x-z plane. The point P on the inner disc is at a distance R from the origin, where OP makes an angle of 30° with the horizontal. Then with respect to the horizontal surface,



(a) the point O has a linear velocity $3R\omega\,\mathbf{i}$
(b) the point P has a linear velocity $\frac{11}{4}R\omega\,\mathbf{i} - \frac{\sqrt{3}}{4}R\omega\,\mathbf{k}$
(c) the point P has a linear velocity $\frac{13}{4}R\omega\,\mathbf{i} - \frac{\sqrt{3}}{4}R\omega\,\mathbf{k}$

(d) the point P has a linear velocity $\left(3-\frac{\sqrt{3}}{4}\right)R\omega\,\mathbf{i}+\frac{1}{4}R\omega\,\mathbf{k}$

39. In the given circuit, the AC source has ω = 100 rad/s. Considering the inductor and capacitor to be ideal, the correct choice(s) is(are)



(a) The current through the circuit, I is 0.3 A

(b) The current through the circuit, I is $0.3\sqrt{2}$ A

(c) The voltage across 100 Ω resistor $= 10\sqrt{2}$ V

(d) The voltage across 50 Ω resistor = 10 V

40. A current carrying infinitely long wire is kept along the diameter of a circular wire loop, without touching it. The correct statement(s) is(are)

(a) The emf induced in the loop is zero if the current is constant

(b) The emf induced in the loop is finite if the current is constant

(c) The emf induced in the loop is zero if the current decreases at a steady rate

(d) The emf induced in the loop is finite if the current decreases at a steady rate

Answers

Paper 1

1. (d) **2.** (d) **3.** (c) **4.** (b) **5.** (c) **6.** (d) **7.** (b) **8.** (a) **9.** (a) **10.** (c)
11. (a,b,c,d) **12.** (b,d) **13.** (c,d) **14.** (a,c) **15.** (a,c)
16. (5) **17.** (7) **18.** (7) **19.** (6) **20.** (3)

Paper 2

21. (a) **22.** (c) **23.** (d) **24.** (*) **25.** (b) **26.** (a) **27.** (b) **28.** (d) **29.** (b) **30.** (c)
31. (d) **32.** (a) **33.** (d) **34.** (c) **35.** (a,b,c) **36.** (d) **37.** (b,d)
38. (a,b) **39.** (a,c) **40.** (a,c)

Note : (*) No option is correct.

Hints & Solutions

Paper 1

1. **For inside points** $(r \le R)$

$E = 0$

$V = \text{constant} = \frac{1}{4\pi\varepsilon_0}\frac{q}{R}$

For outside points $(r \ge R)$

$$E = \frac{1}{4\pi\varepsilon_0}\cdot\frac{q}{r^2} \quad \text{or} \quad E \propto \frac{1}{r^2}$$

and $V = \frac{1}{4\pi\varepsilon_0}\frac{q}{r}$ or $V \propto \frac{1}{r}$

On the surface $(r = R)$

$$V = \frac{1}{4\pi\varepsilon_0}\frac{q}{R}$$

$$E = \frac{1}{4\pi\varepsilon_0}\cdot\frac{q}{R^2} = \frac{\sigma}{\varepsilon_0}$$

where, $\sigma = \frac{q}{4\pi R^2}$ = surface charge density corresponding to above equations the correct graphs are shown in option (d).

2. Fringe width $\beta = \frac{\lambda D}{d}$

or $\beta \propto \lambda$

Now, $\lambda_R > \lambda_G > \lambda_B$

$\therefore$ $\beta_R > \beta_G > \beta_B$

3. 

Net force is at 45° from vertical.

