Solved Paper 2014

JEE Main

Joint Entrance Examination

Time : 3 hrs | MM : 360

Instructions

1. This test consists of 90 questions.
2. There are three parts in the question paper A,B,C consisting of Physics, Chemistry and Mathematics having 30 questions in each part of equal weightage. Each question is allotted 4 marks for correct response.
3. Candidates will be awarded marks as stated above in instruction 2 for correct response of each question. 1/4 mark will be deducted for indicating incorrect response for an item in the answer sheet.
4. There is only one correct response for each question. Filling up more than one response in any question will be treated as wrong response and marks for wrong response will be deducted accordingly as per instructions.

Physics

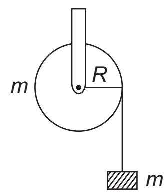
1. The current voltage relation of diode is given by $I = (e^{1000V/T} - 1)$ mA, where the applied voltage V is in volt and the temperature T is in kelvin. If a student makes an error measuring ± 0.01 V while measuring the current of 5 mA at 300K, what will be the error in the value of current in mA?

(a) 0.2 mA (b) 0.02 mA
(c) 0.5 mA (d) 0.05 mA

2. From a tower of height H, a particle is thrown vertically upwards with a speed u. The time taken by the particle to hit the ground, is n times that taken by it to reach the highest point of its path. The relation between H, u and n is

(a) $2gH = n^2u^2$ (b) $gH = (n-2)^2u^2$
(c) $2gH = nu^2(n-2)$ (d) $gH = (n-2)^2u^2$

3. A mass m supported by a massless string wound around a uniform hollow cylinder of mass m and radius R. If the string does not slip on the cylinder, with what acceleration will the mass fall on release?



(a) $\frac{2g}{3}$ (b) $\frac{g}{2}$ (c) $\frac{5g}{6}$ (d) g

4. A block of mass m is placed on a surface with a vertical cross-section given by $y = x^3/6$. If the coefficient of friction is 0.5, the maximum height above the ground at which the block can be placed without slipping is

(a) $\frac{1}{6}$ m (b) $\frac{2}{3}$ m (c) $\frac{1}{3}$ m (d) $\frac{1}{2}$ m

5. When a rubber band is stretched by a distance x, it exerts a restoring force of magnitude $F = ax + bx^2$, where a and b are constants. The work done in stretching the unstretched rubber-band by L is

(a) $aL^2 + bL^3$ (b) $\frac{1}{2}(aL^2 + bL^3)$

(c) $\frac{aL^2}{2} + \frac{bL^3}{3}$ (d) $\frac{1}{2}\left(\frac{aL^2}{2} + \frac{bL^3}{3}\right)$

6. A bob of mass m attached to an inextensible string of length l is suspended from a vertical support. The bob rotates in a horizontal circle with an angular speed ω rad/s about the vertical support. About the point of suspension

(a) angular momentum is conserved
(b) angular momentum changes in magnitude but not in direction
(c) angular momentum changes in direction but not in magnitude
(d) angular momentum changes both in direction and magnitude

7. Four particles, each of mass M and equidistant from each other, move along a circle of radius R under the action of their mutual gravitational attraction, the speed of each particle is

(a) $\sqrt{\frac{GM}{R}}$

(b) $\sqrt{2\sqrt{2}\frac{GM}{R}}$

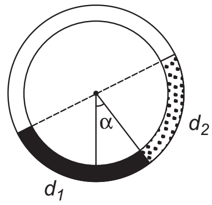
(c) $\sqrt{\frac{GM}{R}(1+2\sqrt{2})}$

(d) $\frac{1}{2}\sqrt{\frac{GM}{R}(1+2\sqrt{2})}$

8. The pressure that has to be applied to the ends of a steel wire of length 10 cm to keep its length constant when its temperature is raised by 100°C is (For steel, Young's modulus is $2 \times 10^{11} \text{Nm}^{-2}$ and coefficient of thermal expansion is $1.1 \times 10^{-5} \text{ K}^{-1}$)

(a) 2.2×10^8 Pa (b) 2.2×10^9 Pa
(c) 2.2×10^7 Pa (d) 2.2×10^6 Pa

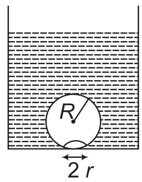
9. There is a circular tube in a vertical plane. Two liquids which do not mix and of densities d_1 and d_2 are filled in the tube. Each liquid subtends 90° angle at centre. Radius joining their interface makes an angle α with vertical. Ratio d_1 / d_2 is



(a) $\frac{1+\sin\alpha}{1-\sin\alpha}$ (b) $\frac{1+\cos\alpha}{1-\cos\alpha}$

(c) $\frac{1+\tan\alpha}{1-\tan\alpha}$ (d) $\frac{1+\sin\alpha}{1-\cos\alpha}$

10. On heating water, bubbles beings formed at the bottom of the vessel detach and rise. Take the bubbles to be spheres of radius R and making a circular contact of radius r with the bottom of the vessel. If $r << R$ and the surface tension of water is T, value of r just before bubbles detach is (density of water is ρ)



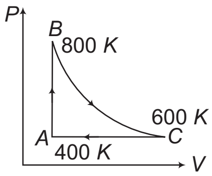
(a) $R^2\sqrt{\frac{\rho_w g}{3T}}$ (b) $R^2\sqrt{\frac{\rho_w g}{6T}}$

(c) $R^2\sqrt{\frac{\rho_w g}{T}}$ (d) $R^2\sqrt{\frac{3\rho_w g}{T}}$

11. Three rods of copper, brass and steel are welded together to form a Y-shaped structure. Area of cross-section of each rod is 4 cm^2. End of copper rod is maintained at 100°C whereas ends of brass and steel are kept at 0°C. Lengths of the copper, brass and steel rods are 46, 13 and 12 cm respectively. The rods are thermally insulated from surroundings except at ends. Thermal conductivities of copper, brass and steel are 0.92, 0.26 and 0.12 in CGS units, respectively. Rate of heat flow through copper rod is

(a) 1.2 cal/s (b) 2.4 cal/s
(c) 4.8 cal/s (d) 6.0 cal/s

12. One mole of diatomic ideal gas undergoes a cyclic process ABC as shown in figure. The process BC is adiabatic. The temperatures at A, B and C are 400 K, 800 K and 600 K, respectively. Choose the correct statement.



(a) The change in internal energy in whole cyclic process is 250 R
(b) The change in internal energy in the process CA is 700 R
(c) The change in internal energy in the process AB is $-350R$
(d) The change in internal energy in the process BC is $-500R$

13. An open glass tube is immersed in mercury in such a way that a length of 8 cm extends above the mercury level. The open end of the tube is then closed and sealed and the tube is raised vertically up by additional 46 cm. What will be length of the air column above mercury in the tube now? (Atmospheric pressure = 76 cm of Hg)
(a) 16 cm (b) 22 cm (c) 38 cm (d) 6 cm

14. A particle moves with simple harmonic motion in a straight line. In first τ sec, after starting from rest it travels a distance a and in next τ sec, it travels $2a$, in same direction, then
(a) amplitude of motion is $3a$
(b) time period of oscillations is 8π
(c) amplitude of motion is $4a$
(d) time period of oscillations is 6π

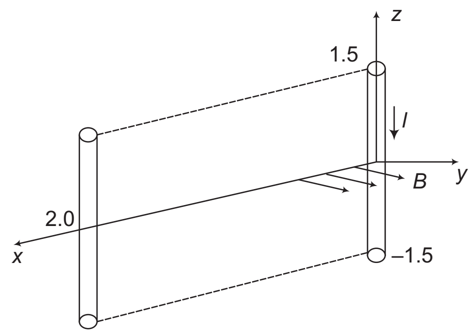
15. A pipe of length 85 cm is closed from one end. Find the number of possible natural oscillations of air column in the pipe whose frequencies lie below 1250 Hz. The velocity of sound in air is 340 m/s.
(a) 12 (b) 8 (c) 6 (d) 4

16. Assume that an electric field $\mathbf{E} = 30x^2\ \hat{\mathbf{i}}$ exists in space. Then, the potential difference $V_A - V_O$, where V_O is the potential at the origin and V_A the potential at x = 2 m, is
(a) 120 J (b) -120 J (c) -80 J (d) 80 J

17. A parallel plate capacitor is made of two circular plates separated by a distance of 5 mm and with a dielectric of dielectric constant 2.2 between them. When the electric field in the dielectric is 3×10^4 V/m, the charge density of the positive plate will be close to
(a) 6×10^{-7} C/m^2 (b) 3×10^{-7} C/m^2
(c) 3×10^{4} C/m^2 (d) 6×10^{4} C/m^2

18. In a large building, there are 15 bulbs of 40 W, 5 bulbs of 100 W, 5 fans of 80 W and 1 heater of 1 kW. The voltage of the electric mains is 220 V. The minimum capacity of the main fuse of the building will be
(a) 8 A (b) 10 A (c) 12 A (d) 14 A

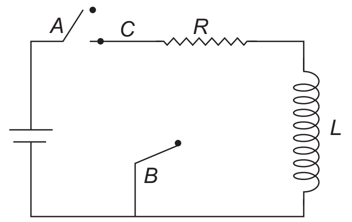
19. A conductor lies along the z-axis at $-1.5 \le z < 1.5$ m and carries a fixed current of 10.0 A in $-a_z$ direction (see figure). For a field $\mathbf{B} = 3.0 \times 10^{-4}\ e^{-0.2x} a_y$ T, find the power required to move the conductor at constant speed to $x = 2.0$ m, $y = 0$ in 5×10^{-3} s. Assume parallel motion along the x-axis.



(a) 1.57 W (b) 2.97 W
(c) 14.85 W (d) 29.7 W

20. The coercivity of a small magnet where the ferromagnet gets demagnetised is 3×10^3 Am^{-1}. The current required to be passed in a solenoid of length 10 cm and number of turns 100, so that the magnet gets demagnetised when inside the solenoid is
(a) 30 mA (b) 60 mA (c) 3 A (d) 6 A

21. In the circuit shown here, the point C is kept connected to point A till the current flowing through the circuit becomes constant. Afterward, suddenly point C is disconnected from point A and connected to point B at time $t = 0$. Ratio of the voltage across resistance and the inductor at $t = L / R$ will be equal to



(a) $\frac{e}{1-e}$ (b) 1

(c) −1 (d) $\frac{1-e}{e}$

22. During the propagation of electromagnetic waves in a medium,

(a) electric energy density is double of the magnetic energy density
(b) electric energy density is half of the magnetic energy density
(c) electric energy density is equal to the magnetic energy density
(d) Both electric and magnetic energy densities are zero

23. A thin convex lens made from crown glass $\left(\mu = \frac{3}{2}\right)$ has focal length f.

When
it is measured in two different liquids having refractive indices $\frac{4}{3}$ and $\frac{5}{3}$. It has the focal lengths f_1 and f_2, respectively. The correct relation between the focal length is

(a) $f_1 = f_2 < f$
(b) $f_1 > f$ and f_2 becomes negative
(c) $f_2 > f$ and f_1 becomes negative
(d) f_1 and f_2 both become negative

24. A green light is incident from the water to the air-water interface at the critical angle (θ). Select the correct statement.

(a) The entire spectrum of visible light will come out of the water at an angle of 90° to the normal
(b) The spectrum of visible light whose frequency is less than that of green light will come out of the air medium.
(c) The spectrum of visible light whose frequency is more than that of green light will come out to the air medium.
(d) The entire spectrum of visible light will come out of the water at various angles to the normal.

25. Two beams, A and B, of plane polarised light with mutually perpendicular planes of polarisation are seen through a polaroid. From the position when the beam A has maximum intensity (and beam B has zero intensity), a rotation of polaroid through 30° makes the two beams appear equally bright. If the initial intensities of the two beams are I_A and I_B respectively, then I_A / I_B equals

(a) 3 (b) $\frac{3}{2}$ (c) 1 (d) $\frac{1}{3}$

26. The radiation corresponding to $3 \to 2$ transition of hydrogen atom falls on a metal surface to produce photoelectrons. These electrons are made to enter a magnetic field of 3×10^{-4} T. If the radius of the largest circular path followed by these electrons is 10.0 mm, the work function of the metal is close to

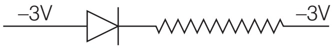
(a) 1.8 eV (b) 1.1 eV (c) 0.8 eV (d) 1.6 eV

27. Hydrogen $({}_1H^1)$, deuterium $({}_1H^2)$, singly ionised helium $({}_2He^4)^+$ and doubly ionised lithium $({}_3Li^8)^{++}$ all have one electron around the nucleus. Consider an electron transition from $n = 2$ to $n = 1$. If the wavelengths of emitted radiation are $\lambda_1, \lambda_2, \lambda_3$ and λ_4, respectively for four elements, then approximately which one of the following is correct?

