JEE Main

Solved Paper 2013

M.Marks 360 **Time 3 Hrs**

Instructions

- This test consists of 90 questions.
- There are three parts in the question paper A, B, C consisting of Physics, Chemistry and Mathematics having 30 questions in each part of equal weightage. Each question is allotted 4 marks for correct response.
- Candidates will be awarded marks as stated above for correct response of each question 1/4 marks will be deducted for indicating incorrect response of each question. No deduction from the total score will be made if no response is indicated for an item in the answer sheet.
- There is only one correct response for each question. Filling up more than one response in any question will be treated as wrong response and marks for wrong response will be deducted according as per instructions.

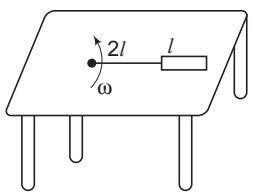
Physics

1. A uniform cylinder of length L and mass M having cross-sectional area A is suspended, with its length vertical from a fixed point by a massless spring such that it is half submerged in a liquid of density σ at equilibrium position. The extension x_0 of the spring when it is in equilibrium is

(a) $\frac{Mg}{k}$ (b) $\frac{Mg}{k}\left(1-\frac{LA\sigma}{M}\right)$

(c) $\frac{Mg}{k}\left(1-\frac{LA\sigma}{2M}\right)$ (d) $\frac{Mg}{k}\left(1+\frac{LA\sigma}{M}\right)$

2. A metallic rod of length l is tied to a string of length 2l and made to rotate with angular speed ω on a horizontal table with one end of the string fixed. If there is a vertical magnetic field B in the region, the emf induced across the ends of the rod is



(a) $\frac{2B\omega l^3}{2}$ (b) $\frac{3B\omega l^3}{2}$

(c) $\frac{4B\omega l^2}{2}$ (d) $\frac{5B\omega l^2}{2}$

3. This question has statement I and statement II. Of the four choices given after the statements, choose the one that best describes the two statements.

Statement I A point particle of mass m moving with speed v collides with stationary point particle of mass M. If the maximum energy loss possible is given as $f\left(\frac{1}{2}mv^2\right)$, then $f = \left(\frac{m}{M+m}\right)$.

Statement II Maximum energy loss occurs when the particles get stuck together as a result of the collision.

(a) Statement I is true, Statement II is true, and Statement II is the correct explanation of Statement I

(b) Statement I is true, Statement II is true, but Statement II is not the correct explanation of Statement I

(c) Statement I is true, Statement II is false

(d) Statement I is false, Statement II is true

4. Let $[\varepsilon_0]$ denote the dimensional formula of the permittivity of vacuum. If M = mass, L = length, T = Time and A = electric current, then

(a) $[\varepsilon_0] = [M^{-1} L^{-3} T^2 A]$

(b) $[\varepsilon_0] = [M^{-1} L^{-3} T^4 A^2]$

(c) $[\varepsilon_0] = [M^{-2} L^2 T^{-1} A^{-2}]$

(d) $[\varepsilon_0] = [M^{-1} L^2 T^{-1} A^2]$

5. A projectile is given an initial velocity of $(\hat{i} + 2\hat{j})$ m/s, where $\hat{i}$ is along the ground and $\hat{j}$ is along the vertical. If $g = 10\ m/s^2$, the equation of its trajectory is

(a) $y = x - 5x^2$ (b) $y = 2x - 5x^2$

(c) $4y = 2x - 5x^2$ (d) $4y = 2x - 25x^2$

6. The amplitude of a damped oscillator decreases to 0.9 times its original magnitude is 5 s. In another 10 s it will decrease to α times its original magnitude, where α equals

(a) 0.7 (b) 0.81

(c) 0.729 (d) 0.6

7. Two capacitors C_1 and C_2 are charged to 120 V and 200 V respectively. It is found that by connecting them together the potential on each one can be made zero. Then

(a) $5C_1 = 3C_2$ (b) $3C_1 = 5C_2$

(c) $3C_1 + 5C_2 = 0$ (d) $9C_1 = 4C_2$

8. A sonometer wire of length 1.5 m is made of steel. The tension in it produces an elastic strain of 1%. What is the fundamental frequency of steel if density and elasticity of steel are 7.7×10^3 kg/m^3 and 2.2×10^{11} N/m^2 respectively?

(a) 188.5 Hz (b) 178.2 Hz

(c) 200.5 Hz (d) 770 Hz

9. A circular loop of radius 0.3 cm lies parallel to a much bigger circular loop of radius 20 cm. The centre of the smaller loop is on the axis of the bigger loop. The distance between their centres is 15 cm. If a current of 2.0 A flows through the bigger loop, then the flux linked with smaller loop is

(a) 9.1×10^{-11} Wb (b) 6×10^{-11} Wb

(c) 3.3×10^{-11} Wb (d) 6.6×10^{-9} Wb

10. Diameter of a plano-convex lens is 6 cm and thickness at the centre is 3 mm. If speed of light in material of lens is 2×10^8 m/s, the focal length of the lens is

(a) 15 cm (b) 20 cm

(c) 30 cm (d) 10 cm

11. What is the minimum energy required to launch a satellite of mass *m* from the surface of a planet of mass M and radius R in a circular orbit at an altitude of 2R?

(a) $\frac{5GmM}{6R}$ (b) $\frac{2GmM}{3R}$

(c) $\frac{GmM}{2R}$ (d) $\frac{GmM}{3R}$

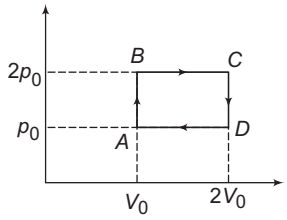
12. A diode detector is used to detect an amplitude modulated wave of 60% modulation by using a condenser of capacity 250 pico farad in parallel with a load resistance 100 kΩ. Find the maximum modulated frequency which could be detected by it.

(a) 10.62 MHz (b) 10.62 kHz
(c) 5.31 MHz (d) 5.31 kHz

13. A beam of unpolarized light of intensity I_0 is passed through a polaroid A and then through another polaroid B which is oriented so that its principal plane makes an angle of 45° relative to that of A. The intensity of the emergent light is
(a) I_0 (b) $I_0/2$
(c) $I_0/4$ (d) $I_0/8$

14. The supply voltage in a room is 120 V. The resistance of the lead wires is 6 Ω. A 60 W bulb is already switched on. What is the decrease of voltage across the bulb, when a 240 W heater is switched on in parallel to the bulb?
(a) zero (b) 2.9 V
(c) 13.3 V (d) 10.04 V

15. The shown p-*V* diagram represents the thermodynamic cycle of an engine, operating with an ideal monoatomic gas. The amount of heat, extracted from the source in a single cycle is

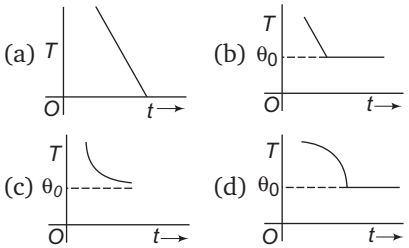


(a) p_0V_0 (b) $\left(\frac{13}{2}\right)p_0V_0$
(c) $\left(\frac{11}{2}\right)p_0V_0$ (d) $4p_0V_0$

16. A hoop of radius r and mass *m* rotating with an angular velocity ω_0 is placed on a rough horizontal surface. The initial velocity of the centre of the hoop is zero. What will be the velocity of the centre of the hoop when it ceases to slip?
(a) $\frac{r\omega_0}{4}$ (b) $\frac{r\omega_0}{3}$
(c) $\frac{r\omega_0}{2}$ (d) $r\omega_0$

17. An ideal gas enclosed in a vertical cylindrical container supports a freely moving piston of mass M. The piston and the cylinder have equal cross-sectional area A. When the piston is in equilibrium, the volume of the gas is V_0 and its pressure is p_0. The piston is slightly displaced from the equilibrium position and released. Assuming that the system is completely isolated from its surrounding, the piston executes a simple harmonic motion with frequency
(a) $\frac{1}{2\pi}\frac{A\gamma p_0}{V_0M}$ (b) $\frac{1}{2\pi}\frac{V_0Mp_0}{A^2\gamma}$
(c) $\frac{1}{2\pi}\sqrt{\frac{A^2\gamma p_0}{MV_0}}$ (d) $\frac{1}{2\pi}\sqrt{\frac{MV_0}{A\gamma p_0}}$

18. If a piece of metal is heated to temperature θ and then allowed to cool in a room which is at temperature θ_0. The graph between the temperature T of the metal and time t will be closed to



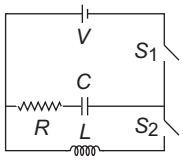
19. This question has Statement I and Statement II. Of the four choices given after the statements, choose the one that best describes the two statements.

Statement I Higher the range, greater is the resistance of ammeter.

Statement II To increase the range of ammeter, additional shunt needs to be used across it.

(a) Statement I is true, Statement II is true and Statement II is the correct explanation of Statement I
(b) Statement I is true, Statement II is true, but Statement II is not the correct explanation of Statement I
(c) Statement I is true, Statement II is false
(d) Statement I is false, Statement II is true

20. In a *L-C-R* circuit as shown below both switches are open initially. Now switch S_1 and S_2, are closed.



(q is charge on the capacitor and $\tau = RC$ is capacitance time constant). Which of the following statement is correct?

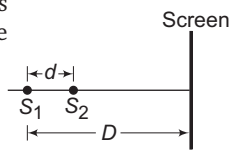
(a) Work done by the battery is half of the energy dissipated in the resistor

(b) At $t = \tau$, $q = CV/2$

(c) At $t = 2\tau$, $q = CV\ (1 - e^{-2})$

(d) At $t = \frac{\tau}{2}$, $q = CV\ (1 - e^{-1})$

21. Two coherent point sources S_1 and S_2 are separated by a small distance d as shown. The fringes obtained on the screen will be



(a) points

(b) straight lines

(c) semi-circle

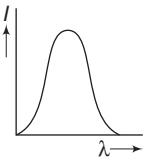
(d) concentric circles

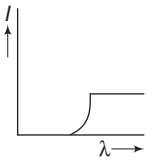
22. The magnetic field in a travelling electromagnetic wave has a peak value of 20 nT. The peak value of electric field strength is

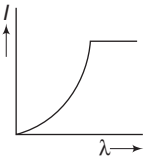
(a) 3 V/m (b) 6 V/m

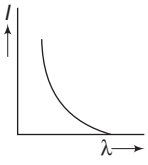
(c) 9 V/m (d) 12 V/m

23. The anode voltage of a photocell is kept fixed. The wavelength λ of the light falling on the cathode is gradually changed. The plate current I of photocell varies as follows

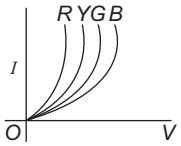
(a) 

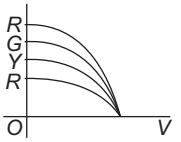
(b) 

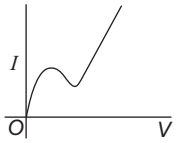
(c) 

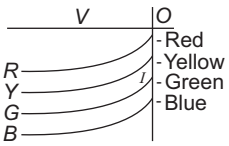
(d) 

24. The *I-V* characteristic of an LED is

(a) 

(b) 

(c) 

(d) 

25. Assume that a drop of liquid evaporates by decrease in its surface energy, so that its temperature remains unchanged. What should be the minimum radius of the drop for this to be possible? The surface tension is T, denstiy of liquid is ρ and L is its latent heat of vaporization