$\therefore$ $qE = mg$

or $\frac{qX}{d} = mg \quad \left(\because E = \frac{X}{d}\right)$

or $X = \frac{mgd}{q}$

$$= \frac{(1.67\times10^{-27})(9.8)(10^{-2})}{(1.6\times10^{-19})}$$

$$\approx 1\times10^{-9}\ \text{V}$$

4. Using the lens formula,

$$\frac{1}{v} - \frac{1}{u} = \frac{1}{F} = \frac{1}{f_1} + \frac{1}{f_2}$$

or $\frac{1}{v} = \frac{1}{u} + \frac{1}{f_1} + \frac{1}{f_2}$

$$= \frac{1}{u} + (n_1 - 1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right) + (n_2 - 1)\left(\frac{1}{R_1'} - \frac{1}{R_2'}\right)$$

Substituting the values, we get

$$\frac{1}{v} = \frac{1}{-40} + (1.5 - 1)\left(\frac{1}{14} - \frac{1}{\infty}\right) + (1.2 - 1)\left(\frac{1}{\infty} - \frac{1}{-14}\right)$$

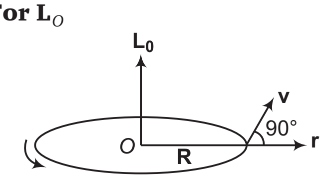
Solving this equations, we get

$$v = +\ 40\ \text{cm}$$

5. Angular momentum of a particle about a point is given by :

$\mathbf{L} = \mathbf{r}\times\mathbf{p} = m(\mathbf{r}\times\mathbf{v})$

For $\mathbf{L}_O$

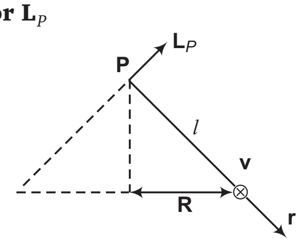


$$|\mathbf{L}| = (mvr\sin\theta) = m(R\omega)(R)\sin 90^\circ$$

$$= mR^2\omega = \text{constant}$$

Direction of $\mathbf{L}_O$ is always upwards. Therefore, complete $\mathbf{L}_O$ is constant, both in magnitude as well as direction.

For $\mathbf{L}_P$



$|\mathbf{L}_P| = (mvr \sin\theta)$

$= (m)(R\omega)(l)\sin 90°$

$= (mRl\omega)$

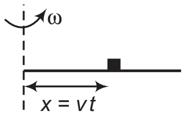
Magnitude of $\mathbf{L}_P$ will remain constant but direction of $\mathbf{L}_P$ keeps on changing.

6. $v_{\text{rms}} = \sqrt{\dfrac{3RT}{M}}$ or $v_{\text{rms}} \propto \dfrac{1}{\sqrt{M}}$

$\therefore \quad \dfrac{(v_{\text{rms}})_{\text{He}}}{(v_{\text{rms}})_{\text{Ar}}} = \sqrt{\dfrac{M_{\text{Ar}}}{M_{\text{He}}}} = \sqrt{\dfrac{40}{4}}$

$= \sqrt{10} \approx 3.16$

7. $|\mathbf{L}|$ or $L = I\omega$ (about axis of rod)



$I = I_{\text{rod}} + mx^2 = I_{\text{rod}} + mv^2t^2$

Here m = mass of insect

$\therefore \quad L = (I_{\text{rod}} + mv^2t^2)\omega$

Now $|\vec{\tau}| = \dfrac{dL}{dt} = (2mv^2t\omega)$ or $|\vec{\tau}| \propto t$

i.e., the graph is straight line passing through origin.

After time T,

$L =$ constant

$\therefore \quad |\vec{\tau}|$ or $\dfrac{dL}{dt} = 0$

8. $\Delta d = \Delta l = \dfrac{0.5}{100}$ mm $= 0.005$ mm

$Y = \dfrac{4MLg}{\pi l d^2}$

$\therefore \quad \left(\dfrac{\Delta Y}{Y}\right)_{\max} = \left(\dfrac{\Delta l}{l}\right) + 2\left(\dfrac{\Delta d}{d}\right)$