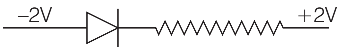
(a) $4\lambda_1 = 2\lambda_2 = 2\lambda_3 = \lambda_4$
(b) $\lambda_1 = 2\lambda_2 = 2\lambda_3 = \lambda_4$
(c) $\lambda_1 = \lambda_2 = 4\lambda_3 = 9\lambda_4$
(d) $\lambda_1 = 2\lambda_2 = 3\lambda_3 = 4\lambda_4$

28. The forward biased diode connection is

(a) 

(b) 

(c) 

(d) 

29. A student measured the length of a rod and wrote it as 3.50 cm. Which instrument did he use to measure it?

(a) A meter scale
(b) A vernier calliper where the 10 divisions in vernier scale matches with 9 divisions in main scale and main scale has 10 divisions in 1 cm
(c) A screw gauge having 100 divisions in the circular scale and pitch as 1 mm
(d) A screw gauge having 50 divisions in the circular scale and pitch as 1 mm

30. Match List I (Electromagnetic wave type) with List II (Its association/application) and select the correct option from the choices given below the lists.

	List I		List II
A.	Infrared waves	1.	To treat muscular strain
B.	Radio waves	2.	For broadcasting
C.	X-rays	3.	To detect fracture of bones
D.	Ultraviolet	4.	Absorbed by the ozone layer of the atmosphere

Codes

	A	B	C	D
(a)	4	3	2	1
(b)	1	2	4	3
(c)	3	2	1	4
(d)	1	2	3	4

Chemistry

31. The correct set of four quantum numbers for the valence electrons of rubidium atom $(Z = 37)$ is

(a) $5, 0, 0, +\frac{1}{2}$ (b) $5, 1, 0, +\frac{1}{2}$
(c) $5, 1, ,1, +\frac{1}{2}$ (d) $5, 0, 1, +\frac{1}{2}$

32. If Z is a compressibility factor, van der Waals' equation at low pressure can be written as

(a) $Z = 1 + \frac{RT}{pb}$ (b) $Z = 1 - \frac{a}{VRT}$
(c) $Z = 1 - \frac{pb}{RT}$ (d) $Z = 1 + \frac{pb}{RT}$

33. CsCl crystallises in body centred cubic lattice. If 'a' its edge length, then which of the following expressions is correct?

(a) $r_{Cs^+} + r_{Cl^-} = 3a$
(b) $r_{Cs^+} + r_{Cl^-} = \frac{3a}{2}$
(c) $r_{Cs^+} + r_{Cl^-} = \frac{\sqrt{3}}{2}a$
(d) $r_{Cs^+} + r_{Cl^-} = \sqrt{3}a$

34. For the estimation of nitrogen, 1.4 g of an organic compound was digested by Kjeldahl's method and the evolved ammonia was absorbed in 60 mL of $\frac{M}{10}$ sulphuric acid. The unreacted acid required 20 mL of $M/10$ sodium hydroxide for complete neutralisation. The percentage of nitrogen in the compound is

(a) 6% (b) 10%
(c) 3% (d) 5%

35. Resistance of 0.2 M solution of an electrolyte is $50\,\Omega$. The specific conductance of the solution of 0.5 M solution of same electrolyte is 1.4 S m^{-1} and resistance of same solution of the same electrolyte is $280\,\Omega$. The molar conductivity of 0.5 M solution of the electrolyte in $S\,m^2mol^{-1}$ is

(a) 5×10^{-4} (b) 5×10^{-3}
(c) 5×10^{3} (d) 5×10^{2}

36. For the complete combustion of ethanol, $C_2H_5OH(l) + 3O_2(g) \longrightarrow 2CO_2(g) + 3H_2O(l)$, the amount of heat produced as measured in bomb calorimeter, is 1364.47 kJ mol^{-1} at 25°C. Assuming ideality the enthalpy of combustion, $\Delta_C H$, for the reaction will be ($R = 8.314$ JK^{-1}mol^{-1})

(a) – 1366.95 kJ mol^{-1}

(b) – 1361.95 kJ mol^{-1}

(c) – 1460.50 kJ mol^{-1}

(d) – 1350.50 kJ mol^{-1}

37. The equivalent conductance of NaCl at concentration C and at infinite dilution are λ_C and λ_∞, respectively. The correct relationship between λ_C and λ_∞ is given as (where, the constant B is positive)

(a) $\lambda_C = \lambda_\infty + (B)C$

(b) $\lambda_C = \lambda_\infty - (B)C$

(c) $\lambda_C = \lambda_\infty - (B)\sqrt{C}$

(d) $\lambda_C = \lambda_\infty + (B)\sqrt{C}$

38. Consider separate solution of 0.500 M C_2H_5OH (aq), 0.100 M $Mg_3(PO_4)_2(aq)$, 0.250 M KBr(aq) and 0.125 M Na_3PO_4 (aq) at 25°C. Which statement is true about these solution, assuming all salts to be strong electrolytes?

(a) They all have the same osmotic pressure

(b) 0.100 M $Mg_3(PO_4)_2(aq)$ has the highest osmotic pressure

(c) 0.125 M $Na_3PO_4(aq)$ has the highest osmotic pressure

(d) 0.500 M C_2H_5OH (aq) has the highest osmotic pressure

39. For the reaction,

$$SO_2(g) + \frac{1}{2}O_2(g) \rightleftharpoons SO_3(g)$$

if $K_p = K_C(RT)^x$

where, the symbols have usual meaning, then the value of x is (assuming ideality)

(a) – 1 (b) $-\frac{1}{2}$

(c) $\frac{1}{2}$ (d) 1

40. For the non-stoichiometric reaction $2A + B \rightarrow C + D$, the following kinetic data were obtained in three separate experiments, all at 298 K.

	Initial concen tration (A)	Initial concentration (B)	Initial rate of formation of C (mol L^{-1}S^{-1})
(i)	0.1 M	0.1 M	1.2×10^{-3}
(ii)	0.1 M	0.2 M	1.2×10^{-3}
(iii)	0.2 M	0.1 M	2.4×10^{-3}

The rate law for the formation of C is

(a) $\frac{dC}{dt} = k[A][B]$ (b) $\frac{dC}{dt} = k[A]^2[B]$

(c) $\frac{dC}{dt} = k[A][B]^2$ (d) $\frac{dC}{dt} = k[A]$

41. Among the following oxoacids, the correct decreasing order of acid strength is

(a) $HOCl > HClO_2 > HClO_3 > HClO_4$

(b) $HClO_4 > HOCl > HClO_2 > HClO_3$

(c) $HClO_4 > HClO_3 > HClO_2 > HOCl$

(d) $HClO_2 > HClO_4 > HClO_3 > HOCl$

42. The metal that cannot be obtained by electrolysis of an aqueous solution of its salts is

(a) Ag (b) Ca

(c) Cu (d) Cr

43. The octahedral complex of a metal ion M^{3+} with four monodentate ligands L_1, L_2, L_3 and L_4 absorb wavelengths in the region of red, green, yellow and blue, respectively. The increasing order of ligand strength of the four ligands is

(a) $L_4 < L_3, L_2 < L_1$

(b) $L_1 < L_3 < L_2 < L_4$

(c) $L_3 < L_2 < L_4 < L_1$

(d) $L_1 < L_2 < L_4 < L_3$

44. Which of the following properties is not shown by NO?

(a) It is diamagnetic in gaseous state

(b) It is a neutral oxide

(c) It combines with oxygen to form nitrogen dioxide

(d) Its bond order is 2.5

45. In which of the following reactions H_2O_2 acts as a reducing agent?

I. $H_2O_2 + 2H^+ + 2e^- \longrightarrow 2H_2O$

II. $H_2O_2 - 2e^- \longrightarrow O_2 + 2H^+$

III. $H_2O_2 + 2e^- \longrightarrow 2OH^-$

IV. $H_2O_2 + 2OH^- - 2e^- \longrightarrow O_2 + 2H_2O$

(a) I and II (b) III and IV
(c) I and III (d) II and IV

46. The correct statement for the molecule, CsI_3 is

(a) it is a covalent molecule
(b) it contains Cs^+ and I_3^-
(c) it contains Cs^{3+} and I^- ions
(d) it contains Cs^+, I^- and lattice I_2 molecule

47. The ratio of masses of oxygen and nitrogen of a particular gaseous mixture is 1 : 4. The ratio of number of their molecule is

(a) 1 : 4 (b) 7 : 32 (c) 1 : 8 (d) 3 : 16

48. Given below are the half-cell reactions

$Mn^{2+} + 2e^- \longrightarrow Mn\ ;\ E° = -1.18\ \text{eV}$

$2\,(Mn^{3+} + e^- \longrightarrow Mn^{2+});\ E° = +\ 1.51\,\text{eV}$

The E° for $3Mn^{2+} \longrightarrow Mn + 2Mn^{3+}$ will be

(a) – 2.69 V; the reaction will not occur
(b) – 2.69 V; the reaction will occur
(c) – 0.33 V; the reaction will not occur
(d) – 0.33 V; the reaction will occur

49. Which series of reactions correctly represents chemical relations related to iron and its compound?

(a) $Fe \xrightarrow{\text{Dil. } H_2SO_4} FeSO_4 \xrightarrow{H_2SO_4, O_2} Fe_2(SO_4)_3 \xrightarrow{\text{Heat}} Fe$

(b) $Fe \xrightarrow{O_2,\text{ Heat}} FeO \xrightarrow{\text{Dil. } H_2SO_4} FeSO_4 \xrightarrow{\text{Heat}} Fe$

(c) $Fe \xrightarrow{Cl_2,\text{ Heat}} FeCl_3 \xrightarrow{\text{Heat, air}} FeCl_2 \xrightarrow{Zn} Fe$

(d) $Fe \xrightarrow{O_2,\text{ Heat}} Fe_3O_4 \xrightarrow{CO,\ 600°C} FeO \xrightarrow{CO,\ 700°C} Fe$

50. The equation which is balanced and represents the correct product(s) is

(a) $Li_2O + 2KCl \longrightarrow 2LiCl + K_2O$

(b) $[CoCl(NH_3)_5]^+ + 5H^+ \longrightarrow Co^{2+} + 5NH_4^+ + Cl^-$

(c) $[Mg(H_2O)_6]^{2+} + (EDTA)^{4-} \xrightarrow{\text{Excess NaOH}} [Mg(EDTA)]^{2+} + 6H_2O$

(d) $CuSO_4 + 4KCN \longrightarrow K_2[Cu(CN)_4] + K_2SO_4$

51. In S_N2 reactions, the correct order of reactivity for the following compounds CH_3Cl, CH_3CH_2Cl, $(CH_3)_2CHCl$ and $(CH_3)_3CCl$ is

(a) $CH_3Cl > (CH_3)_2CHCl > CH_3CH_2Cl > (CH_3)_3CCl$
(b) $CH_3Cl > CH_3CH_2Cl > (CH_3)_2CHCl > (CH_3)_3CCl$
(c) $CH_3CH_2Cl > CH_3Cl > (CH_3)_2CHCl > (CH_3)_3CCl$
(d) $(CH_3)_2CHCl > CH_3CH_2Cl > CH_3Cl > (CH_3)_3CCl$

52. On heating an aliphatic primary amine with chloroform and ethanolic potassium hydroxide, the organic compound formed is

(a) an alkanol
(b) an alkanediol
(c) an alkyl cyanide
(d) an alkyl isocyanide

53. The most suitable reagent for the conversion of

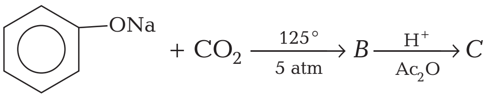
$R—CH_2 — OH \longrightarrow R—CHO$ is

(a) $KMnO_4$
(b) $K_2Cr_2O_7$
(c) CrO_3
(d) PCC (pyridinium chlorochromate)