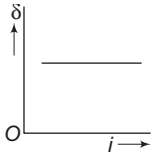
(a) $\rho L/T$ (b) $\sqrt{T/\rho L}$

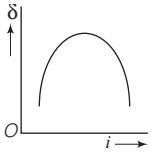
(c) $T/\rho L$ (d) $2T/\rho L$

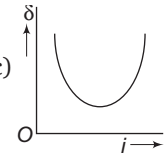
26. In a hydrogen like atom electron makes transition from an energy level with quantum number n to another with quantum number $(n - 1)$. If $n >> 1$, the frequency of radiation emitted is proportional to

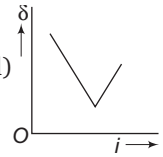
(a) $\frac{1}{n}$ (b) $\frac{1}{n^2}$ (c) $\frac{1}{n^4}$ (d) $\frac{1}{n^3}$

27. The graph between angle of deviation (δ) and angle of incidence (i) for a triangular prism is represented by

(a) 

(b) 

(c) 

(d) 

28. Two charges, each equal to q, are kept at $x = -a$ and $x = a$ on the x-axis. A particle of mass m and charge $q_0 = \frac{q}{2}$ is placed at the origin. If charge q_0 is given a small displacement y ($y << a$) along the y-axis, the net force acting on the particle is proportional to

(a) y (b) $-y$ (c) $\frac{1}{y}$ (d) $-\frac{1}{y}$

29. Two short bar magnets of length 1 cm each have magnetic moments 1.20 Am^2 and 1.00 Am^2 respectively. They are placed on a hoirzontal table parallel to each other with their N poles pointing towards the south. They have a common magnetic equator and are separated by a distance of 20.0 cm. The value of the resultant horizontal magnetic induction at the mid-point O of the line joining their centres is close to (Horizontal component of the earth's magnetic induction is 3.6×10^{-5} Wb/m^2)

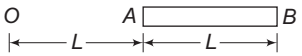
(a) 3.6×10^{-5} Wb/m^2

(b) 2.56×10^{-4} Wb/m^2

(c) 3.50×10^{-4} Wb/m^2

(d) 5.80×10^{-4} Wb/m^2

30. A charge Q is uniformly distributed over a long rod AB of length L as shown in the figure. The electric potential at the point O lying at distance L from the end A is



(a) $\frac{Q}{8\pi\,\varepsilon_0 L}$ (b) $\frac{3Q}{4\pi\,\varepsilon_0 L}$

(c) $\frac{Q}{4\pi\,\varepsilon_0 L \ln 2}$ (d) $\frac{Q \ln 2}{4\pi\,\varepsilon_0 L}$

Chemistry

1. Which of the following complex species is not expected to exhibit optical isomerism?

(a) $[Co(en)_3]^{3+}$

(b) $[Co(en)_2 Cl_2]^+$

(c) $[Co(NH_3)_3Cl_3]$

(d) $[Co(en)(NH_3)Cl_2]^+$

2. Which one of the following molecules is expected to exhibit diamagnetic behaviour?

(a) C_2 (b) N_2

(c) O_2 (d) S_2

3. A solution of (– l)

1-chloro-1-phenylethane in toluene racemises slowly in the presence of a small amount of $SbCl_5$, due to the formation of

(a) carbanion (b) carbene

(c) carbocation (d) free radical

4. Given, $E^\circ_{Cr^{3+}/Cr} = -0.74$ V;

$E^\circ_{MnO_4^-/Mn^{2+}} = 1.51$ V

$E^\circ_{Cr_2O_7^{2-}/Cr^{3+}} = 1.33$ V;

$E^\circ_{Cl/Cl^-} = 1.36$ V

Based on the data given above strongest oxidising agent will be

(a) Cl (b) Cr^{3+}

(c) Mn^{2+} (d) MnO_4^-

5. A piston filled with 0.04 mole of an ideal gas expands reversibly from 50.0 mL to 375 mL at a constant temperature of 37.0°C. As it does so, it absorbs 208 J of heat. The values of q and *W* for the process will be (R = 8.314 J/mol K, ln 7.5 = 2.01)

(a) q = + 208 J, W = – 208 J

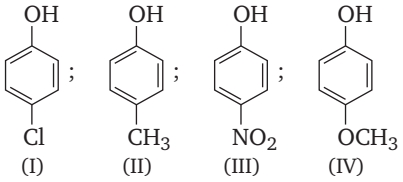
(b) q = – 208 J, W = – 208 J

(c) q = – 208 J, W = + 208 J

(d) q = + 208 J, W = + 208 J

6. The molarity of a solution obtained by mixing 750 mL of 0.5 (M) HCl with 250 mL of 2(M) HCl will be
(a) 0.875 M (b) 1.00 M
(c) 1.75 M (d) 0.0975 M

7. Arrange the following compounds in the order of decreasing acidity



(a) II > IV > I > III (b) I > II > III > IV
(c) III > I > II > IV (d) IV > III > I > II

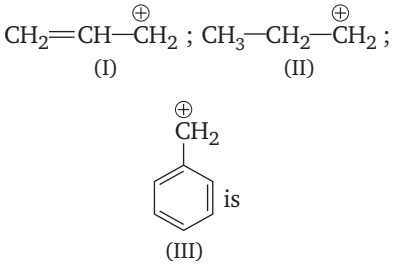
8. For gaseous state, if most probable speed is denoted by C *, average speed by $\overline{C}$ and root square speed by C, then for a large number of molecules, the ratios of these speeds are
(a) C * : $\overline{C}$: C = 1.225 : 1.128 : 1
(b) C * : $\overline{C}$: C = 1.128 : 1.225 : 1
(c) C * : $\overline{C}$: C = 1 : 1.128 : 1.225
(d) C * : $\overline{C}$: C = 1 : 1.225 : 1.128

9. The rate of a reaction doubles when its temperature changes from 300 K to 310 K. Activation energy of such a reaction will be
(R = 8.314 JK^{-1} mol^{-1} and
log 2 = 0.301)
(a) 53.6 kJ mol^{-1} (b) 48.6 kJ mol^{-1}
(c) 58.5 kJ mol^{-1} (d) 60.5 kJ mol^{-1}

10. A compound with molecular mass 180 is acylated with CH_3COCl to get a compound with molecular mass 390. The number of amino groups present per molecule of the former compound is
(a) 2 (b) 5
(c) 4 (d) 6

11. Which of the following arrangements does not represent the correct order of the property stated against it?
(a) $V^{2+} < Cr^{2+} < Mn^{2+} < Fe^{2+}$:
paramagnetic behaviour
(b) $Ni^{2+} < Co^{2+} < Fe^{2+} < Mn^{2+}$:
ionic size
(c) $Co^{3+} < Fe^{3+} < Cr^{3+} < Sc^{3+}$:
stability in aqueous solution
(d) Sc < Ti < Cr < Mn :
number of oxidation states

12. The order of stability of the following carbocations



(a) III > II > I (b) II > III > I
(c) I > II > III (d) III > I > II

13. Consider the following reaction,

$$xMnO_4^- + yC_2O_4^{2-} + zH^+ \longrightarrow xMn^{2+} + 2yCO_2 + \frac{z}{2}H_2O$$

The values of x, y and z in the reaction are, respectively
(a) 5, 2 and 16 (b) 2, 5 and 8
(c) 2, 5 and 16 (d) 5, 2 and 8

14. Which of the following is the wrong statement?
(a) ONCl and ONO^- are not isoelectronic
(b) O_3 molecule is bent
(c) Ozone is violet-black in solid state
(d) Ozone is diamagnetic gas

15. A gaseous hydrocarbon gives upon combustion 0.72 g of water and 3.08 g of CO_2. The empirical formula of the hydrocarbon is
(a) C_2H_4 (b) C_3H_4 (c) C_6H_5 (d) C_7H_8

16. In which of the following pairs of molecules/ions both the species are not likely to exist?
(a) H_2^+, He_2^{2-} (b) H_2^-, He_2^{2-}
(c) H_2^{2+}, He_2 (d) H_2^-, He_2^{2+}

17. Which of the following exists as covalent crystals in the solid state?
(a) Iodine (b) Silicon
(c) Sulphur (d) Phosphorus

18. Synthesis of each molecule of glucose in photosynthesis involves
(a) 18 molecules of ATP
(b) 10 molecules of ATP
(c) 8 molecules of ATP
(d) 6 molecules of ATP

19. The coagulating power of electrolytes having ions Na^+, Al^{3+} and Ba^{2+} for arsenic sulphide sol increases in the order
(a) $Al^{3+} < Ba^{2+} < Na^+$
(b) $Na^+ < Ba^{2+} < Al^{3+}$
(c) $Ba^{2+} < Na^{2+} < Al^{3+}$
(d) $Al^{3+} < Na^+ < Ba^{2+}$

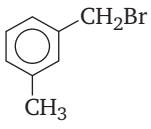
20. Which of the following represents the correct order of increasing first ionization enthalpy for Ca, Ba, S, Se and Ar?
(a) Ca < S < Ba < Se < Ar
(b) S < Se < Ca < Ba < Ar
(c) Ba < Ca < Se < S < Ar
(d) Ca < Ba < S < Se < Ar

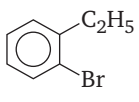
21. Energy of an electron is given by

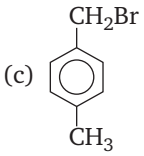
$$E = -2.178 \times 10^{-18}\ J\left(\frac{Z^2}{n^2}\right)$$

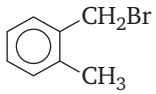
Wavelength of light required to excite an electron in an hydrogen atom from level n = 1 to n = 2 will be
(h = 6.62×10^{-34} Js and
c = 3.0×10^8 ms^{-1})
(a) 1.214×10^{-7} m
(b) 2.816×10^{-7} m
(c) 6.500×10^{-7} m
(d) 8.500×10^{-7} m

22. Compound (A), C_8H_9Br gives a white precipitate when warmed with alcoholic $AgNO_3$. Oxidation of (*A*) gives an acid (*B*), $C_8H_6O_4$. (*B*) easily forms anyhydride on heating. Identify the compound (*A*).