$\left(\dfrac{\Delta l}{l}\right) = \dfrac{0.5/100}{0.25} = 0.02$

and $\quad \dfrac{2\Delta d}{d} = \dfrac{(2)(0.5/100)}{0.5} = 0.02$

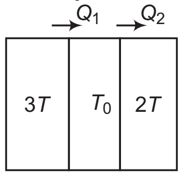
or $\quad \dfrac{\Delta l}{l} = 2 \cdot \dfrac{\Delta d}{d}$

9. t = time of flight of projectile

$= \dfrac{2v \sin\theta}{g} \qquad (\theta = 45°)$

$\therefore \quad v = \dfrac{gt}{2\sin\theta} = \dfrac{10 \times 1}{2 \times 1/\sqrt{2}} = \sqrt{50}$ m/s

10. Let temperature of middle plate in steady state is T_0



$Q_1 = Q_2$

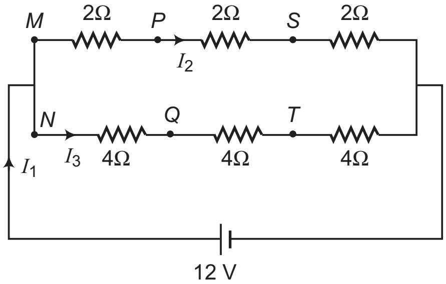
Q = net rate of heat flow

$\therefore \quad \sigma A(3T)^4 - \sigma A T_0^4 = \sigma A T_0^4 - \sigma A(2T)^4$

Solving this equation, we get

$T_0 = \left(\dfrac{97}{2}\right)^{1/4} T$

11. Due to symmetry on upper side and lower side, points P and Q are at same potentials. Similarly, points S and T are at same potentials. Therefore, the simple circuit can be drawn as shown below :



$I_2 = \dfrac{12}{2+2+2} = 2$ A

$I_3 = \dfrac{12}{4+4+4} = 1$ A

$\therefore \quad I_1 = I_2 + I_3 = 3$ A

$I_{PQ} = 0$ because $V_P = V_Q$

Potential drop (from left to right) across each resistance is

$\dfrac{12}{3} = 4$ V

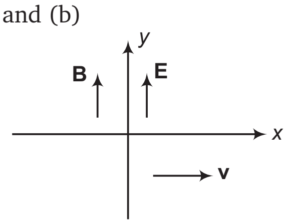
$\therefore \quad V_{MS} = 2 \times 4 = 8$ V

$V_{NQ} = 1 \times 4 = 4$ V

or $\quad V_S < V_Q$

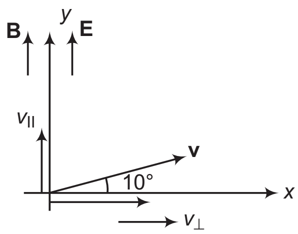
12. At open end phase of pressure wave changes by 180°. So, compression returns as rarefaction. At closed end there is no phase change. So, compression returns as compression and rarefaction as rarefaction.

13. (a) and (b)



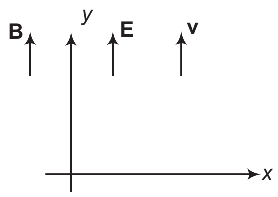
Magnetic field will rotate the particle in a circular path (in x-z plane or perpendicular to **B**). Electric field will exert a constant force on the particle in positive y-direction. Therefore, resultant path is neither purely circular nor helical or the options (a) and (b) both are wrong.

(c)



$v_\perp$ and **B** will rotate the particle in a circular path in x-z plane (or perpendicular to **B**). Further $v_{||}$ and **E** will move the particle (with increasing speed) along positive y-axis (or along the axis of above circular path). Therefore, the resultant path is helical with increasing pitch, along the y-axis (or along **B** and **E**). Therefore option (c) is correct.

(d)



Magnetic force is zero, as θ between **B** and **v** is zero. But electric force will act in y-direction. Therefore, motion is 1-D and uniformly accelerated (towards positive y-direction).