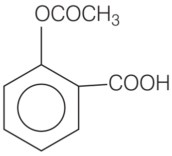
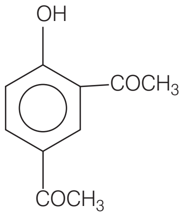
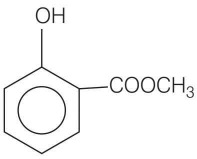
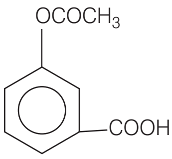
54. The major organic compound formed by the reaction of 1,1,1-trichloroethane with silver powder is

(a) acetylene (b) ethene
(c) 2-butyne (d) 2-butene

55. Sodium phenoxide when heated with CO_2 under pressure at 125°C yields a product which on acetylation produces C.



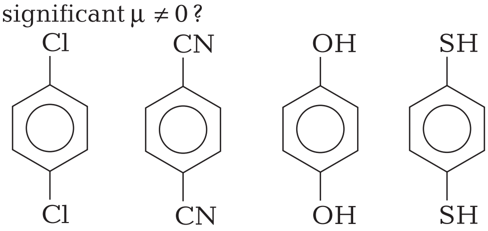
The major product C would be

(a) 
(b) 
(c) 
(d) 

56. Considering the basic strength of amines in aqueous solution, which one has the smallest pK_b value?

(a) $(CH_3)_2NH$ (b) CH_3NH_2
(c) $(CH_3)_3N$ (d) $C_6H_5NH_2$

57. For which of the following molecule significant $\mu \neq 0$?



I II III IV

(a) Only I (b) I and II
(c) Only III (d) III and IV

58. Which one is classified as a condensation polymer?

(a) Dacron (b) Neoprene
(c) Teflon (d) Acrylonitrile

59. Which one of the following bases is not present in DNA?

(a) Quinoline (b) Adenine
(c) Cytosine (d) Thymine

60. In the reaction,

$$CH_3COOH \xrightarrow{LiAlH_4} A \xrightarrow{PCl_5} B \xrightarrow{\text{Alc. KOH}} C$$

The product C is

(a) acetaldehyde (b) acetylene
(c) ethylene (d) acetyl chloride

Mathematics

61. If $X = \{4^n - 3n - 1 : n \in N\}$ and $Y = \{9(n-1) : n \in N\}$; where N is the set of natural numbers, then $X \cup Y$ is equal to

(a) N (b) $Y - X$
(c) X (d) Y

62. If z is a complex number such that $|z| \geq 2$, then the minimum value of $\left|z + \frac{1}{2}\right|$

(a) is equal to $\frac{5}{2}$
(b) lies in the interval (1, 2)
(c) is strictly greater than $\frac{5}{2}$
(d) is strictly greater than $\frac{3}{2}$ but less than $\frac{5}{2}$

63. If $a \in R$ and the equation $-3(x - [x])^2 + 2(x - [x]) + a^2 = 0$ (where, $[x]$ denotes the greatest integer $\leq x$) has no integral solution, then all possible values of a lie in the interval

(a) $(-1, 0) \cup (0, 1)$ (b) $(1, 2)$
(c) $(-2, -1)$ (d) $(-\infty, -2) \cup (2, \infty)$

64. Let α and β be the roots of equation $px^2 + qx + r = 0$, $p \neq 0$. If p, q and r are in AP and $\frac{1}{\alpha} + \frac{1}{\beta} = 4$, then the value of $|\alpha - \beta|$ is

(a) $\frac{\sqrt{61}}{9}$ (b) $\frac{2\sqrt{17}}{9}$ (c) $\frac{\sqrt{34}}{9}$ (d) $\frac{2\sqrt{13}}{9}$

65. If $\alpha, \beta \neq 0$ and $f(n) = \alpha^n + \beta^n$ and

$$\begin{vmatrix} 3 & 1+f(1) & 1+f(2) \\ 1+f(1) & 1+f(2) & 1+f(3) \\ 1+f(2) & 1+f(3) & 1+f(4) \end{vmatrix} = K(1-\alpha)^2(1-\beta)^2(\alpha-\beta)^2,$$

then K is equal to

(a) $\alpha\beta$ (b) $\frac{1}{\alpha\beta}$ (c) 1 (d) -1

66. If A is a 3×3 non-singular matrix such that $AA^T = A^TA$ and $B = A^{-1}A^T$, then BB^T is equal to

(a) $I + B$ (b) I (c) B^{-1} (d) $(B^{-1})^T$

67. If the coefficients of x^3 and x^4 in the expansion of $(1 + ax + bx^2)(1-2x)^{18}$ in powers of x are both zero, then (a, b) is equal to

(a) $\left(16, \frac{251}{3}\right)$ (b) $\left(14, \frac{251}{3}\right)$

(c) $\left(14, \frac{272}{3}\right)$ (d) $\left(16, \frac{272}{3}\right)$

68. The angle between the lines whose direction cosines satisfy the equations $l + m + n = 0$ and $l^2 = m^2 + n^2$ is

(a) $\frac{\pi}{3}$ (b) $\frac{\pi}{4}$

(c) $\frac{\pi}{6}$ (d) $\frac{\pi}{2}$

69. If $(10)^9 + 2(11)^1(10)^8 + 3(11)^2(10)^7 + \ldots + 10(11)^9 = k(10)^9$, then k is equal to

(a) $\frac{121}{10}$ (b) $\frac{441}{100}$

(c) 100 (d) 110

70. Three positive numbers form an increasing GP. If the middle term in this GP is doubled, then new numbers are in AP. Then, the common ratio of the GP is

(a) $\sqrt{2} + \sqrt{3}$ (b) $3 + \sqrt{2}$

(c) $2 - \sqrt{3}$ (d) $2 + \sqrt{3}$

71. $\lim_{x \to 0} \frac{\sin(\pi \cos^2 x)}{x^2}$ is equal to

(a) $\frac{\pi}{2}$ (b) 1

(c) $-\pi$ (d) π

72. If g is the inverse of a function f and $f'(x) = \frac{1}{1+x^5}$, then $g'(x)$ is equal to

(a) $1 + x^5$ (b) $5x^4$

(c) $\frac{1}{1+\{g(x)\}^5}$ (d) $1 + \{g(x)\}^5$

73. If f and g are differentiable functions in $(0, 1)$ satisfying $f(0) = 2 = g(1)$, $g(0) = 0$ and $f(1) = 6$, then for some $c \in]0, 1[$

(a) $2f'(c) = g'(c)$ (b) $2f'(c) = 3g'(c)$

(c) $f'(c) = g'(c)$ (d) $f'(c) = 2g'(c)$

74. If $x = -1$ and $x = 2$ are extreme points of $f(x) = \alpha \log|x| + \beta x^2 + x$, then

(a) $\alpha = -6, \beta = \frac{1}{2}$ (b) $\alpha = -6, \beta = -\frac{1}{2}$

(c) $\alpha = 2, \beta = -\frac{1}{2}$ (d) $\alpha = 2, \beta = \frac{1}{2}$

75. The integral $\int \left(1 + x - \frac{1}{x}\right) e^{x + \frac{1}{x}} dx$ is equal to

(a) $(x-1)e^{x+\frac{1}{x}} + C$ (b) $xe^{x+\frac{1}{x}} + C$

(c) $(x+1)e^{x+\frac{1}{x}} + C$ (d) $-xe^{x+\frac{1}{x}} + C$

76. The integral $\int_0^{\pi} \sqrt{1 + 4\sin^2 \frac{x}{2} - 4\sin\frac{x}{2}}\, dx$ is equal to

(a) $\pi - 4$ (b) $\frac{2\pi}{3} - 4 - 4\sqrt{3}$

(c) $4\sqrt{3} - 4$ (d) $4\sqrt{3} - 4 - \frac{\pi}{3}$

77. The area of the region described by $A = \{(x, y) : x^2 + y^2 \leq 1 \text{ and } y^2 \leq 1 - x\}$ is

(a) $\frac{\pi}{2} + \frac{4}{3}$ (b) $\frac{\pi}{2} - \frac{4}{3}$ (c) $\frac{\pi}{2} - \frac{2}{3}$ (d) $\frac{\pi}{2} + \frac{2}{3}$

78. Let the population of rabbits surviving at a time t be governed by the differential equation $\frac{dp(t)}{dt} = \frac{1}{2}p(t) - 200$. If $p(0) = 100$, then $p(t)$ is equal to

(a) $400 - 300e^{\frac{t}{2}}$ (b) $300 - 200e^{-\frac{t}{2}}$

(c) $600 - 500e^{\frac{t}{2}}$ (d) $400 - 300e^{-\frac{t}{2}}$

79. If PS is the median of the triangle with vertices $P(2,2)$, $Q(6,-1)$ and $R(7,3)$, then equation of the line passing through $(1,-1)$ and parallel to PS is

(a) $4x-7y-11=0$ (b) $2x+9y+7=0$
(c) $4x+7y+3=0$ (d) $2x-9y-11=0$

80. Let a, b, c and d be non-zero numbers. If the point of intersection of the lines $4ax+2ay+c=0$ and $5bx+2by+d=0$ lies in the fourth quadrant and is equidistant from the two axes, then

(a) $2bc-3ad=0$ (b) $2bc+3ad=0$
(c) $2ad-3bc=0$ (d) $3bc+2ad=0$

81. The locus of the foot of perpendicular drawn from the centre of the ellipse $x^2+3y^2=6$ on any tangent to it is

(a) $(x^2-y^2)^2=6x^2+2y^2$
(b) $(x^2-y^2)^2=6x^2-2y^2$
(c) $(x^2+y^2)^2=6x^2+2y^2$
(d) $(x^2+y^2)^2=6x^2-2y^2$

82. Let C be the circle with centre at (1, 1) and radius 1. If T is the circle centred at (0, y) passing through origin and touching the circle C externally, then the radius of T is equal to

(a) $\frac{\sqrt{3}}{\sqrt{2}}$ (b) $\frac{\sqrt{3}}{2}$ (c) $\frac{1}{2}$ (d) $\frac{1}{4}$

83. The slope of the line touching both the parabolas $y^2=4x$ and $x^2=-32y$ is

(a) $\frac{1}{2}$ (b) $\frac{3}{2}$ (c) $\frac{1}{8}$ (d) $\frac{2}{3}$

84. The image of the line $\frac{x-1}{3}=\frac{y-3}{1}=\frac{z-4}{-5}$ in the plane $2x-y+z+3=0$ is the line

(a) $\frac{x+3}{3}=\frac{y-5}{1}=\frac{z-2}{-5}$
(b) $\frac{x+3}{-3}=\frac{y-5}{-1}=\frac{z+2}{5}$
(c) $\frac{x-3}{3}=\frac{y+5}{1}=\frac{z-2}{-5}$
(d) $\frac{x-3}{-3}=\frac{y+5}{-1}=\frac{z-2}{5}$

85. If $[\mathbf{a}\times\mathbf{b}\ \mathbf{b}\times\mathbf{c}\ \mathbf{c}\times\mathbf{a}]=\lambda[\mathbf{a}\ \mathbf{b}\ \mathbf{c}]^2$, then λ is equal to

(a) 0 (b) 1
(c) 2 (d) 3

86. Let A and B be two events such that $P(\overline{A\cup B})=\frac{1}{6}$, $P(A\cap B)=\frac{1}{4}$ and $P(\overline{A})=\frac{1}{4}$, where $\overline{A}$ stands for the complement of the event A. Then, the events A and B are

(a) independent but not equally likely
(b) independent and equally likely
(c) mutually exclusive and independent
(d) equally likely but not independent

87. The variance of first 50 even natural numbers is

(a) $\frac{833}{4}$ (b) 833
(c) 437 (d) $\frac{437}{4}$

88. If $f_k(x)=1/k(\sin^k x+\cos^k x)$, where $x\in R$ and $k\geq 1$, then $f_4(x)-f_6(x)$ is equal to

(a) $\frac{1}{6}$ (b) $\frac{1}{3}$
(c) $\frac{1}{4}$ (d) $\frac{1}{12}$

89. A bird is sitting on the top of a vertical pole 20 m high and its elevation from a point O on the ground is 45°. It flies off horizontally straight away from the point O. After 1s, the elevation of the bird from O is reduced to 30°. Then, the speed (in m/s) of the bird is

(a) $40(\sqrt{2}-1)$
(b) $40(\sqrt{3}-\sqrt{2})$
(c) $20\sqrt{2}$
(d) $20(\sqrt{3}-1)$