(a) 

(b) 

(c) 

(d) 

23. Four successive members of the first row transition elements listed below with atomic numbers. Which one of them is expected to have the highest $E^{\circ}_{M^{3+}/M^{2+}}$ value?
(a) Cr (Z = 24) (b) Mn (Z = 25)
(c) Fe (Z = 26) (d) Co (Z = 27)

24. How many litres of water must be added to 1 L of an aqueous solution of HCl with a pH of 1 to create an aqueous solution with pH of 2 ?
(a) 0.1 L (b) 0.9 L (c) 2.0 L (d) 9.0 L

25. The first ionisation potential of Na is 5.1 eV. The value of electron gain enthalpy of Na^+ will be
(a) – 2.55 eV (b) – 5.1 eV
(c) – 10.2 eV (d) + 2.55 eV

26. An organic compound A upon reacting with NH_3 gives B. On heating, B gives C. C in the presence of KOH reacts with Br_2 to give $CH_3CH_2NH_2$. *A* is
(a) CH_3COOH
(b) $CH_3CH_2CH_2COOH$
(c) $CH_3—\underset{\displaystyle CH_3}{\underset{|}{C}}H—COOH$
(d) CH_3CH_2COOH

27. Stability of the species Li_2, Li_2^- and Li_2^+ increases in the order of
(a) $Li_2 < Li_2^+ < Li_2^-$ (b) $Li_2^- < Li_2^+ < Li_2$
(c) $Li_2 < Li_2^- < Li_2^+$ (d) $Li_2^- < Li_2 < Li_2^+$

28. An unknown alcohol is treated with the "Lucas reagent" to determine whether the alcohol is primary, secondary or tertiary. Which alcohol reacts fastest and by what mechanism?
(a) Secondary alcohol by S_N1
(b) Tertiary alcohol by S_N1
(c) Secondary alcohol by S_N2
(d) Tertiary alcohol by S_N2

29. The gas leaked from a storage tank of the Union Carbide plant in Bhopal gas tragedy was
(a) Methyl isocyanate
(b) Methylamine
(c) Ammonia
(d) Phosgene

30. Experimentally it was found that a metal oxide has formula $M_{0.98}O$. Metal *M*, present as M^{2+} and M^{3+} in its oxide. Fraction of the metal which exists as M^{3+} would be
(a) 7.01% (b) 4.08%
(c) 6.05% (d) 5.08%

Mathematics

1. Distance between two parallel planes $2x + y + 2z = 8$ and $4x + 2y + 4z + 5 = 0$ is
(a) $\frac{3}{2}$ (b) $\frac{5}{2}$ (c) $\frac{7}{2}$ (d) $\frac{9}{2}$

2. At present, a firm is manufacturing 2000 items. It is estimated that the rate of change of production P with respect to additional number of workers x is given by $\frac{dP}{dx} = 100 - 12\sqrt{x}$. If the firm employees 25 more workers, then the new level of production of items is
(a) 2500 (b) 3000 (c) 3500 (d) 4500

3. Let A and B two sets containing 2 elements and 4 elements respectively. The number of subsets of $A \times B$ having 3 or more elements is
(a) 256 (b) 220 (c) 219 (d) 211

4. If the lines
$\frac{x-2}{1} = \frac{y-3}{1} = \frac{z-4}{-k}$ and
$\frac{x-1}{k} = \frac{y-4}{2} = \frac{z-5}{1}$
are coplanar, then k can have
(a) any value
(b) exactly one value
(c) exactly two values
(d) exactly three values

5. If the vectors $AB = 3\hat{i} + 4\hat{k}$ and $AC = 5\hat{i} - 2\hat{j} + 4\hat{k}$ are the sides of a ΔABC, then the length of the median through A is
(a) $\sqrt{18}$ (b) $\sqrt{72}$
(c) $\sqrt{33}$ (d) $\sqrt{45}$

6. The real number k for which the equation, $2x^3 + 3x + k = 0$ has two distinct real roots in [0, 1]
(a) lies between 1 and 2
(b) lies between 2 and 3
(c) lies between – 1 and 0
(d) does not exist

7. The sum of first 20 terms of the sequence 0.7, 0.77, 0.777,…, is
(a) $\frac{7}{81}(179 - 10^{-20})$
(b) $\frac{7}{9}(99 - 10^{-20})$
(c) $\frac{7}{81}(179 + 10^{-20})$
(d) $\frac{7}{9}(99 + 10^{-20})$

8. A ray of light along $x + \sqrt{3}y = \sqrt{3}$ gets reflected upon reaching x-axis, the equation of the reflected ray is
(a) $y = x + \sqrt{3}$
(b) $\sqrt{3}y = x - \sqrt{3}$
(c) $y = \sqrt{3}x - \sqrt{3}$
(d) $\sqrt{3}y = x - 1$

9. The number of values of k, for which the system of equations
$(k + 1)x + 8y = 4k$
$kx + (k + 3)y = 3k - 1$
has no solution, is
(a) infinite (b) 1
(c) 2 (d) 3

10. If the equations $x^2 + 2x + 3 = 0$ and $ax^2 + bx + c = 0$, $a, b, c \in R$, have a common root, then a : b : c is

(a) 1 : 2 : 3 (b) 3 : 2 : 1

(c) 1 : 3 : 2 (d) 3 : 1 : 2

11. The circle passing through (1, – 2) and touching the axis of x at (3, 0) also passes through the point

(a) (– 5, 2) (b) (2, – 5)

(c) (5, – 2) (d) (– 2, 5)

12. If x, y and z are in AP and $\tan^{-1} x$, $\tan^{-1} y$ and $\tan^{-1} z$ are also in AP, then

(a) x = y = z (b) 2x = 3y = 6z

(c) 6x = 3y = 2z (d) 6x = 4y = 3z

13. Consider

Statement I

$(p \wedge \sim q) \wedge (\sim p \wedge q)$ is a fallacy.

Statement II

$(p \to q) \leftrightarrow (\sim q \to \sim p)$ is a tautology.

(a) Statement I is true; Statement II is true; Statement II is a correct explanation for Statement I.

(b) Statement I is true; Statement II is true; Statement II is not a correct explanation for Statement I.

(c) Statement I is true; Statement II is false.

(d) Statement I is false; Statement II is true.

14. If $\int f(x)\,dx = \psi(x)$, then $\int x^5 f(x^3)dx$ is equal to

(a) $\frac{1}{3}[x^3\psi(x^3) - \int x^2\psi(x^3)dx] + C$

(b) $\frac{1}{3}x^3\psi(x^3) - 3\int x^3\psi(x^3)\,dx + C$

(c) $\frac{1}{3}x^3\psi(x^3) - \int x^2\psi(x^3)\,dx + C$

(d) $\frac{1}{3}[x^3\psi(x^3) - \int x^3\psi(x^3)\,dx] + C$

15. $\lim\limits_{x \to 0} \dfrac{(1 - \cos 2x)(3 + \cos x)}{x \tan 4x}$ is equal to

(a) $-\frac{1}{4}$ (b) $\frac{1}{2}$ (c) 1 (d) 2

16. **Statement I** The value of the integral $\int_{\pi/6}^{\pi/3} \frac{dx}{1 + \sqrt{\tan x}}$ is equal to $\pi/6$.

Statement II

$\int_a^b f(x)\,dx = \int_a^b f(a + b - x)\,dx$

(a) Statement I is true; Statement II is true; Statement II is a correct explanation for Statement I.

(b) Statement I is true; Statement II is true; Statement II is not a correct explanation for Statement I.

(c) Statement I is true; Statement II is false.

(d) Statement I is false; Statement II is true.

17. The equation of the circle passing through the foci of the ellipse $\frac{x^2}{16} + \frac{y^2}{9} = 1$ and having centre at (0, 3) is

(a) $x^2 + y^2 - 6y - 7 = 0$

(b) $x^2 + y^2 - 6y + 7 = 0$

(c) $x^2 + y^2 - 6y - 5 = 0$

(d) $x^2 + y^2 - 6y + 5 = 0$

18. A multiple choice examination has 5 questions. Each question has three alternative answers of which exactly one is correct. The probability that a student will get 4 or more correct answers just by guessing is

(a) $\frac{17}{3^5}$ (b) $\frac{13}{3^5}$

(c) $\frac{11}{3^5}$ (d) $\frac{10}{3^5}$

19. The x-coordinate of the incentre of the triangle that has the coordinates of mid-points of its sides as (0, 1), (1, 1) and (1, 0) is

(a) $2 + \sqrt{2}$

(b) $2 - \sqrt{2}$

(c) $1 + \sqrt{2}$

(d) $1 - \sqrt{2}$

20. The term independent of x in expansion of $\left(\frac{x+1}{x^{2/3}-x^{1/3}+1}-\frac{x-1}{x-x^{1/2}}\right)^{10}$ is

(a) 4 (b) 120

(c) 210 (d) 310

21. The area (in square units) bounded by the curves $y=\sqrt{x}$, $2y-x+3=0$, x-axis and lying in the first quadrant is

(a) 9 (b) 36

(c) 18 (d) $\frac{27}{4}$

22. Let T_n be the number of all possible triangles formed by joining vertices of an n-sided regular polygon. If $T_{n+1}-T_n=10$, then the value of n is

(a) 7 (b) 5

(c) 10 (d) 8

23. If z is a complex number of unit modulus and argument θ, then arg $\left(\frac{1+z}{1+\bar{z}}\right)$ is equal to

(a) $-\theta$ (b) $\frac{\pi}{2}-\theta$

(c) θ (d) $\pi-\theta$

24. ABCD is a trapezium such that AB and CD are parallel and BC ⊥ CD. If ∠ ADB = θ, BC = p and CD = q, then AB is equal to

(a) $\frac{(p^2+q^2)\sin\theta}{p\cos\theta+q\sin\theta}$

(b) $\frac{p^2+q^2\cos\theta}{p\cos\theta+q\sin\theta}$

(c) $\frac{p^2+q^2}{p^2\cos\theta+q^2\sin\theta}$

(d) $\frac{(p^2+q^2)\sin\theta}{(p\cos\theta+q\sin\theta)^2}$

25. If $P=\begin{bmatrix}1 & \alpha & 3\\ 1 & 3 & 3\\ 2 & 4 & 4\end{bmatrix}$ is the adjoint of a 3×3 matrix A and $|A|=4$, then α is equal to

(a) 4 (b) 11

(c) 5 (d) 0

26. The intercepts on x-axis made by tangents to the curve, $y=\int_0^x|t|\,dt$, $x\in R$, which are parallel to the line $y=2x$, are equal to

(a) ± 1 (b) ± 2 (c) ± 3 (d) ± 4

27. **Given** A circle, $2x^2+2y^2=5$ and a parabola, $y^2=4\sqrt{5}x$.

Statement I An equation of a common tangent to these curves is $y=x+\sqrt{5}$.

Statement II If the line, $y=mx+\frac{\sqrt{5}}{m}(m\neq0)$ is the common tangent, then m satisfies $m^4-3m^2+2=0$.

(a) Statement I is true; Statement II is true; Statement II is a correct explanation for Statement I.

(b) Statement I is true; Statement II is true; Statement II is not a correct explanation for Statement I.

(c) Statement I is true; Statement II is false.

(d) Statement I is false; Statement II is true.

28. If $y=\sec(\tan^{-1}x)$, then $\frac{dy}{dx}$ at $x=1$ is equal to

(a) $\frac{1}{\sqrt{2}}$ (b) $\frac{1}{2}$ (c) 1 (d) $\sqrt{2}$

29. The expression $\frac{\tan A}{1-\cot A}+\frac{\cot A}{1-\tan A}$ can be written as

(a) sin A cos A + 1

(b) sec A cosec A + 1

(c) tan A + cot A

(d) sec A + cosec A

30. All the students of a class performed poorly in Mathematics. The teacher decided to give grace marks of 10 to each of the students. Which of the following statistical measures will not change even after the grace marks were given?