14. Option (a) is correct due to symmetry.

Option (b) is wrong again due to symmetry.

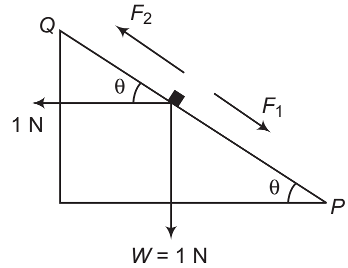
Option (c) is correct because as per Gauss's theorem, net electric flux passing through any closed surface $= \dfrac{q_{in}}{\varepsilon_0}$

Here, $q_{in} = 3q - q - q = q$

$\therefore$ Net electric flux $= \dfrac{q}{\varepsilon_0}$

Option (d) is wrong because there is no symmetry in two given planes.

15. $W = mg = 0.1 \times 10 = 1$ N



F_1 = component of weight
$= 1 \cdot \sin\theta = \sin\theta$

F_2 = component of applied force
$= 1 \cdot \cos\theta = \cos\theta$

Now, at $\theta = 45°$: $F_1 = F_2$ and block remains stationary without the help of friction.

For $\theta > 45°$, $F_1 > F_2$, so friction will act towards Q.

For $\theta < 45°$, $F_2 > F_1$ and friction will act towards P.

16. $B_R = B_T - B_C$

R = Remaining portion

T = Total portion and

C = cavity

$$B_R = \frac{\mu_0 I_T}{2a\pi} - \frac{\mu_0 I_C}{2(3a/2)\pi} \quad \ldots\text{(i)}$$

$$I_T = J(\pi a^2)$$

$$I_C = J\left(\frac{\pi a^2}{4}\right)$$

Substituting the values in Eq. (i), we have

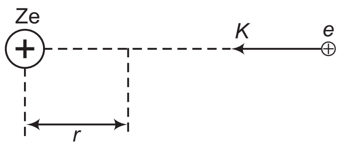
$$B_R = \frac{\mu_o}{a\pi}\left[\frac{I_T}{2} - \frac{I_C}{3}\right]$$

$$= \frac{\mu_o}{a\pi}\left[\frac{\pi a^2 J}{2} - \frac{\pi a^2 J}{12}\right]$$

$$= \frac{5\mu_o aJ}{12}$$

$\therefore \quad N = 5$

17.



$r =$ closest distance $= 10$ fm

From energy conservation, we have

$$K_i + U_i = K_f + U_f$$

or $\quad K + 0 = 0 + \frac{1}{4\pi\varepsilon_0} \cdot \frac{q_1 q_2}{r}$

or $\quad K = \frac{1}{4\pi\varepsilon_0} \cdot \frac{(120\, e)(e)}{r} \quad \ldots(i)$

de-Broglie wavelength

$$\lambda = \frac{h}{\sqrt{2Km}} \quad \ldots(ii)$$

Substituting the given values in above two equations, we get

$$\lambda = 7 \times 10^{-15}\text{m} = 7 \text{ fm}$$

18. If I current flows through the circular loop, then magnetic flux at the location of square loop is

$$B = \frac{\mu_0 I R^2}{2(R^2 + Z^2)^{3/2}}$$

Substituting the value of $Z\ (= \sqrt{3}R)$, we have

$$B = \frac{\mu_0 I}{16R}$$

Now, total flux through the square loop is

$$\phi_T = NBS \cos\theta$$

$$= (2)\left(\frac{\mu_0 T}{16R}\right) a^2 \cos 45°$$

Mutual inductance

$$M = \frac{\phi_T}{I} = \frac{\mu_0 a^2}{2^{7/2} R}$$

$\therefore \quad p = 7$

19. Volume of cylinder per unit length $(l = 1)$ is

$$V = \pi R^2 l = (\pi R^2)$$

$\therefore$ Charge per unit length,

$\lambda =$ (Volume per unit length) $\times$ (Volume charge density)