90. The statement $\sim(p\leftrightarrow\sim q)$ is

(a) equivalent to $p\leftrightarrow q$
(b) equivalent to $\sim p\leftrightarrow q$
(c) a tautology
(d) a fallacy

Answers

1. (a)	**2.** (c)	**3.** (b)	**4.** (a)	**5.** (c)	**6.** (c)	**7.** (d)	**8.** (a)	**9.** (c)	**10.** (*)
11. (c)	**12.** (d)	**13.** (a)	**14.** (d)	**15.** (c)	**16.** (c)	**17.** (a)	**18.** (c)	**19.** (b)	**20.** (c)
21. (c)	**22.** (c)	**23.** (b)	**24.** (d)	**25.** (d)	**26.** (b)	**27.** (c)	**28.** (a)	**29.** (b)	**30.** (d)
31. (a)	**32.** (b)	**33.** (c)	**34.** (b)	**35.** (a)	**36.** (a)	**37.** (c)	**38.** (a)	**39.** (b)	**40.** (d)
41. (c)	**42.** (b)	**43.** (b)	**44.** (a)	**45.** (d)	**46.** (b)	**47.** (b)	**48.** (a)	**49.** (d)	**50.** (b)
51. (b)	**52.** (d)	**53.** (d)	**54.** (c)	**55.** (a)	**56.** (a)	**57.** (d)	**58.** (a)	**59.** (a)	**60.** (c)
61. (d)	**62.** (b)	**63.** (a)	**64.** (d)	**65.** (c)	**66.** (b)	**67.** (d)	**68.** (a)	**69.** (c)	**70.** (d)
71. (d)	**72.** (d)	**73.** (d)	**74.** (c)	**75.** (b)	**76.** (d)	**77.** (a)	**78.** (a)	**79.** (b)	**80.** (c)
81. (c)	**82.** (d)	**83.** (a)	**84.** (a)	**85.** (b)	**86.** (a)	**87.** (b)	**88.** (d)	**89.** (d)	**90.** (a)

NOTE

Solutions

Physics

1. Given, $I = (e^{1000V/T} - 1)\,\text{mA}$,

$$dV = \pm 0.01\ \text{V}$$
$$T = 300\ \text{K}$$
$$I = 5\ \text{mA}$$

So, $I = e^{1000V/T} - 1$

$$I + 1 = e^{1000V/T}$$

Taking log on both sides, we get

$$\log(I + 1) = \frac{1000\,V}{T}$$

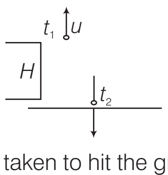
On differentiating, $\frac{dI}{I+1} = \frac{1000}{T}\,dV$

$$dI = \frac{1000}{T} \times (I + 1)\,dV$$

$\Rightarrow$ $dI = \frac{1000}{300} \times (5 + 1) \times 0.01 = 0.2\ \text{mA}$

So, error in the value of current is 0.2 mA.

2. Time taken to reach the maximum height $t_1 = \frac{u}{g}$



If t_2 is the time taken to hit the ground,

i.e., $-H = ut_2 - \frac{1}{2}gt_2^2$

But $t_2 = nt_1$ [Given]

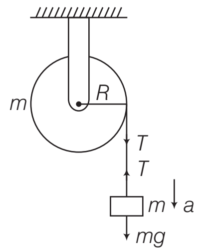
So, $-H = u\frac{nu}{g} - \frac{1}{2}g\frac{n^2u^2}{g^2}$

$$-H = \frac{nu^2}{g} - \frac{1}{2}\frac{n^2u^2}{g}$$

$\Rightarrow$ $2gH = nu^2(n - 2)$

3. For the mass m,

$$mg - T = ma$$



As we know, $a = R\alpha$

So, $mg - T = mR\alpha$ …(i)

Torque about centre of pully

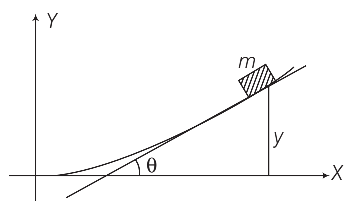
$$T \times R = mR^2\alpha \quad \text{...(ii)}$$

From Eqs. (i) and (ii), we get

$$a = \frac{g}{2}$$

Hence, the acceleration with the mass of a body fall is $\frac{g}{2}$.

4. A block of mass m is placed on a surface with a vertical cross-section, then



$$\tan\theta = \frac{dy}{dx} = \frac{d\left(\frac{x^3}{6}\right)}{dx} = \frac{x^2}{2}$$

At limiting equilibrium, we get

$$\mu = \tan\theta$$

$$0.5 = \frac{x^2}{2} \Rightarrow x^2 = 1 \Rightarrow x = \pm 1$$

Now, putting the value of x in $y = \frac{x^3}{6}$, we get

When $x = 1$	When $x = -1$
$y = \frac{(1)^3}{6} = \frac{1}{6}$	$y = \frac{(-1)^3}{6} = \frac{-1}{6}$

So, the maximum height above the ground at which the block can be placed without slipping is $\frac{1}{6}$ m.

5. **We know that change in potential energy of a system corresponding to a conservative internal force as**

$$U_f - U_i = -W = -\int_i^f \mathbf{F}.d\mathbf{r}$$

Given, $F = ax + bx^2$

We know that work done in stretching the rubber band by L is

$$|dW| = |Fdx|$$

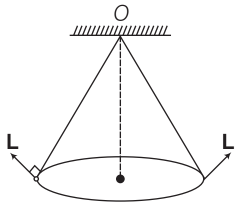
$$|W| = \int_O^L (ax + bx^2)dx$$

$$= \left[\frac{ax^2}{2}\right]_O^L + \left[\frac{bx^3}{3}\right]_O^L$$

$$= \left[\frac{aL^2}{2} - \frac{a \times (0)^2}{2}\right] + \left[\frac{b \times L^3}{3} - \frac{b \times (0)^3}{3}\right]$$

$$|W| = \frac{aL^2}{2} + \frac{bL^3}{3}$$

6. Angular momentum of the pendulum about the suspension point O is



$\mathbf{L} = m(\mathbf{r} \times \mathbf{v})$

Then, $\mathbf{v}$ can be resolved into two components, radial component r_{rad} and tangential component r_{tan}. Due to v_{rad}, L will be tangential and due to v_{tan}, L will be radially outwards as shown.

So, net angular momentum will be as shown in figure whose magnitude will be constant ($|L| = mvl$). But its direction will change as shown in the figure.

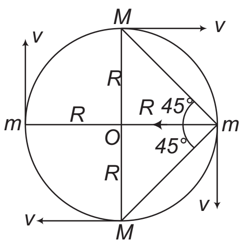
$$\mathbf{L} = m(\mathbf{r} \times \mathbf{v})$$

where, r = radius of circle

7. Net force acting on any one particle M,

$$= \frac{GM^2}{(2R)^2} + \frac{GM^2}{(R\sqrt{2})^2}\cos 45° + \frac{GM^2}{(R\sqrt{2})^2}\cos 45°$$

$$= \frac{GM^2}{R^2}\left(\frac{1}{4} + \frac{1}{\sqrt{2}}\right)$$



This force will equal to centripetal force.

So, $$\frac{Mv^2}{R} = \frac{GM^2}{R^2}\left(\frac{1}{4} + \frac{1}{\sqrt{2}}\right)$$

$$v = \sqrt{\frac{GM}{4R}(1 + 2\sqrt{2})} = \frac{1}{2}\sqrt{\frac{GM}{R}(2\sqrt{2} + 1)}$$

Hence, speed of each particle in a circular motion is

$$\frac{1}{2}\sqrt{\frac{GM}{R}(2\sqrt{2} + 1)}.$$

8. **If the deformation is small, then the stress in a body is directly proportional to the corresponding strain.**

According to Hooke's law *i.e.,*

$$\text{Young's modulus } (Y) = \frac{\text{Tensile stress}}{\text{Tensile strain}}$$

So, $$Y = \frac{F/A}{\Delta L/L} = \frac{FL}{A\Delta L}$$

If the rod is compressed, then compressive stress and strain appear. Their ratio Y is same as that for tensile case.

Given, length of a steel wire (L) = 10cm,

Temperature (θ) = 100°C

As length is constant.

$$\therefore \quad \text{Strain} = \frac{\Delta L}{L} = \alpha\Delta\theta$$

Now, pressure = stress = $Y \times$ strain

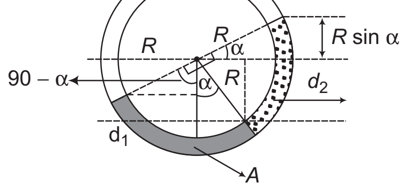
[Given, $Y = 2 \times 10^{11} \text{N/m}^2$ and $\alpha = 1.1. \times 10^{-5} \text{K}^{-1}$]

$= 2 \times 10^{11} \times 1.1 \times 10^{-5} \times 100$

$= 2.2 \times 10^{8}$ Pa

9. Equating pressure at A, we get

$R \sin\alpha \, d_2 + R \cos\alpha \, d_2 + R(1 - \cos\alpha) d_1 = R(1 - \sin\alpha) d_1$



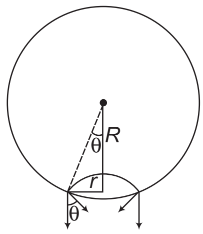
$(\sin\alpha + \cos\alpha) d_2 = d_1(\cos\alpha - \sin\alpha)$

$\Rightarrow \quad \dfrac{d_1}{d_2} = \dfrac{1 + \tan\alpha}{1 - \tan\alpha}$

10. The bubble will detach if,

Buoyant force ≥ Surface tension force

$\frac{4}{3}\pi R^3 \rho_w g \geq \int T \times dl \sin\theta$



$\int \sin\theta \; T \times dl = T(2pr) \sin\theta$

$(\rho_w)\left(\frac{4}{3}\pi R^3\right) g \geq (T)(2\pi r)\sin\theta$

$\sin\theta = \dfrac{r}{R}$

Solving, $\quad r = \sqrt{\dfrac{2\rho_w R^4 g}{3T}} = R^2\sqrt{\dfrac{2\rho_w g}{3T}}$

No option matches with the correct answer.

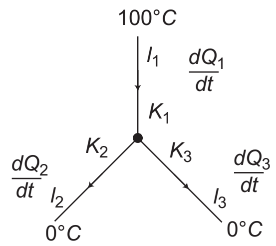
11. In thermal conduction, it is found that in steady state the heat current is directly proportional to the area of cross-section A which is proportional to the change in temperature $(T_1 - T_2)$.

Then, $\quad \dfrac{\Delta Q}{\Delta t} = \dfrac{KA(T_1 - T_2)}{x}$

According to thermal conductivity, we get

$$\frac{dQ_1}{dt} = \frac{dQ_2}{dt} + \frac{dQ_3}{dt}$$

$$\frac{0.92(100 - T)}{46} = \frac{0.26(T - 0)}{13} + \frac{0.12(T - 0)}{12}$$



$\Rightarrow \quad T = 40° \text{ C}$

$\therefore \quad \dfrac{dQ_1}{dt} = \dfrac{0.92 \times 4(100 - 40)}{40} = 4.8$ cal/s

12. According to first law of thermodynamics, we get

(*i*) Change in internal energy from A to B *i.e.*, ΔU_{AB}

$\Delta U_{AB} = nC_V(T_B - T_A)$

$= 1 \times \dfrac{5R}{2}(800 - 400) = 1000 R$

(*ii*) Change in internal energy from B to C

$\Delta U_{BC} = nC_V(T_C - T_B)$

$= 1 \times \dfrac{5R}{2}(600 - 800) = -500 R$

(*iii*) $\Delta U_{\text{isothermal}} = 0$

(*iv*) Change in internal energy from C to A *i.e.*, ΔU_{CA}

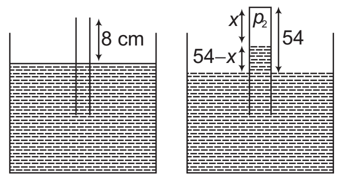
$\Delta U_{CA} = nC_V(T_A - T_C)$

$= 1 \times \dfrac{5R}{2}(400 - 600) = -500 \; R$

13. In this question, the system is accelerating horizontally *i.e.*, no component of acceleration in vertical direction. Hence, the pressure in the vertical direction will remain unaffected.

i.e., $\quad p_1 = p_0 + \rho g h$

Again, we have to use the concept that the pressure in the same level will be same.