(a) Mean (b) Median

(c) Mode (d) Variance

Answers

Physics

1. (c)	**2.** (d)	**3.** (d)	**4.** (b)	**5.** (b)	**6.** (c)	**7.** (b,c)	**8.** (b)	**9.** (a)	**10.** (c)
11. (a)	**12.** (b)	**13.** (c)	**14.** (d)	**15.** (b)	**16.** (c)	**17.** (c)	**18.** (c)	**19.** (d)	**20.** (c)
21. (d)	**22.** (b)	**23.** (d)	**24.** (a)	**25.** (d)	**26.** (d)	**27.** (c)	**28.** (a)	**29.** (b)	**30.** (d)

Chemistry

1. (c)	**2.** (a.b)	**3.** (c)	**4.** (d)	**5.** (a)	**6.** (a)	**7.** (c)	**8.** (c)	**9.** (a)	**10.** (b)
11. (a)	**12.** (d)	**13.** (c)	**14.** (*)	**15.** (d)	**16.** (c)	**17.** (b)	**18.** (a)	**19.** (b)	**20.** (c)
21. (a)	**22.** (d)	**23.** (d)	**24.** (d)	**25.** (b)	**26.** (d)	**27.** (b)	**28.** (b)	**29.** (a)	**30.** (b)

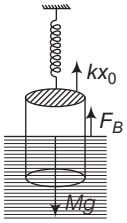
Mathematics

1. (c)	**2.** (c)	**3.** (c)	**4.** (c)	**5.** (c)	**6.** (d)	**7.** (c)	**8.** (b)	**9.** (b)	**10.** (a)
11. (c)	**12.** (a)	**13.** (b)	**14.** (c)	**15.** (d)	**16.** (d)	**17.** (a)	**18.** (c)	**19.** (b)	**20.** (c)
21. (a)	**22.** (b)	**23.** (c)	**24.** (a)	**25.** (b)	**26.** (a)	**27.** (b)	**28.** (a)	**29.** (b)	**30.** (d)

Hints & Solutions

Physics

1. In equilibrium,



Upward force = Downward force

$$kx_0 + F_B = mg$$

Here, kx_0 is restoring force of spring and F_B is buoyancy force.

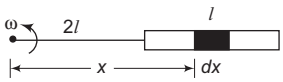
$$kx_0 + \sigma\frac{L}{2}Ag = Mg$$

$$x_0 = \frac{Mg - \frac{\sigma LAg}{2}}{k}$$

$$= \frac{Mg}{k}\left(1 - \frac{\sigma LA}{2M}\right)$$

2. $e = \int_{2l}^{3l} (\omega x)\, B dx = B\omega \frac{[(3l)^2 - (2l)^2]}{2}$

$$= \frac{5Bl^2\omega}{2}$$



3. Maximum energy loss

$$= \frac{p^2}{2m} - \frac{p^2}{2(m+M)} \qquad \left(\because KE = \frac{p^2}{2m}\right)$$

Before collision the mass *m* and after collision the mass is m + M

$$= \frac{p^2}{2m}\left[\frac{M}{(m+M)}\right]$$

$$= \frac{1}{2}mv^2\left\{\frac{M}{m+M}\right\} \qquad \left(f = \frac{M}{m+M}\right)$$

4. From Coulomb's law $F = \frac{1}{4\pi\varepsilon_0}\frac{q_1 q_2}{R^2}$

$$\varepsilon_0 = \frac{q_1 q_2}{4\pi F R^2}$$

Substituting the units we have,

$$\varepsilon_0 = \frac{C^2}{N\text{-}m^2} = \frac{[AT]^2}{[MLT^{-2}]\,[L^2]}$$

$$= [M^{-1}L^{-3}T^4A^2]$$

5. Initial velocity $= (\mathbf{i} + 2\mathbf{j})$ m/s

Magnitude of initial velocity $u = \sqrt{(1)^2 + (2^2)} = \sqrt{5}$ m/s

Equation of trajectory of projectile is

$$y = x\tan\theta - \frac{gx^2}{2u^2}(1 + \tan^2\theta)$$

$$\left[\tan\theta = \frac{y}{x} = \frac{2}{1} = 2\right]$$

$$\therefore \quad y = x \times 2 - \frac{10\,(x)^2}{2(\sqrt{5})^2}[1 + (2)^2]$$

$$= 2x - \frac{10\,(x^2)}{2\times 5}(1 + 4)$$

$$= 2x - 5x^2$$

6. Amplitude decreases exponentially. In 5 s it remains 0.9 times. Therefore in total 15 s it will remains (0.9) (0.9) (0.9) = 0.729 times its original value.

7. Polarity should be mentioned in the question. Potential on each of them can be zero if, $q_{net} = 0$

or $\quad q_1 \pm q_2 = 0$

or $\quad 120\,C_1 \pm 200\,C_2 = 0$

or $\quad 3C_1 \pm 5C_2 = 0$

8. Fundamental frequency of sonometer wire

$$f = \frac{v}{2l} = \frac{1}{2l}\sqrt{\frac{T}{\mu}}$$

$$= \frac{1}{2l}\sqrt{\frac{T}{Ad}}$$

Here, μ = mass per unit length of wire.

Also, Young's modulus of elasticity

$$Y = \frac{Tl}{A\Delta l}$$

$$\Rightarrow \quad \frac{T}{A} = \frac{Y\Delta l}{l} \Rightarrow f = \frac{1}{2l}\sqrt{\frac{Y\Delta l}{ld}}$$

$l = 1.5$ m, $d = 7.7 \times 10^3$ kg/m^3

$Y = 2.2 \times 10^{11}$ N/m^2

After substituting the values we get,

$$f \approx 178.2 \text{ Hz}$$

9. Magnetic field at the centre of smaller loop

$$B = \frac{\mu_0 i R_2^2}{2(R_2^2 + x^2)^{3/2}}$$

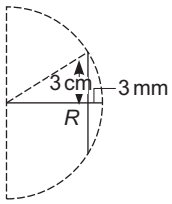
Area of smaller loop $S = \pi R_1^2$

$\therefore$ Flux through smaller loop $\phi = BS$

Substituting the values we get,

$$\phi \approx 9.1 \times 10^{-11} \text{ Wb}$$

10. By Pythagoras theorem



$$R^2 = (3)^2 + (R - 0.3)^2$$

$$\Rightarrow \quad R \approx 15 \text{ cm}$$

Refractive index of material of lens

$$\mu = \frac{c}{v}$$

Here c = speed of light in vacuum

$$= 3 \times 10^8 \text{ m/s}$$

v = speed of light in material of lens

$$= 2 \times 10^8 \text{ m/s}$$

$$= \frac{3\times 10^8}{2\times 10^8} = \frac{3}{2}$$

From lens maker's formula

$$\frac{1}{f} = (\mu - 1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$$

Here, $R_1 = R$ and $R_2 = \infty$ (For plane surface)

$$\frac{1}{f} = \left(\frac{3}{2} - 1\right)\left(\frac{1}{15}\right)$$

$$\Rightarrow \quad f = 30 \text{ cm}$$

11. E = Energy of satellite – energy of mass on the surface of planet

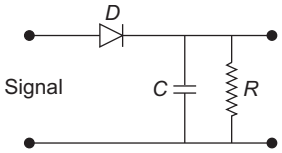
$$= -\frac{GMm}{2r} - \left(-\frac{GMm}{R}\right)$$

Here, r = R + 2R = 3R

Substituting in about equation we get,

$$E = \frac{5GMm}{6R}$$

12.



$$\tau = RC = 100 \times 10^3 \times 250 \times 10^{-12} \text{ s}$$

$$= 2.5 \times 10^{-5} \text{ s}$$

The higher frequency which can be detected with tolerable distortion is

$$f = \frac{1}{2\pi m_a RC}$$

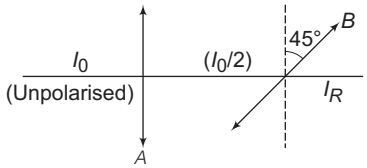
$$= \frac{1}{2\pi \times 0.6 \times 2.5 \times 10^{-5}} \text{ Hz}$$

$$= \frac{100 \times 10^4}{25 \times 1.2\pi} \text{ Hz}$$

$$= \frac{4}{1.2\pi} \times 10^4 \text{ Hz}$$

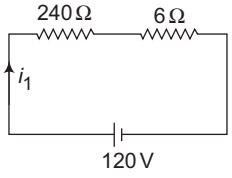
$$= 10.61 \text{ kHz}$$

13. Relation between intensities is



$$I_R = \left(\frac{I_0}{2}\right)\cos^2(45°) = \frac{I_0}{2} \times \frac{1}{2} = \frac{I_0}{4}$$

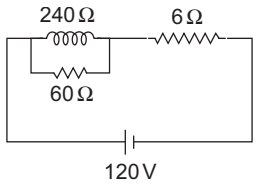
14. $P = \frac{V^2}{R}$



$$R = \frac{120 \times 120}{60} = 240\ \Omega$$

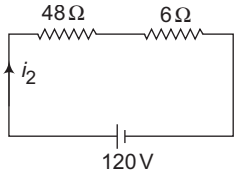
$$R_{eq} = 240 + 6 = 246\ \Omega$$

$$\Rightarrow \quad i_1 = \frac{V}{R_{eq}} = \frac{120}{246}$$



$$V_1 = \frac{240}{246} \times 120 = 117.073 \text{ V}$$

$$\Rightarrow \quad i_2 = \frac{120}{48 + 6}$$



$$V_2 = \frac{48}{54} \times 120$$

$$= 106.66 \text{ V}$$

$$V_1 - V_2 = 10.04 \text{ V}$$

15. Heat is extracted from the source means heat is given to the system (or gas). or Q is positive. This is positive only along the path ABC.

Heat supplied

$$\therefore \quad Q_{ABC} = \Delta U_{ABC} + W_{ABC}$$

$$= nC_V (T_f - T_i)$$

+ Area under p-V graph

$$= n\left(\frac{3}{2}R\right)(T_C - T_A) + 2p_0V_0$$

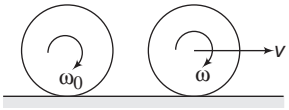
$$= \frac{3}{2}(nRT_C - nRT_A) + 2p_0V_0$$

$$= \frac{3}{2}(p_CV_C - p_AV_{A)} + 2p_0V_0$$

$$= \frac{3}{2}(4p_0V_0 - p_0V_0) + 2p_0V_0$$

$$= \frac{13}{2}p_0V_0$$

16.



$$\omega = v/r$$

From conservation of angular momentum about bottommost point

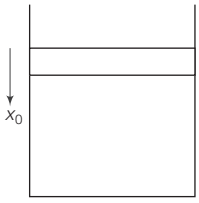
$$mr^2\omega_0 = mvr + mr^2 \times \frac{v}{r}$$

$$\Rightarrow \qquad v = \frac{\omega_0 r}{2}$$

17. In equilibrium,

$$p_0A = Mg \qquad \ldots(i)$$

when slightly displaced downwards,



$$dp = -\gamma\left(\frac{p_0}{V_0}\right)dV$$

$$\left(\text{As in adiabatic process, } \frac{dp}{dV} = -\gamma\frac{p}{V}\right)$$

$\therefore$ Restoring force,

$$F = (dp)A = -\left(\frac{\gamma p_0}{V_0}\right)(A)(Ax)$$

$$F \propto -x$$

Therefore, motion is simple harmonic comparing with $F = -Kx$ we have :

$$K = \frac{\gamma p_0 A^2}{V_0}$$

$$\therefore \qquad f = \frac{1}{2\pi}\sqrt{\frac{K}{m}} = \frac{1}{2\pi}\sqrt{\frac{\gamma p_0 A^2}{MV_0}}$$

18. According to Newton's cooling law, option (c) is correct answer.

19. Statement I is false and Statement II is true.

20. For charging of capacitor

$$q = CV\ (1 - e^{t/\tau})$$

at $t = 2\tau \Rightarrow q = CV\ (1 - e^{-2})$

21. It will be concentric circles.

22. Peak value of electric field

$$E_0 = B_0c = 20 \times 10^{-9} \times 3 \times 10^8$$

$$= 6\ V/m$$

23. As λ is increased, there will be a value of λ above which photoelectron will cease to come out. So, photocurrent will be zero.