$= (\pi R^2 \rho)$

Now at *P*

$$E_R = E_T - E_C$$

$R =$ Remaining portion

$T =$ Total portion and

$C =$ cavity

$$\therefore \quad E_R = \frac{\lambda}{2\pi\varepsilon_0(2R)} - \frac{1}{4\pi\varepsilon_0}\frac{Q}{(2R)^2}$$

$Q =$ charge on sphere

$$= \frac{4}{3}\pi\left(\frac{R}{2}\right)^3 \rho = \frac{\pi R^3 \rho}{6}$$

Substituting the values, we have

$$E_R = \frac{(\pi R^2 \rho)}{4\pi\varepsilon_0 R} - \frac{1}{4\pi\varepsilon_0} \cdot \frac{(\pi R^3 \rho/6)}{4R^2}$$

$$= \frac{23\rho R}{96\varepsilon_0} = \frac{23\rho R}{(16)(6)\,\varepsilon_0}$$

$\therefore \quad k = 6$

20. $T =$ Total portion

$R =$ Remaining portion and

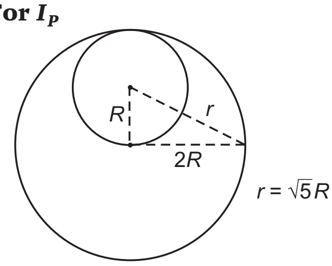
$C =$ Cavity and

let $\sigma =$ mass per unit area.

Then, $\quad m_T = \pi(2R)^2\sigma = 4\pi R^2 \sigma$

$m_C = \pi(R)^2\sigma = \pi R^2 \sigma$

For I_P



$$I_R = I_T - I_C$$
$$= \frac{3}{2} m_T (2R)^2 - \left[\frac{1}{2} m_C R^2 + m_C r^2\right]$$
$$= \frac{3}{2}(4\pi R^2 \sigma)(4R^2) - \left[\frac{1}{2}(\pi R^2\sigma) + (\pi R^2 \sigma)(5R^2)\right]$$
$$= (18.5\,\pi R^4 \sigma)$$

For I_O $I_R = I_T - I_C$
$$= \frac{1}{2} m_T (2R)^2 - \frac{3}{2} m_C R^2$$
$$= \frac{1}{2}(4\pi R^2\sigma)(4R^2) - \frac{3}{2}(\pi R^2 \sigma)(R^2)$$
$$= 6.5\,\pi R^4 \sigma$$
$$\therefore \quad \frac{I_P}{I_O} = \frac{18.5\pi R^4\sigma}{6.5\pi R^4 \sigma} = 2.846$$

Therefore, the nearest integer is 3.

Paper 2

21. Let V_1 = total material volume of shell
V_2 = total inside volume of shell and
x = fraction of V_2 volume filled with water.

In floating condition,

Total weight = Upthrust

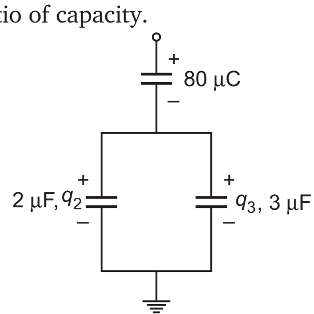
$$\therefore \quad V_1 \rho_c g + (xV_2)(1)g = \left(\frac{V_1 + V_2}{2}\right)(1)g$$

or $$x = 0.5 + (0.5 - \rho_c)\frac{V_1}{V_2}$$

From here we can see that,

$$x > 0.5 \text{ if } \rho_c < 0.5$$

22. Between $3\mu F$ and $2\mu F$ (in parallel) total charge of $80\,\mu C$ will distribute in direct ratio of capacity.



$$\therefore \quad \frac{q_3}{q_2} = \frac{3}{2}$$

or $$q_3 = \left(\frac{3}{3+2}\right)(80) = 48\,\mu C$$

23. The process may be assumed to be isobaric.

$$\therefore \quad Q = n C_p \Delta T = (2)\left(\frac{5}{2}R\right)(5)$$
$$= 5 \times 8.31 \times 5$$
$$= 207.75 \text{ J} \approx 208 \text{ J}$$

24. Language of question is not very clear. For example, disc is rotating. Its different points have different velocities. Relative velocity of pebble with respect to which point, it is not clear. Further, actual initial positions of P and Q are also not given.