For air trapped in tube, $p_1V_1 = p_2V_2$

$p_1 = p_{\text{atm}} = \rho g 76$

$V_1 = A \cdot 8 \qquad$ [A = area of cross-section]

$p_2 = p_{\text{atm}} - \rho g(54 - x) = \rho g(22 + x)$

$V_2 = A \cdot x$

$\rho g 76 \times 8A = \rho g(22 + x)Ax$

$x^2 + 22x - 78 \times 8 = 0$

$\Rightarrow \quad x = 16$ cm

14. In SHM, a particle starts from rest, we have

i.e., $x = A\cos\omega t$, at $t = 0$, $x = A$

When $t = \tau$, then

$$x = A - a \quad \text{....(i)}$$

When $t = 2\tau$, then

$$x = A - 3a \quad \text{....(ii)}$$

On comparing Eqs. (i) and (ii), we get

$$A - a = A\cos\omega\tau$$

$$A - 3a = A\cos 2\omega\tau$$

As $\cos 2\omega\tau = 2\cos^2\omega\tau - 1$

$$\Rightarrow \frac{A - 3a}{A} = 2\left(\frac{A - a}{A}\right)^2 - 1$$

$$\Rightarrow \frac{A - 3a}{A} = \frac{2A^2 + 2a^2 - 4Aa - A^2}{A^2}$$

$$A^2 - 3aA = A^2 + 2a^2 - 4Aa$$

$$a^2 = 2aA$$

$$A = 2a$$

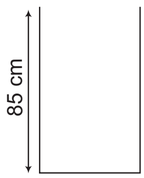
Now, $A - a = A\cos\omega\tau$

$$\Rightarrow \cos\omega\tau = 1/2$$

$$\frac{2\pi}{T}\tau = \frac{\pi}{3}$$

$$T = 6\pi$$

15. For closed organ pipe



$$= \frac{(2n+1)v}{4l} \quad [n = 0, 1, 2, \ldots]$$

$$\frac{(2n+1)v}{4l} < 1250$$

$$(2n+1) < 1250 \times \frac{4 \times 0.85}{340}$$

$$(2n+1) < 12.5 \quad 2n < 11.50$$

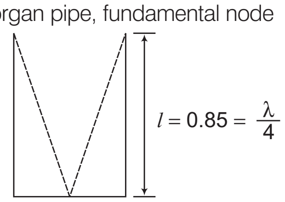
$$n < 5.25$$

So, $n = 0, 1, 2, 3, \ldots, 5$

So, we have 6 possibilities.

Alternate method

In closed organ pipe, fundamental node



i.e., $\frac{\lambda}{4} = 0.85 \Rightarrow \lambda = 4 \times 0.85$

As we know, $\nu = \frac{c}{\lambda} \Rightarrow \frac{340}{4 \times 0.85} = 100$ Hz

∴ Possible frequencies = 100 Hz, 300 Hz, 500 Hz, 700 Hz, 900 Hz, 1100 Hz below 1250 Hz.

16. As we know, potential difference $V_A - V_O$ is

$$dV = -E\,dx$$

$$\int_{V_O}^{V_A} dV = -\int_0^2 30x^2\,dx$$

$$V_A - V_O = -30 \times \left[\frac{x^3}{3}\right]_0^2 = -10 \times [2^3 - (0)^3]$$

$$= -10 \times 8 = -80 \text{ J}$$

17. When free space between parallel plates of capacitor, $E = \frac{\sigma}{\varepsilon_0}$

When dielectric is introduced between parallel plates of capacitor, $E' = \frac{\sigma}{K\varepsilon_0}$

Electric field inside dielectric

$$\frac{\sigma}{K\varepsilon_0} = 3 \times 10^4$$

where, K = dielectric constant of medium = 2.2

ε_0 = permitivity of free space = 8.85×10^{-12}

$$\Rightarrow \sigma = 2.2 \times 8.85 \times 10^{-12} \times 3 \times 10^4$$

$$= 6.6 \times 8.85 \times 10^{-8} = 5.841 \times 10^{-7}$$

$$= 6 \times 10^{-7} \text{ C/m}^2$$

18. Total power (P) consumed

$$= (15 \times 40) + (5 \times 100) + (5 \times 80) + (1 \times 1000) = 2500 \text{ W}$$

As we know,

Power, $P = VI$

$$\Rightarrow I = \frac{2500}{220}\text{ A} = \frac{125}{11} = 11.3 \text{ A}$$

Minimum capacity should be 12 A.

19. When force exerted on a current carrying conductor

$$F_{ext} = BIL$$

Average power = $\frac{\text{Work done}}{\text{Time taken}}$

$$P = \frac{1}{t}\int_0^2 F_{ext.} \cdot dx = \frac{1}{t}\int_0^2 B(x)IL\,dx$$

$$= \frac{1}{5 \times 10^{-3}}\int_0^2 3 \times 10^{-4} e^{-0.2x} \times 10 \times 3\,dx$$

$$= 9\,[1 - e^{-0.4}]$$

$$= 9\left[1 - \frac{1}{e^{0.4}}\right]$$

$$= 2.967$$

$$\approx 2.97 \text{ W}$$

20. For solenoid, the magnetic field needed to be magnetised the magnet.

$$B = \mu_0 nI$$

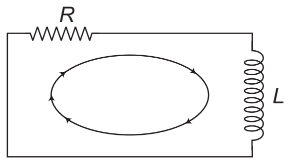
$$\left[\text{Where, } n = 100, l = 10\text{cm} = \frac{10}{100}\text{m} = 0.1 \text{ m}\right]$$

$$\Rightarrow \quad 3 \times 10^3 = \frac{100}{0.1} \times I$$

$$I = 3\text{A}$$

21. After connecting C to B hanging the switch, the circuit will act like an L-R discharging circuit.

Applying Kirchhoff's loop equation,



$$V_R + V_L = 0$$

$$\Rightarrow \quad V_R = -V_L$$

$$\frac{V_R}{V_L} = -1$$

22. Both the energy densities are equal *i.e.*, energy is equally divided between electric and magnetic field.

23. It is based on lens maker's formula and its magnification

i.e., $\frac{1}{f} = (\mu - 1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$.

According to lens maker's formula, when the lens in the air.

$$\frac{1}{f} = \left(\frac{3}{2} - 1\right)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$$

$$\frac{1}{f} = \frac{1}{2x} \Rightarrow f = 2x$$

Here, $\left(\frac{1}{x} = \frac{1}{R_1} - \frac{1}{R_2}\right)$

In case of liquid, where refractive index is $\frac{4}{3}$ and $\frac{5}{3}$, we get

Focal length in first liquid

$$\frac{1}{f_1} = \left(\frac{\mu_s}{\mu_{l_1}} - 1\right)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$$

$$\frac{1}{f_1} = \left(\frac{\frac{3}{2}}{\frac{4}{3}} - 1\right)\frac{1}{x}$$

$\Rightarrow f_1$ is positive.

$$\frac{1}{f_1} = \frac{1}{8x} = \frac{1}{4(2x)} = \frac{1}{4f} \Rightarrow f_1 = 4f$$

Focal length in second liquid

$$\frac{1}{f_2} = \left(\frac{\mu_s}{\mu_{l_2}} - 1\right)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$$

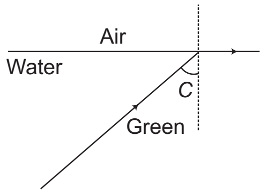
$$\Rightarrow \quad \frac{1}{f_2} = \left(\frac{\frac{3}{2}}{\frac{5}{3}} - 1\right)\left(\frac{1}{x}\right)$$

$\Rightarrow$ f_2 is negative.

24. For total internal reflection of light take place, following conditions must be obeyed.

(*i*) The ray must travel from denser to rarer medium.

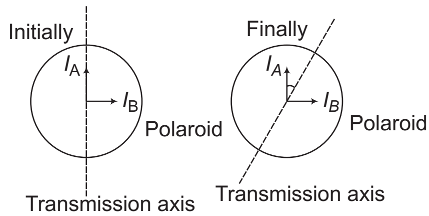
(*ii*) Angle of incidence (θ) must be greater than or equal to critical angle (C) *i.e.*, $C = \sin^{-1}\left[\frac{\mu_{\text{rarer}}}{\mu_{\text{denser}}}\right]$



Here, $\sin C = \frac{1}{n_{\text{water}}}$ and $n_{\text{water}} = a + \frac{b}{\lambda^2}$

If frequency is less $\Rightarrow \lambda$ is greater and hence, RI $n_{(\text{water})}$ is less and therefore, critical angle increases. So, they do not suffer reflection and come out at angle less than 90°.

25. By law of Malus *i.e.*, $I = I_0 \cos^2\theta$



Now, $\quad I_{A'} = I_A \cos^2 30°$

$\quad I_{B'} = I_B \cos^2 60°$

As, $\quad I'_A = I'_B$

$$I_A \cos^2 30° = I_B \cos^2 60°$$

$$\Rightarrow \quad I_A \frac{3}{4} = I_B \frac{1}{4}$$

$$\frac{I_A}{I_B} = \frac{1}{3}$$

26. The problem is based on frequency dependence of photoelectric emission. When incident light with certain frequency (greater than on the threshold frequency is focus on a metal surface) then some electrons are emitted from the metal with substantial initial speed.

When an electron moves in a circular path, then

$$r = \frac{mv}{eB} \Rightarrow \frac{r^2e^2B^2}{2} = \frac{m^2v^2}{2}$$

$$\text{KE}_{\max} = \frac{(mv)^2}{2m} \Rightarrow \frac{r^2e^2B^2}{2m} = (\text{KE})_{\max}$$

Work function of the metal (W),

i.e., $W = h\nu - KE_{max}$

$$1.89 - \phi = \frac{r^2e^2B^2}{2m}\frac{1}{2}\text{eV} = \frac{r^2eB^2}{2m}\text{eV}$$

[$h\nu \rightarrow$ 1.89 eV, for the transition on from third to second orbit of H-atom]

$$= \frac{100\times10^{-6}\times1.6\times10^{-19}\times9\times10^{-8}}{2\times9.1\times10^{-31}}$$

$$\phi = 1.89 - \frac{1.6\times9}{2\times9.1}$$

$= 1.89 - 0.79$

$= 1.1$ eV

27. For hydrogen atom, we get

$$\frac{1}{\lambda} = Rz^2\left(\frac{1}{1^2} - \frac{1}{2^2}\right)$$

$$\frac{1}{\lambda_1} = R(1)^2\left(\frac{3}{4}\right)$$

$$\frac{1}{\lambda_2} = R(1)^2\left(\frac{3}{4}\right)$$

$$\frac{1}{\lambda_3} = R(2)^2\left(\frac{3}{4}\right)$$

$$\frac{1}{\lambda_4} = R(3)^2\left(\frac{3}{4}\right)$$

$$\frac{1}{\lambda_1} = \frac{1}{4\lambda_3} = \frac{1}{9\lambda_4} = \frac{1}{\lambda_2}$$

28. For forward bias, p-side must be a higher potential than n-side.

So, is forward biased.

29. If student measures 3.50 cm, it means that there in an uncertainly of order 0.01 cm.

For vernier scale with 1 MSD = 1 mm

and 9 MSD = 10 VSD

LC of Vernier calliper

$= 1\text{MSD} - 1\text{VSD}$

$$= \frac{1}{10}\left(1 - \frac{9}{10}\right)$$

$$= \frac{1}{100}\text{ cm}$$

30. (a) Infrared rays are used to treat muscular strain.

(b) Radiowaves are used for broadcasting purposes.

(c) X-rays are used to detect fracture of bones.

(d) Ultraviolet rays are absorbed by ozone.

Chemistry

31. Given, atomic number of Rb, $Z = 37$

Thus, its electronic configuration is [Kr]$5s^1$. Since the last electron or valence electron enter in $5s$ subshell.

So, the quantum numbers are $n = 5$, $l = 0$, (for s orbital) $m = 0\,(\because m = +l \text{ to } -l)$, $s = +\frac{1}{2}$ or $-\frac{1}{2}$

32. To solve this problem, the stepwise approach required *i.e.,*

(*i*) Write the van der Waals' equation, then apply the condition that at low pressure, volume become high,

i.e., $V - b \simeq V$

(*ii*) Now calculate the value of compressibility factor (Z).