24. For same value of current higher value of voltage is required for higher frequency.

25. Decrease in surface energy = heat required in vaporization.

$$\therefore \qquad T\,(dS) = L(dm)$$

$$\therefore T(2)(4\pi r)\,dr = L(4\pi r^2 dr)\rho$$

$$\therefore \qquad r = \frac{2T}{\rho L}$$

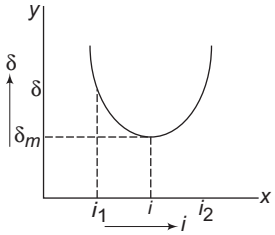
26. $\Delta E = h\nu$

$$\nu = \frac{\Delta E}{h} = k\left[\frac{1}{(n-1)^2} - \frac{1}{n^2}\right]$$

$$= \frac{k2n}{n^2(n-1)^2}$$

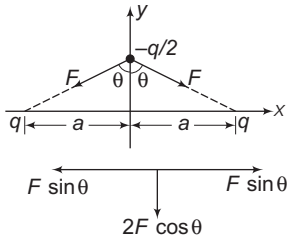
$$\approx \frac{2k}{n^3} \propto \frac{1}{n^3}$$

27. We know that the angle of deviation depends upon the angle of incidence. If we determine experimentally, the angles of deviation corresponding to different angles of incidence and then plot i (on-x-axis) and δ (on-y-axis), we get a curve as shown in figure.



Clearly if angle of incidence is gradually increased, from a small value, the angle of deviation first decreases, becomes minimum for a particular angle of incidence and then begins to increase.

28.

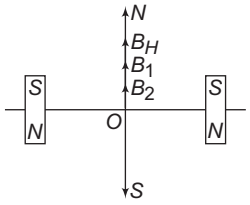


$F_{net} = 2F\cos\theta$

$$F_{net} = \frac{2kq\left(\frac{q}{2}\right)}{(\sqrt{y^2 + a^2})^2} \cdot \frac{y}{\sqrt{y^2 + a^2}}$$

$$F_{net} = \frac{2kq\left(\frac{q}{2}\right)y}{(y^2 + a^2)^{3/2}} \Rightarrow \frac{kq^2y}{a^3} \propto y$$

29. $B_{net} = B_1 + B_2 + B_H$

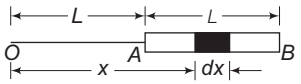


$$B_{net} = \frac{\mu_0}{4\pi}\frac{(M_1 + M_2)}{r^3} + B_H$$

$$= \frac{10^{-7}\ (1.2 + 1)}{(0.1)^3} + 3.6 \times 10^{-5}$$

$$= 2.56 \times 10^{-4}\ \text{Wb/m}^2$$

30.



$$V = \int_L^{2L} \frac{kdQ}{x}$$

$$= \int_L^{2L} \frac{k\left(\frac{Q}{L}\right)dx}{x}$$

$$= \frac{Q}{4\pi\varepsilon_0 L}\int_L^{2L}\left(\frac{1}{x}\right)dx$$

$$= \frac{Q}{4\pi\varepsilon_0 L}[\log_e x]_L^{2L}$$

$$= \frac{Q}{4\pi\varepsilon_0 L}[\log_e 2L - \log_e L]$$

$$= \frac{Q}{4\pi\varepsilon_0 L}\ln(2)$$

Chemistry

1. Optical isomerism is exhibited by only those complexes which lacks elements of symmetry.

$[Co(NH_3)_3Cl_3]$ shows facial as well as meridional isomerism. But both the forms contain plane of symmetry. Thus, this complex does not exhibit optical isomerism.

2. $C_2\ (6 + 6 = 12) = \sigma 1s^2, \overset{*}{\sigma}\ 1s^2, \sigma 2s^2, \overset{*}{\sigma}\ 2s^2,\ \pi 2p_x^2 \approx \pi 2p_y^2$

Since, all the electrons are paired, it is a diamagnetic species.

$N_2\ (7 + 7 = 14) = \sigma 1s^2, \overset{*}{\sigma}\ 1s^2, \sigma 2s^2, \overset{*}{\sigma}\ 2s^2,\ \pi 2p_x^2 \approx \pi 2p_y^2, \sigma 2p_z^2$

It is also a diamagnetic species because of the absence of unpaired electrons.

$O_2\ (8 + 8 = 16)$

or $S_2 = \sigma 1s^2, \overset{*}{\sigma}\ 1s^2, \sigma 2s^2, \overset{*}{\sigma}\ 2s^2, \sigma 2p_z^2, \pi 2p_x^2 \approx \pi 2p_y^2$

$$\overset{*}{\pi}\ 2p_x^1 \approx \overset{*}{\pi}\ 2p_y^1$$

Due to the presence of two unpaired electrons O_2 and S_2 both are paramagnetic molecules.

3. $Cl—\underset{\underset{(-)}{Ph}}{CH}—CH_3 \xrightarrow[\text{Toluene}]{SbCl_5}$

$$Ph—\overset{\oplus}{\underset{\underset{\text{(planar)}}{\text{(Carbocation)}}}{C}}H—CH_3 + SbCl_6^-$$

$$\longrightarrow Ph—\underset{\underset{Cl}{|}}{CH}—CH_3$$

(d and l) mixture

$+ SbCl_5$

4. Higher the SRP, better is oxidising agent. Among the given $E^{\circ}_{MnO_4^-/Mn^{2+}}$ is highest, hence MnO_4^- is the strongest oxidising agent.

5. The process is isothermal expansion, hence

$$Q = -W$$

$$\Delta E = 0$$

$$W = 2.303\, nRT \log \frac{V_2}{V_1}$$

$$= -2.303 \times 0.04 \times 8.314 \times 310 \times \log \frac{335}{50}$$

$$= -208\ J$$

$$Q = +208\ J$$

$$W = -208\ J \text{ (expansion work)}$$

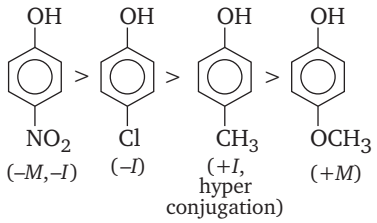
6. $$M_f = \frac{M_1V_1 + M_2V_2}{V_1 + V_2}$$

$$= \frac{750 \times 0.5 + 250 \times 2}{750 + 250}$$

$$= \frac{875}{1000}$$

$$= 0.875\ M$$

7.



Electron releasing group decreases while electron withdrawing group increases acidic strength by destabilising and stabilising the phenoxide ion formed respectively.

8. C^* = most probable speed = $\sqrt{\frac{2RT}{M}}$

speed = $\sqrt{\frac{8RT}{\pi M}}$

$\bar{C}$ = Mean square speed corrected as

rms = $\sqrt{\frac{3RT}{M}}$,

$$\overset{*}{C} < \bar{C} < C$$

$$\overset{*}{C} : \bar{C} : C = 1 : \sqrt{\frac{4}{\pi}} : \sqrt{\frac{3}{2}}$$

$$= 1 : 1.128 : 1.225$$

As no option correspond to mean square speed, it is understood as misprint. It should be root mean square speed.

9. From Arrhenius equation,

$$\log \frac{k_2}{k_1} = \frac{-E_a}{2.303\,R}\left(\frac{1}{T_2} - \frac{1}{T_1}\right)$$

Given, $\frac{k_2}{k_1} = 2$; $T_2 = 310$ K

$T_1 = 300$ K

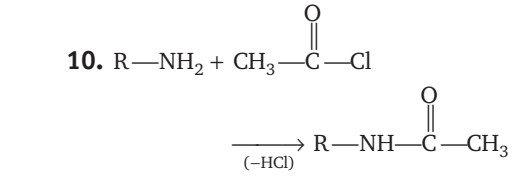
On putting values,

$$\Rightarrow \log 2 = \frac{-E_a}{2.303 \times 8.314}\left(\frac{1}{310} - \frac{1}{300}\right)$$

$$\Rightarrow \quad E_a = 53598.6\ J/mol$$

$$= 53.6\ kJ/mol$$

10.



Since each $—COCH_3$ group displace one H atom in the reaction of one mole of $CH_3—\overset{\overset{O}{\|}}{C}—Cl$ with one $—NH_2$ group, the molecular mass increases with 42 unit. Since the mass increases by $(390 - 180) = 210$ hence the number of $—NH_2$ group is $\frac{210}{42} = 5$.

11. (a) V^{2+} = 3 unpaired electrons

Cr^{2+} = 4 unpaired electrons

Mn^{2+} unpaired electrons

Fe^{2+} = 4 unpaired electrons

Hence, the order of paramagnetic behaviour should be

$$V^{2+} < Cr^{2+} = Fe^{2+} < Mn^{2+}$$

(b) Ionic size decreases from left to right in the same period.

(c) (As per data from NCERT)

$Co^{3+}/Co^{2+} = 1.97;$

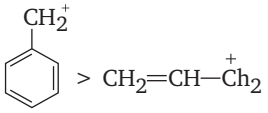
$Fe^{3+}/Fe^{2+} = 0.77;$

$Cr^{3+}/Cr^{2+} = -0.41$

Sc^{3+} is highly stable (It does not show + 2).

(d) The oxidation states increases as we go from group 3 to group 7 in the same period.

12. The order of stability of carbocation will be



$> CH_2{=}CH{-}\overset{+}{C}H_2$

Benzyl (more resonance stabilised) — Allyl (resonance stabilised)

$> CH_2{-}CH_2{-}\overset{+}{C}H_2$

Propyl (stabilised by $+I$ effect)

13. The half equations of the reaction are

$MnO_4^- \longrightarrow Mn^{2+}$

$C_2O_4^{2-} \longrightarrow CO_2$

The balanced half equations are

$MnO_4^- + 8H^+ + 5e^- \longrightarrow Mn^{2+} + 4H_2O$

$C_2O_4^{2-} \longrightarrow 2CO_2 + 2e^-$

On equating number of electrons, we get

$2MnO_4^- + 16H^+ + 10e^- \longrightarrow 2Mn^{2+} + 8H_2O$

$5C_2O_4^{2-} \longrightarrow 10CO_2 + 10e^-$

On adding both the equations, we get

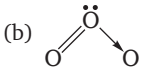
$2MnO_4^- + 5C_2O_4^- + 16H^+ \longrightarrow 2Mn^{2+} + 2 \times 5CO_2 + \frac{16}{2} H_2O$

Thus x, y and z are 2, 5 and 16 respectively.

14. (a) $ONCl = 8 + 7 + 17 = 32e^-$

$ONO^- = 8 + 7 + 8 + 1 = 24\,e^-$

(correct)

(b) 

Central O atom is sp^2 hybridised with 1 lone pair, so bent shape (correct).

(c) In solid state, ozone is violet-black ozone does not exist in solid state.

(d) O_3 has no unpaired electrons, so diamagnetic (correct).

*No option is correct.

15. 18 g H_2O contains 2gH

$\therefore$ 0.72 g H_2O contains 0.08 gH

44 g CO_2 contains 12 gC

$\therefore$ 3.08 g CO_2 contains 0.84 gC

$\therefore \quad C : H = \frac{0.84}{12} : \frac{0.08}{1}$

$= 0.07 : 0.08$

$= 7 : 8$

$\therefore$ Empirical formula = C_7H_8

16. Species having zero or negative bond order do not exist.

$H_2^{2+}(1 + 1 - 2 = 0) = \sigma 1s^0$

Bond order = 0

$He_2\,(2 + 2 = 4) = \sigma 1s^2, \overset{*}{\sigma}\, 1s^2$

Bond order $= \frac{N_b - N_a}{2}$

$= \frac{2-2}{2} = 0$

So, both H_2^{2+} and He_2 do not exist.