25. With end correction,

$$f = n\left[\frac{v}{4(l+e)}\right], \quad \text{where } n = 1, 3, \ldots$$
$$= n\left[\frac{v}{4(l + 0.6\,r)}\right]$$

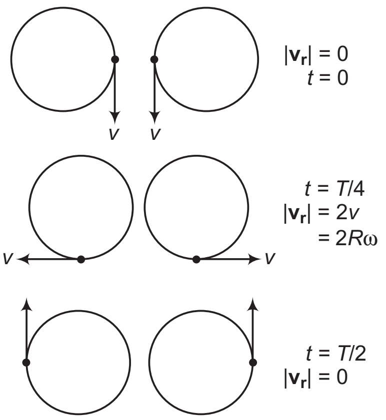
because, $e = 0.6\,r$, where r is radius of pipe.

For first resonance, $n = 1$

$$\therefore \quad f = \frac{v}{4(l + 0.6\,r)}$$

or $$l = \frac{v}{4f} - 0.6r$$
$$= \left[\left(\frac{336 \times 100}{4 \times 512}\right) - 0.6 \times 2\right]\text{cm}$$
$$= 15.2 \text{ cm}$$

26. Language of question is wrong because relative speed is not the correct word. Relative speed between two is always zero. The correct word is magnitude of relative velocity.



$$v = R\omega$$

Corresponding to above values, the correct graph is (a).

27. Area of the given loop is

A = (area of two circles of radius $\frac{a}{2}$ and area of a square of side a)

$$= 2\pi\left(\frac{a}{2}\right)^2 + a^2$$

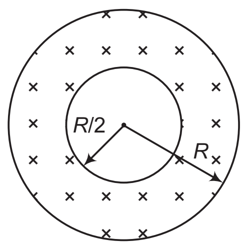
$$= \left(\frac{\pi}{2} + 1\right) a^2$$

$$|\mathbf{M}| = IA = \left(\frac{\pi}{2} + 1\right) a^2 I$$

From screw law direction of **M** is outwards or in positive z-direction.

$$\therefore \quad \mathbf{M} = \left(\frac{\pi}{2} + 1\right) a^2 I\,\hat{\mathbf{k}}$$

28.



r = distance of a point from centre.

For $r \le R/2$ Using Ampere's circuital law,

$$\oint \mathbf{B} \cdot d\mathbf{l} \quad \text{or} \quad Bl = \mu_0 (I_{in})$$

or $\quad B(2\pi r) = \mu_0 (I_{in})$

or $\quad B = \frac{\mu_0}{2\pi} \frac{I_{in}}{r} \qquad \ldots$(i)

Since $I_{in} = 0 \quad \therefore \quad B = 0$

For $\frac{R}{2} \le r \le R$

$$I_{in} = \left[\pi r^2 - \pi\left(\frac{R}{2}\right)^2\right]\sigma$$

Here σ = current per unit area.

Substituting in Eq. (i), we have

$$B = \frac{\mu_0}{2\pi} \frac{\left[\pi r^2 - \pi \frac{R^2}{4}\right]\sigma}{r}$$

$$= \frac{\mu_0 \sigma}{2r}\left(r^2 - \frac{R^2}{4}\right)$$

At $r = \frac{R}{2}$, $B = 0$

At $r = R$, $B = \frac{3\mu_0 \sigma R}{8}$

For $r \ge R$

$$I_{in} = I_{Total} = I \text{ (say)}$$

Therefore, substituting in Eq. (i), we have

$$B = \frac{\mu_0}{2\pi} \cdot \frac{I}{r} \quad \text{or} \quad B \propto \frac{1}{r}$$

29. Since value of n in meta-material is negative.

$$\therefore \qquad v = \frac{c}{|n|}$$

30. According to the paragraph, refracted ray in meta-material should be on same side of normal.

31-32. (i) Every particle of the disc is rotating in a horizontal circle.