$[Z = pV/RT]$

According to van der Waals' equation,

$$\left(p + \frac{a}{V^2}\right)(V - b) = RT$$

At low pressure, $\left(p + \frac{a}{V^2}\right)V = RT$

$$pV + \frac{a}{V} = RT$$

$$pV = RT - \frac{a}{V}$$

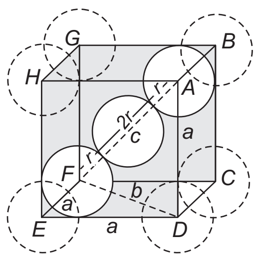
Divide both side by RT, $\frac{pV}{RT} = 1 - \frac{a}{RTV}$

33. In CsCl, Cl^- lie at corners of simple cube and Cs^+ at the body centre, Hence, along the body diagonal, Cs^+ and Cl^- touch each other so

$$r_{Cs^+} + r_{Cl^-} = 2r$$

Calculation of r

In ΔEDF,



Body centred cubic unit cell

$$FD = b = \sqrt{a^2 + a^2} = \sqrt{2}a$$

In ΔAFD,

$$c^2 = a^2 + b^2 = a^2 + (\sqrt{2}a)^2 = a^2 + 2a^2$$

$$c^2 = 3a^2$$

$$c = \sqrt{3}a$$

As ΔAFD is an equilateral triangle.

$$\therefore \quad \sqrt{3}a = 4r \Rightarrow r = \frac{\sqrt{3}a}{4}$$

Hence, $r_{Cs^+} + r_{Cl^-} = 2r = 2 \times \frac{\sqrt{3}}{4}a = \frac{\sqrt{3}}{2}a$

34. This problem is based on the estimation of percentage of N in organic compound using Kjeldahl's method. Use the concept of stoichiometry and follow the steps given below to solve the problem.

(*i*) Write the balanced chemical reaction for the conversion of N present in organic compound to ammonia, ammonia to ammonium sulphate and ammonium sulphate to sodium sulphate.

(*ii*) Calculate millimoles (m moles) of N present in organic compound followed by mass of N present in organic compound using the concept of stoichiometry.

(*iii*) At last, calculate % of N present in organic compound using formula

$$\% \text{ of N} = \frac{\text{Mass of N} \times 100}{\text{Mass of organic compound}}$$

Mass of organic compound = 1.4 g

Let it contain x mmole of N atom.

$$\text{Organic compound} \longrightarrow \underset{x \text{ mmole}}{NH_3}$$

$$2NH_3 + \underset{\substack{\text{6 m mole}\\ \text{initially taken}}}{H_2SO_4} \longrightarrow (NH_4)_2SO_4 \quad \ldots\text{(i)}$$

$$H_2SO_4 + 2\,NaOH \longrightarrow Na_2SO_4 + 2\,H_2O \quad \ldots\text{(ii)}$$

2 mmole NaOH reacted.

Hence, mmoles of H_2SO_4 reacted in Eq. (ii) = 1

$\Rightarrow$ mmoles of H_2SO_4 reacted from Eq. (i)

$= 6 - 1 = 5$ mmoles

$\Rightarrow$ mmoles of NH_3 in Eq. (i) $= 2 \times 5$

$= 10$ mmoles

$\Rightarrow$ mmoles of N atom in the organic compound

$= 10$ mmoles

$\Rightarrow$ Mass of N $= 10 \times 10^{-3} \times 14 = 0.14$ g

$$\% \text{ of N} = \frac{\text{Mass of N present in organic compound}}{\text{Mass of organic compound}} \times 100$$

$$\Rightarrow \quad \% \text{ of N} = \frac{0.14}{1.4} \times 100 = 10\%$$

35. In order to solve the problem, calculate the value of cell constant of the first solution and then use this value of cell constant to calculate the value of k of second solution. Afterwards, finally calculate molar conductivity using values of k and m.

For first solution,

$$k = 1.4 \text{ Sm}^{-1},\ R = 50\ \Omega,\ M = 0.2$$

Specific conductance $(\kappa) = \frac{1}{R} \times \frac{l}{A}$

$$1.4 \text{ Sm}^{-1} = \frac{1}{50} \times \frac{l}{A}$$

$$\frac{l}{A} = 50 \times 1.4 \text{ m}^{-1}$$

For second solution,

$$R = 280, \frac{l}{A} = 50 \times 1.4 \text{ m}^{-1}$$

$$\kappa = \frac{1}{280} \times 1.4 \times 50 = \frac{1}{4}$$

Now, molar conductivity

$$\lambda_m = \frac{\kappa}{1000 \times m} = \frac{1/4}{1000 \times 0.5} = \frac{1}{2000}$$

$$= 5 \times 10^{-4} \text{ Sm}^2 \text{ mol}^{-1}$$

36. $C_2H_5OH(l) + 3O_2(g) \longrightarrow 2CO_2(g) + 3H_2O(l)$

$\Delta U = -1364.47$ kJ/mol

$\Delta H = \Delta U + \Delta n_g RT$

$\Delta n_g = -1$

$$\Delta H = -1364.47 + \frac{-1 \times 8.314 \times 298}{1000}$$

[Here, value of R in unit of J must be converted into kJ]

$= -1364.47 - 2.4776$

$= -1366.94$ kJ/mol

During solving such problem, students are advised to keep much importance in unit conversion. As here, value of R ($8.314 \text{ J K}^{-1}\text{mol}^{-1}$) in $\text{JK}^{-1}\text{mol}^{-1}$ must be converted into kJ by dividing the unit by 1000.

37. According to Debye Huckel Onsager equation,

$$\lambda_C = \lambda_\infty - B\sqrt{C}$$

where, λ_C = limiting equivalent conductivity at concentration C

λ_∞ = limiting equivalent conductivity at infinite dilution

C = concentration

38. This problem includes concept of colligative properties (osmotic pressure here) and van't Hoff factor. Calculate the effective molarity of each solution.

i.e., effective molarity

= van't Hoff factor × molarity

0.5 M $C_2H_5OH(aq)$ $\quad i = 1$

Effective molarity = 0.5

0.25 M KBr (aq) $\quad i = 2$

Effective molarity = 0.5 M

0.1 M $Mg_3(PO_4)_2(aq)$ $\quad i = 5$

Effective molarity = 0.5 M

0.125 M $Na_3PO_4(aq)$ $i = 4$

Effective molarity = 0.5 M

Hence, all colligative properties are same.

NOTE

$Mg_3(PO_4)_2$

39. For the given reaction, $\Delta n_g = n_p - n_R$

where, n_p = number of moles of products

n_R = number of moles of reactants

$$K_p = K_c (RT)^{\Delta n_g}$$

$$\Delta n_g = -\frac{1}{2}$$

40. This problem can be solved by determining the order of reaction w.r.t. each reactant and then writing rate law equation of the given equation accordingly as

$$r = \frac{dC}{dt} = k[A]^x[B]^y$$

where, x = order of reaction w.r.t. A

y = order of reaction w.r.t. B

$$1.2 \times 10^{-3} = k(0.1)^x(0.1)^y$$

$$1.2 \times 10^{-3} = k(0.1)^x(0.2)^y$$

$$2.4 \times 10^{-3} = k(0.2)^x(0.1)^y$$

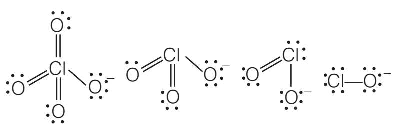
$$R = k[A]^1[B]^0$$

As shown above, rate of reaction remains constant as the concentration of reactant (B) changes from 0.1 M to 0.2 M and becomes double when concentration of A change from 0.1 to 0.2 (*i.e.,* doubled).

41. Decreasing order of strength of oxoacids

$HClO_4 > HClO_3 > HClO_2 > HOCl$

Reason Consider the structures of conjugate bases of each oxyacids of chlorine.



Negative charge is more delocalised on ClO_4^- due to resonance, hence ClO_4^- is more stable (and less basic).

Hence, we can say as the number of oxygen atom(s) around Cl-atom increases as oxidation number of Cl-atom increases and thus, the ability of loose the H^+ increases.

42. Higher the position of element in the electro-chemical series more difficult is the reduction of its cations.

If $Ca^{2+}(aq)$ is electrolysed, water is reduced in preference to it. Hence, it cannot be reduced electrolytically from their aqueous solution.

$$Ca^{2+}(aq) + H_2O \longrightarrow Ca^{2+} + OH^- + H_2\uparrow$$

43. Arrange the complex formed by different ligands L_1, L_2, L_3 and L_4, according to wavelength of their absorbed light, then use the following relation to answer the question.

Ligand field strength

$$\propto \text{Energy of light absorbed} \propto \frac{1}{\text{Wavelength of light absorbed}}$$

λ	L_1	L_2	L_3	L_4
Absorbed light	Red	Green	Yellow	Blue

Wavelength of absorbed light decreases

∴ Increasing order of energy of wavelengths absorbed reflect greater extent of crystal-field splitting, hence higher field strength of the ligand.

Energy Blue (L_4) > green (L_2) > yellow (L_3) > red (L_1)

∴ $L_4 > L_2 > L_3 > L_1$ in field strength of ligands.

44. NO is paramagnetic in gaseous state because in gaseous state, it has one unpaired electron.

Total number of electron present = 7 + 8

$= 15\,e^-$

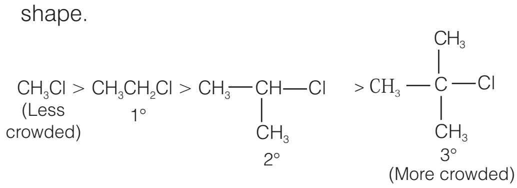
Hence, there must be the presence of unpaired electron in gaseous state while in liquid state, it dimerises due to unpaired electron.

45. Release of electron is known as reduction. So, H_2O_2 acts as reducing agent when it releases electrons.

Here, in reactions (II) and (IV), H_2O_2 releases two electron, hence reactions (II) and (IV) is known as reduction.

In reactions (I) and (III), two electrons are being added, so (I) and (III) represents oxidation.

46. I_3^- is an ion made up of I_2 and I^- which has linear shape.



while Cs^+ is an alkali metal cation.

47. $\dfrac{n_{O_2}}{n_{N_2}} = \dfrac{\dfrac{(m_{O_2})}{(M_{O_2})}}{\dfrac{(m_{N_2})}{(M_{N_2})}}$

where, m_{O_2} = given mass of O_2
m_{N_2} = given mass of N_2
M_{O_2} = molecular mass of O_2
M_{N_2} = molecular mass of N_2
n_{O_2} = number of moles of O_2
n_{N_2} = number of moles of N_2

$$= \left[\frac{m_{O_2}}{m_{N_2}}\right]\frac{28}{32} = \frac{1}{4} \times \frac{28}{32} = \frac{7}{32}$$

48. Standard electrode potential of reaction $[E^0]$ can be calculated as

$$E^0_{cell} = E_R - E_P$$

where, E_R = SRP of reactant
E_P = SRP of product

If $E^0_{cell} = +$ve, then reaction is spontaneous otherwise non-spontaneous.

$$Mn^{3+} \xrightarrow{E_1^\circ = 1.51V} Mn^{2+}$$

$$Mn^{2+} \xrightarrow{E_2^\circ = -1.18V} Mn$$

∴ For Mn^{2+} disproportionation,

$$E^\circ = -1.51\,V - 1.18\,V = -2.69\,V < 0$$

49. **Analyse each reaction given in the question and choose the correct answer on the basis of oxidation state and stability of iron compounds. Use the concept of Ellingham diagram to solve this problem.**

The correct reactions are as follows

(a) $Fe + \text{dil } H_2SO_4 \longrightarrow FeSO_4 + H_2$

$$H_2SO_4 + 2FeSO_4 + \frac{1}{2}O_2 \longrightarrow Fe_2(SO_4)_3 + H_2O$$

$$Fe_2(SO_4)_3 \xrightarrow{\Delta} Fe_2O_3\,(s) + 3SO_3\uparrow$$

The given reaction is incorrect in question

(b) $Fe \xrightarrow[\Delta]{O_2} FeO$

[It could also be Fe_2O_3 or Fe_3O_4]

$$FeO + H_2SO_4 \longrightarrow FeSO_4 + H_2O$$

$$2FeSO_4 \xrightarrow{\Delta} Fe_2O_3 + SO_2 + SO_3$$

(c) $Fe \xrightarrow[Cl_2]{\Delta} FeCl_3 \xrightarrow[Air]{\Delta}$ No reaction

[It cannot give $FeCl_2$]

(d) $Fe \xrightarrow[\Delta]{O_2} Fe_3O_4 \xrightarrow[600^\circ C]{CO} FeO \xrightarrow[700^\circ C]{CO} Fe$

This is correct reaction.