17. Silicon exists as covalent crystal in solid state. (Network like structure, like diamond).

18. $6CO_2 + 12NADPH + 18\ ATP \longrightarrow C_6H_{12}O_6 + 12NADP + 18\ ADP$

19. According to Hardy Schulze rule, greater the charge on oppositely charged ion, greater is its coagulating power. Since arsenic sulphide is a negatively charged sol, thus, the order of coagulating power is $Na^+ < Ba^{2+} < Al^{3+}$.

20. Ionisation energy increases along a period from left to right and decreases down a group. The position of given elements in the periodic table is as

2	16	18
Ca	S	Ar
Ba	Se	

Thus, the order of increasing ΔH_{IE_1} is
$Ba < Ca < Se < S < Ar$

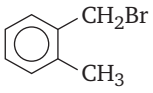
21. $\Delta E = 2.178 \times 10^{-18}\left(\frac{1}{1^2} - \frac{1}{2^2}\right) = \frac{hc}{\lambda}$

$$2.178 \times 10^{-18}\left(\frac{1}{1^2} - \frac{1}{2^2}\right) = \frac{6.62 \times 10^{-34} \times 3.0 \times 10^{8}}{\lambda}$$

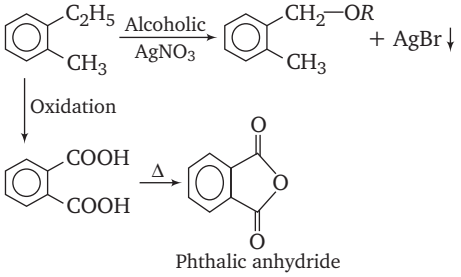
$\therefore \lambda \approx 1.21 \times 10^{-7}$ m

22. Compound A gives a precipitate with alcoholic $AgNO_3$, so it must contains Br in side chain. On oxidation, it gives $C_8H_6O_4$, which shows the presence of two alkyl chains attached directly with the benzene nucleus.

Since compound B gives anhydride on heating, the two alkyl substituent must occupy adjacent (1, 2) position. Thus, *A* must be



and the reactions are as follows



Phthalic anhydride

23. SRP value normally increases from left to right in the period of d-block elements. Some SRP value are exceptionally higher due to stability of product ion. *e.g.,*

$$E^{\circ}_{Mn^{3+}/Mn^{2+}} = +1.57\ V;$$

$$E^{\circ}_{Co^{3+}/Co^{2+}} = +1.97\ V$$

Thus, $E^{\circ}_{M^{3+}/M^{2+}}$ is highest for Co.

24. $pH = 1 \therefore [H^+] = 10^{-1} = 0.1$ M

$pH = 2 \ \therefore [H^+] = 10^{-2} = 0.01$ M

For dilution of HCl $M_1V_1 = M_2V_2$

$$0.1 \times 1 = 0.01 \times V_2$$

$$V_2 = 10\ L$$

Volume of water to be added

$$= 10 - 1 = 9\ L$$

25. $Na \longrightarrow Na^+ + e^-$ First IE

$Na^+ + e^- \longrightarrow Na$

Electron gain enthalpy of Na^+ is reverse of (IE)

Because reaction is reverse so

$$\Delta H(eq) = -5.1\ eV$$

26. $CH_3\underset{(A)}{CH_2—C(=O)—OH} \xrightarrow{NH_3}$

$\underset{(B)}{CH_3CH_2COONH_4} \xrightarrow{\Delta}$

$\underset{(C)}{CH_3—CH_2—C(=O)—NH_2}$

$\xrightarrow[\text{Hofmann bromamide reaction}]{Br_2, KOH} CH_3—CH_2—NH_2$

27. $Li_2\ (3 + 3 = 6) = \sigma 1s^2,\ \overset{*}{\sigma}\ 1s^2, \sigma 2s^2$

Bond order $= \frac{N_b - N_a}{2} = \frac{4-2}{2} = 1$

$Li_2^+\ (3 + 3 - 1 = 5) = \sigma 1s^2, \overset{*}{\sigma}\ 1s^2, \sigma 2s^1$ B

ond order $= \frac{3-2}{2} = \frac{1}{2} = 0.5$

$Li_2^-\ (3 + 3 + 1 = 7) = \sigma 1s^2,\ \overset{*}{\sigma} 1s^2,$

$\sigma 2s^2\ \ \overset{*}{\sigma}\ 2s^1$

Bond order $= \frac{4-3}{2} = \frac{1}{2} = 0.5$

Stability order is $Li_2 > Li_2^+ > Li_2^-$

(because Li_2^- has more number of electrons in antibonding orbitals which destabilises the species)

28. The reaction of alcohol with Lucas reagent is mostly an S_N1 reaction and the rate of reaction is directly proportional to the stability of carbocation formed in the reaction. Since 3° R—OH forms 3° carbocation (most stable) hence it will react fastest.

29. Methyl isocyanate $CH_3—N=C=O$ (MIC gas) gas leaked from the storage tank of the union carbide plant in Bhopal gas tragedy.

30. From the valency of M^{2+} and M^{3+}, it is clear that three M^{2+} ions will be replaced by M^{3+} causing a loss of one M^{3+} ion. Total loss of than from one molecule of Mo = 1 − 0.98 = 0.02

Total M^{3+} present in one molecule of

$$Mo = 2 \times 0.02 = 0.04$$

That M^{2+} and M^{3+} = 0.98

$$\text{Thus} \quad \%\ \text{of}\ M^{3+} = \frac{0.04 \times 100}{0.98}$$

$$= 4.08\%$$

Mathematics

1. Given planes are

$$2x + y + 2z - 8 = 0$$

and $2x + y + 2z + \frac{5}{2} = 0$

Distance between two planes

$$= \frac{|c_1 - c_2|}{\sqrt{a^2 + b^2 + c^2}} = \left|\frac{-8 - \frac{5}{2}}{\sqrt{2^2 + 1^2 + 2^2}}\right|$$

$$= \frac{\frac{21}{2}}{3} = \frac{7}{2}$$

2. Given, $\frac{dP}{dx} = (100 - 12\sqrt{x})$

$$\Rightarrow \quad dP = (100 - 12\sqrt{x})\, dx$$

On integrating both sides, we get

$$\int dP = \int (100 - 12\sqrt{x})\, dx$$

$$P = 100x - 8x^{3/2} + C$$

When $x = 0$, then $P = 2000$

$$\Rightarrow \quad C = 2000$$

Now, when $x = 25$, then is

$$P = 100 \times 25 - 8 \times (25)^{3/2} + 2000$$

$$= 2500 - 8 \times 125 + 2000$$

$$= 4500 - 1000 = 3500$$

3. Given, $n(A) = 2$, $n(B) = 4$

$$\therefore \quad n(A \times B) = 8$$

The number of subsets of $A \times B$ having 3 or more elements

$$= {}^8C_3 + {}^8C_4 + \ldots + {}^8C_8$$

$$= ({}^8C_0 + {}^8C_1 + {}^8C_2 + {}^8C_3 + \ldots + {}^8C_8) - ({}^8C_0 + {}^8C_1 + {}^8C_2)$$

$$\therefore \quad 2^n = {}^nC_0 + {}^nC_1 + \ldots + {}^nC_n$$

$$= 2^8 - {}^8C_0 - {}^8C_1 - {}^8C_2$$

$$= 256 - 1 - 8 - 28 = 219$$

4. Condition for two lines are coplanar.

$$\begin{vmatrix} x_1 - x_2 & y_1 - y_2 & z_1 - z_2 \\ l_1 & m_1 & n_1 \\ l_2 & m_2 & n_2 \end{vmatrix} = 0$$

where, (x_1, y_1, z_1) and (x_2, y_2, z_2) are the points lie on a line (i) and (ii) respectively and $< l_1, m_1, n_1 >$ and $< l_2, m_2, n_2 >$ are the direction cosines of the line (i) and line (ii) respectively.

$$\therefore \quad \begin{vmatrix} 2-1 & 3-4 & 4-5 \\ 1 & 1 & -k \\ k & 2 & 1 \end{vmatrix} = 0$$

$$\Rightarrow \quad \begin{vmatrix} 1 & -1 & -1 \\ 1 & 1 & -k \\ k & 2 & 1 \end{vmatrix} = 0$$

$$\Rightarrow \quad 1(1 + 2k) + (1 + k^2) - (2 - k) = 0$$

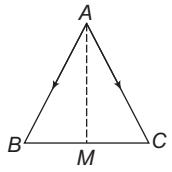
$$\Rightarrow \quad k^2 + 2k + k = 0$$

$$\Rightarrow \quad k^2 + 3k = 0 \Rightarrow k = 0, -3$$

If 0 appears in the denominator, then the correct way of representing the equation of straight line is

$$\frac{x - 2}{1} = \frac{y - 3}{1};\ z = 4$$

5. We know that, the sum of three vectors of a triangle is zero.



$\therefore$ $AB + BC + CA = 0$

$\Rightarrow$ $BC = AC - AB$

$\Rightarrow$ $BM = \dfrac{AC - AB}{2}$

($\because$ M is a mid-point of BC)

Also, $AB + BM + MA = 0$

(by properties of a triangle)

$\Rightarrow$ $AB + \dfrac{AC - AB}{2} = AM$

$\Rightarrow$ $AM = \dfrac{AB + AC}{2}$

$$= \frac{3\hat{i} + 4\hat{k} + 5\hat{i} - 2\hat{k} + 4\hat{k}}{2}$$

$$= 4\hat{i} - \hat{j} + 4\hat{k}$$

$\Rightarrow |AM| = \sqrt{4^2 + 1^2 + 4^2} = \sqrt{33}$

6. Let $f(x) = 2x^3 + 3x + k$

On differentiating w.r.t x, we get

$f'(x) = 6x^2 + 3 > 0, \forall\ x \in R$

$\Rightarrow f(x)$ is strictly increasing function.

$\Rightarrow f(x) = 0$ has only one real root, so two roots are not possible.