(ii) Actual velocity of any particle is horizontal.

(iii) Magnitude of velocity of any particle is

$$v = r\omega$$

where, r is the perpendicular distance of that particle from actual axis of rotation (z-axis).

(iv) When it is broken into two parts then actual velocity of any particle is resultant of two velocities

$$v_1 = r_1\omega_1 \text{ and } v_2 = r_2\omega_2$$

Here,

r_1 = perpendicular distance of centre of mass from z-axis.

ω_1 = angular speed of rotation of centre of mass from z-axis.

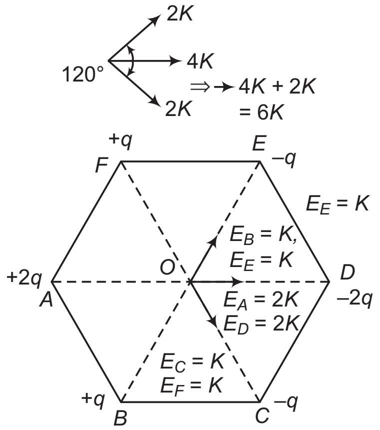
r_2 = distance of particle from centre of mass and

ω_2 = angular speed of rotation of the disc about the axis passing through centre of mass.

(v) Net v will be horizontal, if v_1 and v_2 both are horizontal. Further, v_1 is already horizontal, because centre of mass is rotating about a vertical z-axis. To make v_2 also horizontal, second axis should also be vertical.

33-34. Maximum kinetic energy of anti-neutrino is nearly (0.8×10^6) eV.

35. (a)



Resultant of $2K$ and $2K$ (at 120°) is also $2K$ towards $4K$. Therefore, net electric field is $6K$.

(b) $V_0 = \dfrac{1}{4\pi\varepsilon_0}$

$$\left[\frac{q_A}{L} + \frac{q_B}{L} + \frac{q_C}{L} + \frac{q_D}{L} + \frac{q_E}{L} + \frac{q_F}{L}\right]$$

$$= \frac{1}{4\pi\varepsilon_0 L}(q_A + \ldots + q_F)$$

$$= 0$$

Because $q_A + q_B + q_C + q_D + q_E + q_F = 0$

(c) Only line PR, potential is same (= 0).

36. $I_P > I_Q$

In case of pure rolling,

$$a = \frac{g \sin\theta}{1 + I/mR^2}$$

$a_Q > a_P$ as its moment of inertia is less. Therefore, Q reaches first with more linear speed and more translational kinetic energy.

Further, $\omega = \dfrac{v}{R}$

or $\omega \propto v$

$\therefore$ $\omega_Q > \omega_P$ as $v_P > v_Q$

37. Surface area of Q is four times. Therefore, radius of Q is two times. Volume is eight times. Therefore, mass of Q is also eight times.

So, let $M_P = M$ and $R_P = r$

Then, $M_Q = 8$ M and $R_Q = 2r$

Now, mass of R is $(M_P + M_Q)$ or 9 M. Therefore, radius of R is $(9)^{1/3}r$. Now, escape velocity from the surface of a planet is given by

$$v = \sqrt{\frac{2GM}{r}}$$

(r = radius of that planet)

$$\therefore \quad v_P = \sqrt{\frac{2GM}{r}}$$

$$v_Q = \sqrt{\frac{2G(8M)}{(2r)}}$$

$$v_R = \sqrt{\frac{2G(9M)}{(9)^{1/3}r}}$$

From here we can see that,

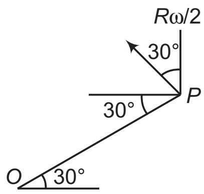
$$\frac{v_P}{v_Q} = \frac{1}{2}$$

and $v_R > v_Q > v_P$

38. Velocity of point O is

$$v_0 = (3R\omega)\,\hat{\mathbf{i}}$$

$\mathbf{v}_{PO}$ is $\frac{R\cdot\omega}{2}$ in the direction shown in figure. In vector form