50. **This problem is based on conceptual mixing of properties of lithium oxide and preparation, properties of coordination compounds. To answer this question, keep in mind that on adding acid ammine complexes get destroyed.**

(a) $Li_2O + KCl \longrightarrow 2LiCl + K_2O$

This is wrong equation, since a stronger base K_2O cannot be generated by a weaker base Li_2O.

(b) $[CoCl(NH_3)_5]^+ + 5H^+ \longrightarrow Co^{2+}(aq) + 5NH_4^+ + Cl^-$

This is correct. All ammine complexes can be destroyed by adding $H^\oplus$. Hence, on adding acid to $[CoCl(NH_3)_5]$, It gets converted to Co^{2+} (*aq*) NH_4^+ and Cl^-.

(c) $[Mg(H_2O)_6]^{2+} + EDTA^{4-} \xrightarrow[excess]{OH^-} [Mg(EDTA)]^{2+} + 6H_2O$

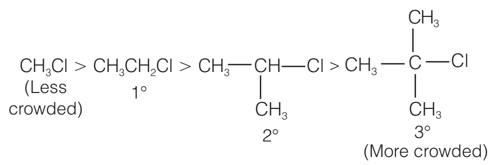
This is wrong, since the formula of complex must be $[Mg(EDTA)]^{2+}$ as EDT.

(d) The 4th reaction is incorrect. It can be correctly represented as

$$2CuSO_4 + 10KCN \longrightarrow 2K_3[Cu(CN)_4] + 2K_2SO_4 + (CN)_2\uparrow$$

51. **Steric hindrance (crowding) is the basis of S_N2 reaction, by using which we can arrange the reactant in correct order of their reactivity towards S_N2 reaction.**

$$\text{Rate of } S_N2 \propto \frac{1}{\text{Steric crowding of 'C'}}$$



As steric hinderance (crowding) increases, rate of S_N2 reaction decreases.

NOTE S_N2

52. This reaction is an example of carbylamine reaction which includes conversion of amine to isocyanide.

$$\underset{1^\circ \text{ amine}}{R—\ddot{N}H_2} + CHCl_3 \xrightarrow[KOH]{C_2H_5OH} \underset{\text{Alkyl isocyanide}}{R—\overset{+}{N}\equiv\overset{-}{C}:}$$

NOTE

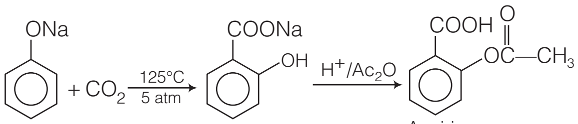
53. $R — CH_2OH \xrightarrow{PCC} R — CH = O$

Pyridinium chlorochromate is the mild oxidising agent which causes conversion of alcohol to aldehyde stage. While others causes conversion of alcohol to acid.

54. The reaction is

$$2CH_3—CCl_3 \xrightarrow[\Delta]{6Ag} \underset{\text{But-2-yne}}{CH_3—C\equiv C—CH_3} + 6AgCl$$

55. It is a Kolbe Schmidt reaction.



Aspirin
(pain killer)

The second step of the reaction is an example of acetylation reaction.

56. **This problem can be solved by using the concept of effect of steric hindrance, hydration and H-bonding in basic strength of amines.**

Order of basic strength of aliphatic amine in aqueous solution is as follows (order of K_b)

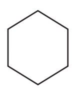
$$(CH_3)_2\ddot{N}H > CH_3\ddot{N}H_2 > (CH_3)_3\ddot{N} > C_6H_5\ddot{N}H_2$$

As we know, $pK_b = -\log K_b$

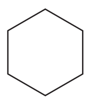
So, $(CH_3)_2\ddot{N}H$ will have smallest pK_b value.

In case of phenyl amine, N is attached to sp^2 hybridised carbon, hence it has highest pK_b and least basic strength.

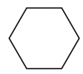
57. **Draw the structure of organic compounds indicating net dipole moment which includes lone pair and bond angle also.**



—OH groups and —SH groups do not cancel their dipole moment as they exist in different confirmation.



58. Dacron is a condensation polymer of ethylene glycol and methyl terepthalate. Formation of dacron can be shown as



Here, elimination of MeOH occurs as a by product. So, this reaction is known as condensation polymerisation.

59. Quinoline is an alkaloid, it is not present in DNA. DNA has four nitrogen bases in adenine, guanine, cytosine and thymine.

60. **This problem is based on successive reduction, chlorination and elimination reaction. To solve such problem, use the function of the given reagents.**

(*i*) $LiAlH_4$ causes reduction

(*ii*) PCl_5 causes chlorination

(*iii*) Alc. KOH causes elimination reaction

$$CH_3COOH \xrightarrow{LiAlH_4} \underset{(A)}{CH_3CH_2OH}$$

$$\xrightarrow{PCl_5} \underset{(B)}{CH_3CH_2Cl} \xrightarrow[-HCl]{Alc.KOH} \underset{\substack{(C)\\ \text{Ethylene}}}{CH_2{=}CH_2}$$

Mathematics

61. We have, $X = \{4^n - 3n - 1 : n \in N\}$

$X = \{0, 9, 54, 243, ...\}$ [Put $n = 1, 2, 3, ...$]

$Y = \{9(n-1) : n \in N\}$

$Y = \{0, 9, 18, 27, ...\}$ [Put $n = 1, 2, 3, ...$]

It is clear that $X \subset Y$.

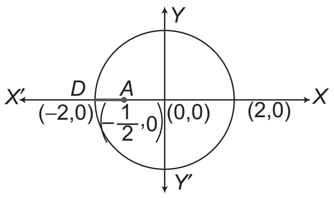
$\therefore \quad X \cup Y = Y$

62. $|z| \geq 2$ is the region on or outside circle whose centre is (0, 0) and radius is 2.

Minimum $\left|z + \frac{1}{2}\right|$ is distance of z, which lie on circle $|z| = 2$ from $\left(-\frac{1}{2}, 0\right)$.

$\therefore$ Minimum $\left|z + \frac{1}{2}\right| =$ Distance of $\left(-\frac{1}{2}, 0\right)$ from (–2, 0)

$$= \sqrt{\left(-2+\frac{1}{2}\right)^2 + 0} = \frac{3}{2} = \sqrt{\left(\frac{-1}{2}+2\right)^2 + 0} = \frac{3}{2}$$



Geometrically Min $\left|z+\frac{1}{2}\right| = AD$

Hence, minimum value of $\left(z+\frac{1}{2}\right)$ lies in the interval (1, 2).

63. **Put $t = x - [x] = \{X\}$, which is a fractional part function and lie between $0 \le \{X\} < 1$ and then solve it.**

Given, $a \in R$ and equation is

$-3\{x-[x]\}^2 + 2\{x-[x]\} + a^2 = 0$

Let $t = x - [x]$, then equation is

$$-3t^2 + 2t + a^2 = 0 \Rightarrow t = \frac{1 \pm \sqrt{1+3a^2}}{3}$$

$\because \quad t = x - [x] = \{X\}$ [Fractional part]

$\therefore \quad 0 \le t \le 1$

$$0 \le \frac{1 \pm \sqrt{1+3a^2}}{3} \le 1$$

Taking positive sign, we get

$$0 \le \frac{1+\sqrt{1+3a^2}}{3} < 1 \qquad [\because \{x\} > 0]$$

$\Rightarrow \sqrt{1+3a^2} < 2 \Rightarrow 1 + 3a^2 < 4 \Rightarrow a^2 - 1 < 0$

$\Rightarrow (a+1)(a-1) < 0$

+ +
−1 1

$a \in (-1, 1)$

For no integral solution of a, we consider the interval $(-1, 0) \cup (0, 1)$.

NOTE $a = 0$

64. **If $ax^2 + bx + c = 0$ has roots α and β, then $\alpha + \beta = \frac{-b}{a}$ and $\alpha\beta = \frac{c}{a}$. Find the values of $\alpha + \beta$ and $\alpha\beta$ and then put in $(\alpha - \beta)^2 = (\alpha + \beta)^2 - 4\alpha\beta$ to get required value.**

Given, α and β are roots of $px^2 + qx + r = 0$, $p \ne 0$.

$$\therefore \quad \alpha + \beta = \frac{-q}{p}, \quad \alpha\beta = \frac{r}{p} \qquad \text{...(i)}$$

Since, p, q and r are in AP.

$$\therefore \quad 2q = p + r \qquad \text{...(ii)}$$

Also, $\frac{1}{\alpha} + \frac{1}{\beta} = 4$

$\Rightarrow \frac{\alpha+\beta}{\alpha\beta} = 4$

$\Rightarrow \alpha + \beta = 4\alpha\beta$

$\Rightarrow \frac{-q}{p} = \frac{4r}{p}$ [From Eq. (i)]

$\Rightarrow q = -4r$

On putting the value of q in Eq. (ii), we get

$\Rightarrow 2(-4r) = p + r \Rightarrow p = -9r$

Now, $\alpha + \beta = \frac{-q}{p} = \frac{4r}{p} = \frac{4r}{-9r} = -\frac{4}{9}$

and $\alpha\beta = \frac{r}{p} = \frac{r}{-9r} = \frac{1}{-9}$

$\therefore \quad (\alpha-\beta)^2 = (\alpha+\beta)^2 - 4\alpha\beta = \frac{16}{81} + \frac{4}{9} = \frac{16+36}{81}$

$\Rightarrow (\alpha-\beta)^2 = \frac{52}{81} \Rightarrow |\alpha - \beta| = \frac{2}{9}\sqrt{13}$

65. **Use the property that, two determinants can be multiplied row-to-row or row-to-column, to write the given determinant as the product of two determinants and then expand.**

Given, $f(n) = \alpha^n + \beta^n$

$f(1) = \alpha + \beta$, $f(2) = \alpha^2 + \beta^2$,

$f(3) = \alpha^3 + \beta^3$, $f(4) = \alpha^4 + \beta^4$

Let
$$\Delta = \begin{vmatrix} 3 & 1+f(1) & 1+f(2) \\ 1+f(1) & 1+f(2) & 1+f(3) \\ 1+f(2) & 1+f(3) & 1+f(4) \end{vmatrix}$$

$$\Rightarrow \Delta = \begin{vmatrix} 3 & 1+\alpha+\beta & 1+\alpha^2+\beta^2 \\ 1+\alpha+\beta & 1+\alpha^2+\beta^2 & 1+\alpha^3+\beta^3 \\ 1+\alpha^2+\beta^2 & 1+\alpha^3+\beta^3 & 1+\alpha^4+\beta^4 \end{vmatrix}$$

$$= \begin{vmatrix} 1\cdot1+1\cdot1+1\cdot1 & 1\cdot1+1\cdot\alpha+1\cdot\beta & 1\cdot1+1\cdot\alpha^2+1\cdot\beta^2 \\ 1\cdot1+1\cdot\alpha+1\cdot\beta & 1\cdot1+\alpha\cdot\alpha+\beta\cdot\beta & 1\cdot1+\alpha\cdot\alpha^2+\beta\cdot\beta^2 \\ 1\cdot1+1\cdot\alpha^2+1\cdot\beta^2 & 1\cdot1+\alpha^2\cdot\alpha+\beta^2\cdot\beta & 1\cdot1+\alpha^2\cdot\alpha^2+\beta^2\cdot\beta^2 \end{vmatrix}$$

$$= \begin{vmatrix} 1 & 1 & 1 \\ 1 & \alpha & \beta \\ 1 & \alpha^2 & \beta^2 \end{vmatrix} \begin{vmatrix} 1 & 1 & 1 \\ 1 & \alpha & \beta \\ 1 & \alpha^2 & \beta^2 \end{vmatrix} = \begin{vmatrix} 1 & 1 & 1 \\ 1 & \alpha & \beta \\ 1 & \alpha^2 & \beta^2 \end{vmatrix}^2$$