7. Let $S = 0.7 + 0.77 + 0.777 + \ldots$

$$= \frac{7}{10} + \frac{77}{10^2} + \frac{777}{10^3} + \ldots$$

$$= 7\left[\frac{1}{10} + \frac{11}{10^2} + \frac{111}{10^3} + \ldots \text{ upto 20 terms}\right]$$

$$= \frac{7}{9}\left[\frac{9}{10} + \frac{99}{100} + \frac{999}{1000} + \ldots \text{ upto 20 terms}\right]$$

$$= \frac{7}{9}\left[\left(1 - \frac{1}{10}\right) + \left(1 - \frac{1}{10^2}\right) + \left(1 - \frac{1}{10^3}\right) + \ldots + \text{upto 20 terms}\right]$$

$$= \frac{7}{9}\left[(1 + 1 + \ldots + \text{upto 20 terms}) - \left(\frac{1}{10} + \frac{1}{10^2} + \frac{1}{10^3} + \ldots \text{ upto 20 terms}\right)\right]$$

$$= \frac{7}{9}\left[20 - \frac{\frac{1}{10}\left\{1 - \left(\frac{1}{10}\right)^{20}\right\}}{1 - \frac{1}{10}}\right]$$

$$\left[\because \sum_{i=1}^{20} = 20 \text{ and sum of n terms of GP } S_n = \frac{a(1 - r^n)}{1 - r} \text{ when } (r < 1)\right]$$

$$= \frac{7}{9}\left[20 - \frac{1}{9}\left\{1 - \left(\frac{1}{10}\right)^{20}\right\}\right]$$

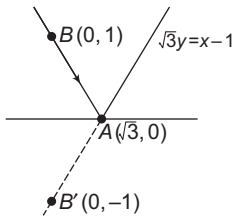
$$= \frac{7}{9}\left[\frac{179}{9} + \frac{1}{9}\left(\frac{1}{10}\right)^{20}\right]$$

$$= \frac{7}{81}[179 + (10)^{-20}]$$

8. Take any point B (0, 1) on given line.

Equation of AB′

$$y - 0 = \frac{-1 - 0}{0 - \sqrt{3}}(x - \sqrt{3})$$



$\Rightarrow$ $-\sqrt{3}y = -x + \sqrt{3}$

$\Rightarrow$ $x - \sqrt{3}y = \sqrt{3}$

$\Rightarrow$ $\sqrt{3}y = x - \sqrt{3}$

9. Given equations can be written in matrix form

$AX = B$

where, $A = \begin{bmatrix} k+1 & 8 \\ k & k+3 \end{bmatrix}$, $X = \begin{bmatrix} x \\ y \end{bmatrix}$

and $B = \begin{bmatrix} 4k \\ 3k - 1 \end{bmatrix}$

For no solution, $|A| = 0$ and
$(\text{adj } A)\, B \neq 0$

Now, $|A| = \begin{vmatrix} k+1 & 8 \\ k & k+3 \end{vmatrix} = 0$

$\Rightarrow \quad (k+1)(k+3) - 8k = 0$

$\Rightarrow \quad k^2 + 4k + 3 - 8k = 0$

$\Rightarrow \quad k^2 - 4k + 3 = 0$

$\Rightarrow \quad (k-1)(k-3) = 0$

$\Rightarrow \quad k = 1, k = 3$

Now, $\text{adj } A = \begin{bmatrix} k+3 & -8 \\ -k & k+1 \end{bmatrix}$

Now, $(\text{adj } A)\, B = \begin{bmatrix} k+3 & -8 \\ -k & k+1 \end{bmatrix} \begin{bmatrix} 4 & k \\ 3 & k-1 \end{bmatrix}$

$$= \begin{bmatrix} (k+3)(4k) - 8(3k-1) \\ -4k^2 + (k+1)(3k-1) \end{bmatrix}$$

$$= \begin{bmatrix} 4k^2 - 12k + 8 \\ -k^2 + 2k - 1 \end{bmatrix}$$

Put $k = 1$

$$(\text{adj } A)\, B = \begin{bmatrix} 4 - 12 + 8 \\ -1 + 2 - 1 \end{bmatrix}$$

$$= \begin{bmatrix} 0 \\ 0 \end{bmatrix} \text{ not true}$$

Put $k = 3$

$$(\text{adj } A)\, B = \begin{bmatrix} 36 - 36 + 8 \\ -9 + 6 - 1 \end{bmatrix}$$

$$= \begin{bmatrix} 8 \\ -4 \end{bmatrix} \neq 0 \text{ true.}$$

Hence, required value of k is 3.

Alternate Solution

Condition for the system of equations has no solution,

$$\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$$

$\therefore \quad \dfrac{k+1}{k} = \dfrac{8}{k+3} \neq \dfrac{4k}{3k-1}$

Take $\dfrac{k+1}{k} = \dfrac{8}{k+3}$

$\Rightarrow \quad k^2 + 4k + 3 = 8k$

$\Rightarrow \quad k^2 - 4k + 3 = 0$

$\Rightarrow \quad (k-1)(k-3) = 0$

$k = 1, 3$

If $k = 1$, then $\dfrac{8}{1+3} \neq \dfrac{4 \cdot 1}{2}$, false

And if $k = 3$, then $\dfrac{8}{6} \neq \dfrac{4 \cdot 3}{9-1}$, true

Therefore $k = 3$

Hence, only one value of k exist.

10. Given equations are

$$x^2 + 2x + 3 = 0 \quad \ldots(i)$$

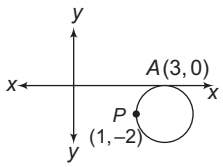
and $\quad ax^2 + bx + c = 0 \quad \ldots(ii)$

Since, Eq. (i) has imaginary roots so, Eq. (ii) will also have both roots same as Eq. (i).

Thus, $\dfrac{a}{1} = \dfrac{b}{2} = \dfrac{c}{3}$

Hence, a : b : c is 1 : 2 : 3

11. Let the equation of circle be



$$(x-3)^2 + (y-0)^2 + \lambda y = 0$$

As it passes through $(1, -2)$

$\therefore \quad (1-3)^2 + (-2)^2 + \lambda(-2) = 0$

$\Rightarrow \quad 4 + 4 - 2\lambda = 0 \Rightarrow \lambda = 4$

$\therefore$ Equation of circle is

$$(x-3)^2 + y^2 + 4y = 0$$

By hit and trial method, we see that point $(5, -2)$ satisfies equation of circle.

12. Since, x, y and z are in AP.

$\therefore \quad 2y = x + z$

Also, $\tan^{-1} x$, $\tan^{-1} y$ and $\tan^{-1} z$ are in AP.

$\therefore \quad 2\tan^{-1} y = \tan^{-1} x + \tan^{-1}(z)$

$\Rightarrow \quad \tan^{-1}\left(\dfrac{2y}{1-y^2}\right) = \tan^{-1}\left(\dfrac{x+z}{1-xz}\right)$

$\Rightarrow \quad \dfrac{x+z}{1-y^2} = \dfrac{x+z}{1-xz} \Rightarrow y^2 = xz$

Since x, y and z are in AP as well as in GP

$\therefore \quad x = y = z$

13. Statement II

$(p \to q) \leftrightarrow (\sim q \to \sim p)$

$\equiv (p \to q) \leftrightarrow (p \to q)$

which is always true, so statement II is true.

Statement I $(p \wedge \sim q) \wedge (\sim p \wedge q)$

$\equiv p \wedge \sim q \wedge \sim p \wedge q$

$\equiv p \wedge \sim p \wedge \sim q \wedge q$

$\equiv f \wedge f \equiv f$

Hence, it is a fallacy statement.

So, statement I is true.

Alternate Solution

Statement II

$(p \to q) \leftrightarrow (\sim q \to \sim p)$

$\sim q \to \sim p$ is contrapositive of $p \to q$ hence $(p \to q) \leftrightarrow (p \to q)$ will be a tautology.

Statement I $(p \wedge \sim q) \wedge (\sim p \wedge q)$

p	q	$\sim p$	$\sim q$	$p \wedge \sim q$	$\sim p \wedge q$	$(p \wedge \sim q) \wedge (\sim p \wedge q)$
T	T	F	F	F	F	F
T	F	F	T	T	F	F
F	T	T	F	F	T	F
F	F	T	T	F	F	F

Hence, it is a fallacy.

14. Given, $\int f(x)\,dx = \psi(x)$

Let $I = \int x^5 f(x^3)\,dx$

Put $x^3 = t$

$\Rightarrow \quad x^2 dx = \dfrac{dt}{3}$ ...(i)

$\therefore \quad I = \dfrac{1}{3}\int t\, f(t)\,dt$

$= \dfrac{1}{3}\left[t \cdot \int f(t)dt - \int \left\{\dfrac{d}{dt}(t)\int f(t)dt\right\}dt\right]$ (by parts)

$= \dfrac{1}{3}[t\,\psi(t) - \int \psi(t)\,dt]$

$= \dfrac{1}{3}[x^3\psi(x^3) - 3\int x^2\psi(x^3)\,dx] + C$ [$\because$ from Eq. (i)]

$= \dfrac{1}{3}x^3\psi(x^3) - \int x^2\psi(x^3)\,dx + C$

15. Let $I = \lim\limits_{x\to 0} \dfrac{(1-\cos 2x)}{x^2}\dfrac{(3+\cos x)}{1} \cdot \dfrac{x}{\tan 4x}$

$= \lim\limits_{x\to 0} \dfrac{2\sin^2 x}{x^2}\cdot\dfrac{3+\cos x}{1}\cdot\dfrac{x}{\tan 4x}$

$= 2\lim\limits_{x\to 0}\left(\dfrac{\sin x}{x}\right)^2 \cdot \lim\limits_{x\to 0}(3+\cos x) \cdot \lim\limits_{x\to 0}\dfrac{4x}{4\tan 4x}$

$= 2\cdot(1)^2\cdot(3+\cos 0°)\cdot\dfrac{1}{4}(1)$

$= 2\cdot 1\cdot(3+1)\cdot\dfrac{1}{4} = 2\cdot 4\cdot\dfrac{1}{4} = 2$

16. Let $I = \int_{\pi/6}^{\pi/3} \dfrac{dx}{1+\sqrt{\tan x}}$ ...(i)

$\therefore \quad I = \int_{\pi/6}^{\pi/3} \dfrac{dx}{1+\sqrt{\tan\left(\dfrac{\pi}{2}-x\right)}}$

$= \int_{\pi/6}^{\pi/3} \dfrac{dx}{1+\sqrt{\cot x}}$

$\Rightarrow \quad I = \int_{\pi/6}^{\pi/3} \dfrac{\sqrt{\tan x}\,dx}{1+\sqrt{\tan x}}$ ...(ii)

On adding Eqs. (i) and (ii), we get

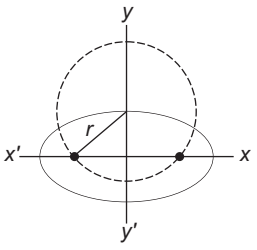
$2I = \int_{\pi/6}^{\pi/3} dx \quad \Rightarrow \quad 2I = [x]_{\pi/6}^{\pi/3}$

$\Rightarrow \quad I = \dfrac{1}{2}\left[\dfrac{\pi}{3} - \dfrac{\pi}{6}\right] = \dfrac{\pi}{12}$

Statement I is false.

But $\int_a^b f(x)\,dx = \int_a^b f(a+b-x)\,dx$ is a true statement by property of definite integrals.

17. Given equation of ellipse is $\dfrac{x^2}{16} + \dfrac{y^2}{9} = 1$



Here, a = 4, b = 3, $e = \sqrt{1 - \frac{9}{16}} \Rightarrow \frac{\sqrt{7}}{4}$

$\therefore$ Foci is $(\pm ae, 0) = \left(\pm 4 \times \frac{\sqrt{7}}{4}, 0\right)$

$= (\pm \sqrt{7}, 0)$

$\therefore$ Radius of the circle,

$r = \sqrt{(ae)^2 + b^2} = \sqrt{7 + 9} = \sqrt{16} = 4$

Now, equation of circle is

$(x - 0)^2 + (y - 3)^2 = 16$

$\therefore \quad x^2 + y^2 - 6y - 7 = 0$

18. Probability of guessing a correct answer, $p = \frac{1}{3}$ and probability of guessing a wrong answer, $q = \frac{2}{3}$

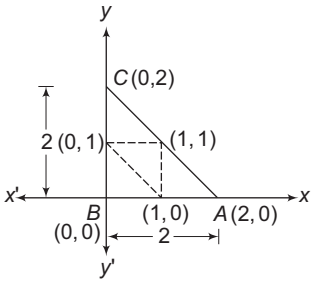
$\therefore$ The probability of guessing a 4 or more correct answer

$= {}^5C_4\left(\frac{1}{3}\right)^4 \cdot \frac{2}{3} + {}^5C_5\left(\frac{1}{3}\right)^5$

$= 5 \cdot \frac{2}{3^5} + \frac{1}{3^5} = \frac{11}{3^5}$

19. Given mid-points of a triangle are (0, 1), (1, 1) and (1, 0). Plotting these points on a graph paper and make a triangle.