$$\mathbf{v}_{PO} = -\frac{R\omega}{2}\sin 30°\,\hat{\mathbf{i}} + \frac{R\omega}{2}\cos 30°\,\hat{\mathbf{k}}$$

$$= -\frac{R\omega}{4}\,\hat{\mathbf{i}} + \frac{\sqrt{3}R\omega}{4}\,\hat{\mathbf{k}}$$

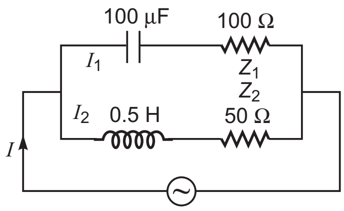
But $\mathbf{v}_{PO} = \mathbf{v}_P - \mathbf{v}_O$

$$\therefore \quad \mathbf{v}_P = \mathbf{v}_{PO} + \mathbf{v}_O$$

$$= \left(-\frac{R\omega}{4}\,\hat{\mathbf{i}} + \frac{\sqrt{3}R\omega}{4}\,\hat{\mathbf{k}}\right) + 3R\omega\,\hat{\mathbf{i}}$$

$$= \frac{11}{4}R\omega\,\hat{\mathbf{i}} + \frac{\sqrt{3}}{4}R\omega\,\hat{\mathbf{k}}$$

39.



Circuit 1

$$X_C = \frac{1}{\omega C} = 100\ \Omega$$

$\therefore$

$$Z_1 = \sqrt{(100)^2 + (100)^2} = 100\sqrt{2}\ \Omega$$

$$\phi_1 = \cos^{-1}\left(\frac{R_1}{Z_1}\right) = 45°$$

In this circuit current leads the voltage.

$$I_1 = \frac{V}{Z_1} = \frac{20}{100\sqrt{2}} = \frac{1}{5\sqrt{2}}\ \text{A}$$

$$V_{100\,\Omega} = (100)\,I_1 = (100)\frac{1}{5\sqrt{2}}\ \text{V}$$

$$= 10\sqrt{2}\ \text{V}$$

Circuit 2

$$X_L = \omega L = (100)(0.5) = 50\ \Omega$$

$$Z_2 = \sqrt{(50)^2 + (50)^2} = 50\sqrt{2}\ \Omega$$

$$\phi_2 = \cos^{-1}\left(\frac{R_2}{Z_2}\right) = 45°$$

In this circuit voltage leads the current.

$$I_2 = \frac{V}{Z_2} = \frac{20}{50\sqrt{2}} = \frac{\sqrt{2}}{5}\ \text{A}$$

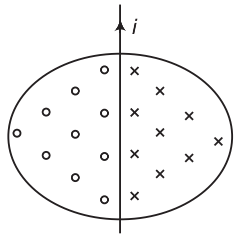
$$V_{50\,\Omega} = (50)\,I_2 = 50\left(\frac{\sqrt{2}}{5}\right) = 10\sqrt{2}\ \text{V}$$

Further, I_1 and I_2 have a mutual phase difference of 90°.

$$\therefore I = \sqrt{I_1^2 + I_2^2} = \sqrt{\frac{1}{50} + \frac{4}{50}} = \frac{1}{\sqrt{10}}\ \text{A}$$

$$\approx 0.3\ \text{A}$$

40.



Due to the current in the straight wire, net magnetic flux from the circular loop is zero. Because in half of the circle magnetic field is inwards and in other half, magnetic field is outwards. Therefore, change in current will not cause any change in magnetic flux from the loop. Therefore, induced emf under all conditions through the circular loop is zero.