So, the sides of a triangle will be 2, 2 and $\sqrt{2^2 + 2^2}$ i. e., $2\sqrt{2}$



x-coordinate of incentre

$= \frac{2 \times 0 + 2\sqrt{2} \cdot 0 + 2 \cdot 2}{2 + 2 + 2\sqrt{2}}$

$= \frac{2}{2 + \sqrt{2}} \times \frac{2 - \sqrt{2}}{2 - \sqrt{2}} = 2 - \sqrt{2}$

20. $\therefore \left[\frac{x + 1}{x^{2/3} - x^{1/3} + 1} - \frac{(x - 1)}{x - x^{1/2}}\right]^{10}$

$= \left[\frac{(x^{1/3})^3 + 1^3}{x^{2/3} - x^{1/3} + 1} - \frac{\{(\sqrt{x})^2 - 1\}}{\sqrt{x}\,(\sqrt{x} - 1)}\right]^{10}$

$= \left[\frac{(x^{1/3} + 1)(x^{2/3} + 1 - x^{1/3})}{x^{2/3} - x^{1/3} + 1} - \frac{\{(\sqrt{x})^2 - 1\}}{\sqrt{x}\,(\sqrt{x} - 1)}\right]^{10}$

$= \left[(x^{1/3} + 1) - \frac{(\sqrt{x} + 1)}{\sqrt{x}}\right]^{10}$

$= (x^{1/3} - x^{-1/2})^{10}$

$\therefore$ The general term is

$T_{r+1} = {}^{10}C_r (x^{1/3})^{10 - r} (- x^{-1/2})^r$

$= {}^{10}C_r (-1)^r x^{\frac{10 - r}{3} - \frac{r}{2}}$

For independent of x, put

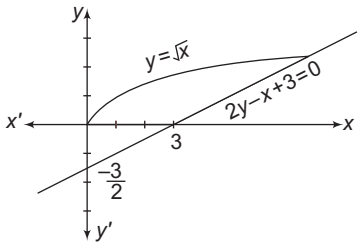
$\frac{10 - r}{3} - \frac{r}{2} = 0 \Rightarrow \quad 20 - 2r - 3r = 0$

$\Rightarrow \qquad 20 = 5r \quad \Rightarrow \quad r = 4$

$\therefore \quad T_5 = {}^{10}C_4 = \frac{10 \times 9 \times 8 \times 7}{4 \times 3 \times 2 \times 1} = 210$

21. Given curves are $y = \sqrt{x}$ …(i)

and $2y - x + 3 = 0$ …(ii)



On solving Eqs. (i) and (ii), we get

$2\sqrt{x} - (\sqrt{x})^2 + 3 = 0$

$\Rightarrow \quad (\sqrt{x})^2 - 2\sqrt{x} - 3 = 0$

$\Rightarrow \quad (\sqrt{x} - 3)(\sqrt{x} + 1) = 0$

$\Rightarrow \qquad \sqrt{x} = 3,$

$(\because \sqrt{x} = -1$ is not possible)

$\therefore \qquad y = 3$

$\therefore$ Required area $= \int_0^3 (x_2 - x_1)\, dy$

$$= \int_0^3 \{(2y + 3) - y^2\}\, dy$$

$$= \left[y^2 + 3y - \frac{y^3}{3}\right]_0^3$$

$$= 9 + 9 - 9 = 9$$

22. Given, $T_n = {}^nC_3$

$$T_{n+1} = {}^{n+1}C_3$$

$\therefore\ T_{n+1} - T_n = {}^{n+1}C_3 - {}^nC_3 = 10$

(given)

$\Rightarrow \quad {}^nC_2 + {}^nC_3 - {}^nC_3 = 10$

$(\because\ {}^nC_r + {}^nC_{r+1} = {}^{n+1}C_{r+1})$

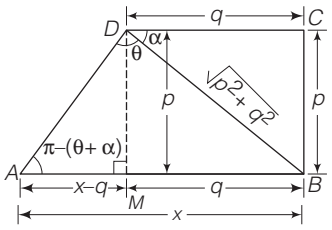
$\Rightarrow \quad {}^nC_2 = 10 \quad \Rightarrow \quad n = 5$

23. Given, $|z| = 1$, $\arg z = \theta \therefore z = e^{i\theta}$

But $\quad \bar{z} = \dfrac{1}{z}$

$\therefore \quad \arg\left(\dfrac{1+z}{1+\dfrac{1}{z}}\right) = \arg(z) = \theta$

24. Let AB = x



In ΔDAM, $\tan(\pi - \theta - \alpha) = \dfrac{p}{x - q}$

$\Rightarrow \quad \tan(\theta + \alpha) = \dfrac{p}{q - x}$

$\Rightarrow \quad q - x = p\cot(\theta + \alpha)$

$\Rightarrow \quad x = q - p\cot(\theta + \alpha)$

$$= q - p\left(\frac{\cot\theta\cot\alpha - 1}{\cot\alpha + \cot\theta}\right)$$

$$\left(\because \cot\alpha = \frac{q}{p}\right)$$

$$= q - p\left(\frac{\frac{q}{p}\cot\theta - 1}{\frac{q}{p} + \cot\theta}\right)$$

$$= q - p\left(\frac{q\cot\theta - p}{q + p\cot\theta}\right)$$

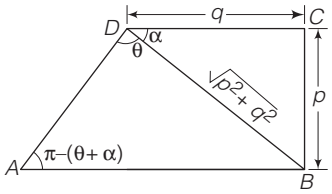
$$= q - p\left(\frac{q\cos\theta - p\sin\theta}{q\sin\theta + p\cos\theta}\right)$$

$$\Rightarrow \quad x = \frac{q^2\sin\theta + pq\cos\theta - pq\cos\theta + p^2\sin\theta}{p\cos\theta + q\sin\theta}$$

$$\Rightarrow AB = \frac{(p^2 + q^2)\sin\theta}{p\cos\theta + q\sin\theta}$$

Alternate Solution

Applying Sine rule in Δ ABD,



$$\frac{AB}{\sin\theta} = \frac{\sqrt{p^2 + q^2}}{\sin\{\pi - (\theta + \alpha)\}}$$

$$\Rightarrow \quad \frac{AB}{\sin\theta} = \frac{\sqrt{p^2 + q^2}}{\sin(\theta + \alpha)}$$

$$\Rightarrow \quad AB = \frac{\sqrt{p^2 + q^2}\sin\theta}{\sin\theta\cos\alpha + \cos\theta\sin\alpha}$$

$$= \frac{(p^2 + q^2)\sin\theta}{q\sin\theta + p\cos\theta}$$

$$\left(\because \cos\alpha = \frac{q}{\sqrt{p^2 + q^2}}\right)$$

$$= \frac{(p^2 + q^2)\sin\theta}{p\cos\theta + q\sin\theta}$$

and $\quad \sin\alpha = \dfrac{p}{\sqrt{p^2 + q^2}}$

25. Given, $P = \begin{bmatrix} 1 & \alpha & 3 \\ 1 & 3 & 3 \\ 2 & 4 & 4 \end{bmatrix}$

$\therefore \quad |P| = 1(12 - 12) - \alpha(4 - 6) + 3(4 - 6)$

$$= 2\alpha - 6$$

$\because \quad P = \text{adj}(A)$ (given)

$\therefore \quad |P| = |\text{adj } A| = |A|^2 = 16$

$(\because |\text{adj } A| = |A|^{n-1})$

$\therefore \quad 2\alpha - 6 = 16$

$\Rightarrow \quad 2\alpha = 22 \Rightarrow \alpha = 11$

26. Given, $y = \int_0^x |t|\, dt$

$\therefore \quad \frac{dy}{dx} = |x| \cdot 1 - 0 = |x|$

(by Leibnitz rule)

$\because$ Tangent to the curve $y = \int_0^x (t)\, dt$, $x \in R$ are parallel to the line $y = 2x$

$\therefore$ Slope of both are equal

$\Rightarrow \quad x = \pm 2$

$\therefore$ Points, $y = \int_0^{\pm 2} |t|\, dt = \pm 2$

$\therefore$ Equation of tangent is

$$y - 2 = 2(x - 2)$$

and $\quad y + 2 = 2(x + 2)$

For x-intercept put $y = 0$, we get

$$0 - 2 = 2(x - 2)$$

and $\quad 0 + 2 = 2(x + 2) \Rightarrow x = \pm 1$

27. Equation of circle can be rewritten as

$$x^2 + y^2 = \frac{5}{2},$$

centre $\rightarrow (0,)$ and radius $\rightarrow \sqrt{\frac{5}{2}}$

Let common tangent be

$$y = mx + \frac{\sqrt{5}}{m}$$

$\Rightarrow \quad m^2x - my + \sqrt{5} = 0$

The perpendicular from centre to the tangent is equal to radius of the circle.

$$\therefore \quad \frac{\frac{\sqrt{5}}{m}}{\sqrt{1 + m^2}} = \sqrt{\frac{5}{2}}$$

$$\Rightarrow \quad m\sqrt{1 + m^2} = \sqrt{2}$$

$$\Rightarrow \quad m^2(1 + m^2) = 2$$

$$\Rightarrow \quad m^4 + m^2 - 2 = 0$$

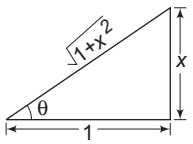
$$\Rightarrow \quad (m^2 + 2)(m^2 - 1) = 0$$

$$\Rightarrow \quad m = \pm 1$$

$(\because m^2 + 2 \neq 0$, as $m \in R)$

$\therefore y = \pm (x + \sqrt{5})$, both statements are correct as $m = \pm 1$ satisfies the given equation of statement II.

28. Given, $y = \sec(\tan^{-1} x)$



Let $\quad \tan^{-1} x = \theta \Rightarrow x = \tan\theta$

$\therefore \quad y = \sec\theta = \sqrt{1 + x^2}$

On differentiating w.r.t. x, we get

$$\frac{dy}{dx} = \frac{1}{2\sqrt{1 + x^2}} \cdot 2x$$

At $x = 1$, $\quad \frac{dy}{dx} = \frac{1}{\sqrt{2}}$

29. Given expression is

$$\frac{\tan A}{1 - \cot A} + \frac{\cot A}{1 - \tan A}$$

$$= \frac{\sin A}{\cos A} \times \frac{\sin A}{\sin A - \cos A} + \frac{\cos A}{\sin A} \times \frac{\cos A}{\cos A - \sin A}$$

$$= \frac{1}{\sin A - \cos A}\left\{\frac{\sin^3 A - \cos^3 A}{\cos A \sin A}\right\}$$

$$= \frac{\sin^2 A + \sin A \cos A + \cos^2 A}{\sin A \cos A}$$

$$= \frac{1 + \sin A \cos A}{\sin A \cos A}$$

$$= 1 + \sec A \operatorname{cosec} A$$

30. If initially all marks were x_i, then

$$\sigma_1^2 = \frac{\sum (x_i - \bar{x})^2}{N}$$

Now, each is increased by 10

$$\therefore \sigma_2^2 = \frac{\sum (x_i + 10) - (\bar{x} + 10)]^2}{N} = \sigma_1^2$$

So, variance will not change whereas mean, median and mode will increase by 10.