On expanding, we get

$$\Delta = (1-\alpha)^2(1-\beta)^2(\alpha-\beta)^2$$

But given, $\Delta = K(1-\alpha)^2(1-\beta)^2(\alpha-\beta)^2$

Hence, $K(1-\alpha)^2(1-\beta)^2(\alpha-\beta)^2$

$$= (1-\alpha)^2(1-\beta)^2(\alpha-\beta)^2$$

$\therefore \quad K = 1$

66. **Use the following properties of transpore $(AB)^T = B^T A^T$, $(A^T)^T = A$ and $A^{-1}A = I$ and simplify.**

If A is non-singular matrix, then $|A| \neq 0$

$$AA^T = A^T A \text{ and } B = A^{-1}A^T$$

$$BB^T = (A^{-1}A^T)(A^{-1}A^T)^T$$
$$= A^{-1}A^T A(A^{-1})^T \quad [\because (AB)^T = B^T A^T]$$
$$= A^{-1}AA^T(A^{-1})^T \quad [\because AA^T = A^T A]$$
$$= IA^T(A^{-1})^T \quad [\because A^{-1}A = I]$$
$$= A^T(A^{-1})^T$$
$$= (A^{-1}A)^T \quad [\because (AB)^T = B^T A^T]$$
$$= I^T = I$$

67. **To find the coefficient of x^3 and x^4, use the formula of coefficient of x^r in $(1-x)^n$ is $(-1)^r\, {}^nC_r$ and then simplify.**

In expansion of $(1 + ax + bx^2)(1 - 2x)^{18}$,

Coefficient of x^3 = Coefficient of x^3 in $(1-2x)^{18}$
+ Coefficient of x^2 in $a(1-2x)^{18}$
+ Coefficient of x in $b(1-2x)^{18}$

$$= -{}^{18}C_3 \cdot 2^3 + a\,{}^{18}C_2 \cdot 2^2 - b\,{}^{18}C_1 \cdot 2$$

Given, coefficient of $x^3 = 0$

$$= {}^{18}C_3 \cdot 2^3 + a\,{}^{18}C_2 \cdot 2^2 - b\,{}^{18}C_1 \cdot 2 = 0$$

$$\Rightarrow \quad -\frac{18 \times 17 \times 16}{3 \times 2} \cdot 8 + a \cdot \frac{18 \times 17}{2} \cdot 2^2 - b \cdot 18 \cdot 2 = 0$$

$$\Rightarrow \quad 17a - b = \frac{34 \times 16}{3} \quad \text{...(i)}$$

Similarly, coefficient of x^4

$${}^{18}C_4 \cdot 2^4 - a \cdot {}^{18}C_3 2^3 + b \cdot {}^{18}C_2 \cdot 2^2 = 0$$

$$\therefore \quad 32a - 3b = 240 \quad \text{...(ii)}$$

On solving Eqs. (i) and (ii), we get

$$a = 16, \quad b = \frac{272}{3}$$

68. We know that angle between two lines is

$$\cos\theta = \frac{a_1a_2 + b_1b_2 + c_1c_2}{\sqrt{a_1^2 + b_1^2 + c_1^2}\sqrt{a_2^2 + b_2^2 + c_2^2}}$$

$$l + m + n = 0$$
$$\Rightarrow \quad l = -(m+n)$$
$$\Rightarrow \quad (m+n)^2 = l^2$$
$$\Rightarrow \quad m^2 + n^2 + 2mn = m^2 + n^2 \quad [\because l^2 = m^2 + n^2, \text{ given}]$$
$$\Rightarrow \quad 2mn = 0$$

When $m = 0 \Rightarrow l = -n$

Hence, (l, m, n) is $(1, 0, -1)$.

When $n = 0$, then $l = -m$

Hence, (l, m, n) is $(1, 0, -1)$.

$$\therefore \quad \cos\theta = \frac{1 + 0 + 0}{\sqrt{2} \times \sqrt{2}} = \frac{1}{2} \quad \Rightarrow \quad \theta = \frac{\pi}{3}$$

69. Given,

$$k \cdot 10^9 = 10^9 + 2(11)^1(10)^8 + 3(11)^2(10)^7 + \dots + 10(11)^9$$

$$k = 1 + 2\left(\frac{11}{10}\right) + 3\left(\frac{11}{10}\right)^2 + \dots + 10\left(\frac{11}{10}\right)^9 \quad \text{...(i)}$$

$$\left(\frac{11}{10}\right)k = 1\left(\frac{11}{10}\right) + 2\left(\frac{11}{10}\right)^2 + \dots + 9\left(\frac{11}{10}\right)^9 + 10\left(\frac{11}{10}\right)^{10} \quad \text{...(ii)}$$

On subtracting Eq. (ii) from Eq. (i), we get

$$k\left(1 - \frac{11}{10}\right) = 1 + \frac{11}{10} + \left(\frac{11}{10}\right)^2 + \dots + \left(\frac{11}{10}\right)^9 - 10\left(\frac{11}{10}\right)^{10}$$

$$\Rightarrow \quad k\left(\frac{10-11}{10}\right) = \frac{1\left[\left(\frac{11}{10}\right)^{10} - 1\right]}{\left(\frac{11}{10} - 1\right)} - 10\left(\frac{11}{10}\right)^{10}$$

$$\left[\because \text{In GP, sum of } n \text{ terms} = \frac{a(r^n - 1)}{r - 1}, \text{when } r > 1\right]$$

$$\Rightarrow \quad -k = 10\left[10\left(\frac{11}{10}\right)^{10} - 10 - 10\left(\frac{11}{10}\right)^{10}\right]$$

$$\therefore \quad k = 100$$

70. Let a, ar, ar^2 be in GP $(r > 1)$.

On multiplying middle term by 2, $a, 2ar, ar^2$ are in AP.

$$\Rightarrow \quad 4ar = a + ar^2$$
$$\Rightarrow \quad r^2 - 4r + 1 = 0$$
$$\Rightarrow \quad r = \frac{4 \pm \sqrt{16 - 4}}{2} = 2 \pm \sqrt{3}$$
$$\Rightarrow \quad r = 2 + \sqrt{3} \quad [\because \text{AP is increasing}]$$

71. $$\lim_{x \to 0} \frac{\sin(\pi\cos^2 x)}{x^2}$$

$$= \lim_{x \to 0} \frac{\sin \pi(1 - \sin^2 x)}{x^2} = \lim_{x \to 0} \frac{\sin(\pi - \pi\sin^2 x)}{x^2}$$

$$= \lim_{x \to 0} \frac{\sin(\pi \sin^2 x)}{x^2} \quad [\because \sin(\pi - \theta) = \sin\theta]$$

$$= \lim_{x \to 0} \frac{\sin \pi \sin^2 x}{\pi \sin^2 x} \times (\pi)\left(\frac{\sin^2 x}{x^2}\right) = \pi \left[\because \lim_{\theta \to 0} \frac{\sin\theta}{\theta} = 1\right]$$

72. Here, 'g' is the inverse of f(x).

$\Rightarrow \quad fog(x) = x$

On differentiating w.r.t. x, we get

$f'\{g(x)\} \times g'(x) = 1$

$$g'(x) = \frac{1}{f'(g(x))} = \frac{1}{\dfrac{1}{1+\{g(x)\}^5}} \qquad \left[\because f'(x) = \frac{1}{1+x^5}\right]$$

$g'(x) = 1 + \{g(x)\}^5$

73. Given, $f(0) = 2 = g(1), g(0) = 0$ and $f(1) = 6$

f and g are differentiable in (0, 1).

Let $\quad h(x) = f(x) - 2g(x) \qquad \ldots$(i)

$h(0) = f(0) - 2g(0)$

$h(0) = 2 - 0$

$h(0) = 2$

and $\quad h(1) = f(1) - 2g(1) = 6 - 2(2)$

$h(1) = 2, \ h(0) = h(1) = 2$

Hence, using Rolle's theorem,

$h'(c) = 0$, such that $c \in (0, 1)$

Differentiating Eq. (i) at c, we get

$\Rightarrow \quad f'(c) - 2g'(c) = 0$

$\Rightarrow \quad f'(c) = 2g'(c)$

74. Here, $x = -1$ and $x = 2$ are extreme points of $f(x) = \alpha \log|x| + \beta x^2 + x$, then

$$f'(x) = \frac{\alpha}{x} + 2\beta x + 1$$

$f'(-1) = -\alpha - 2\beta + 1 = 0 \qquad \ldots$(i)

[At extreme point, $f'(x) = 0$]

$f'(2) = \dfrac{\alpha}{2} + 4\beta + 1 = 0 \qquad \ldots$(ii)

On solving Eqs. (i) and (ii), we get

$\alpha = 2, \ \beta = -\dfrac{1}{2}$

75. $\displaystyle\int\left(1 + x - \frac{1}{x}\right)e^{x+\frac{1}{x}}dx$

$$= \int e^{x+\frac{1}{x}}dx + \int x\left(1 - \frac{1}{x^2}\right)e^{x+\frac{1}{x}}dx$$

$$= \int e^{x+\frac{1}{x}}dx + xe^{x+\frac{1}{x}} - \int \frac{d}{dx}(x)e^{x+\frac{1}{x}}dx$$

$$= \int e^{x+\frac{1}{x}}dx + xe^{x+\frac{1}{x}} - \int e^{x+\frac{1}{x}}\,dx$$

$$\left[\because \int\left(1 - \frac{1}{x^2}\right)e^{x+\frac{1}{x}}dx = e^{x+\frac{1}{x}}\right]$$

$$= \int e^{x+1/x}dx + xe^{x+1/x} - \int ex^{+1/x}dx$$

$$= xe^{x+\frac{1}{x}} + C$$

76. Use the formula, $|x - a| = \begin{cases} x - a, & x \ge a \\ -(x - a), & x < a \end{cases}$

to break given integral in two parts and then integrate separately.

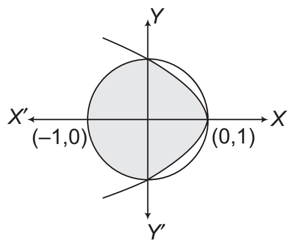
$$\int_0^{\pi}\sqrt{\left(1 - 2\sin\frac{x}{2}\right)^2}dx = \int_0^{\pi}\left|1 - 2\sin\frac{x}{2}\right|dx$$

$$= \int_0^{\frac{\pi}{3}}\left(1 - 2\sin\frac{x}{2}\right)dx - \int_{\frac{\pi}{3}}^{\pi}\left(1 - 2\sin\frac{x}{2}\right)dx$$

$$= \left(x + 4\cos\frac{x}{2}\right)_0^{\frac{\pi}{3}} - \left(x + 4\cos\frac{x}{2}\right)_{\frac{\pi}{3}}^{\pi}$$

$$= 4\sqrt{3} - 4 - \frac{\pi}{3}$$

77. Given, $A = \{(x, y) : x^2 + y^2 \le 1 \text{ and } y^2 \le 1 - x\}$



$$\text{Required area} = \frac{1}{2}\pi r^2 + 2\int_0^1 (1 - y^2)dy$$

$$= \frac{1}{2}\pi(1)^2 + 2\left(y - \frac{y^3}{3}\right)_0^1$$

$$= \frac{\pi}{2} + \frac{4}{3}$$

78. Given differential equation $\dfrac{dp}{dt} - \dfrac{1}{2}p(t) = -200$ is a linear differential equation.

Here, $p(t) = \dfrac{-1}{2}, Q(t) = -200$

$$\text{IF} = e^{\int -\left(\frac{1}{2}\right)dt} = e^{-\frac{t}{2}}$$

Hence, solution is

$$p(t).\text{IF} = \int Q(t)\ \text{IF}dt$$

$$p(t)\cdot e^{-\frac{t}{2}} = \int -200\cdot e^{-\frac{t}{2}}dt$$

$$p(t)\cdot e^{-\frac{t}{2}} = 400e^{-\frac{t}{2}} + K$$

$\Rightarrow \quad p(t) = 400 + ke^{-1/2}$

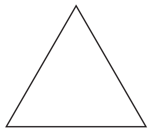
If $p(0) = 100$, then $k = -300$

$\Rightarrow \quad p(t) = 400 - 300e^{\frac{t}{2}}$

79. Coordinate of $S = \left(\frac{7+6}{2}, \frac{3-1}{2}\right) = \left(\frac{13}{2}, 1\right)$

[$\because S$ is mid-point of line QR]

Slope of the line PS is $\frac{-2}{9}$.



Required equation passes through (1, – 1) and parallel to PS is

$$y + 1 = \frac{-2}{9}(x - 1)$$

$\Rightarrow 2x + 9y + 7 = 0$

80. Let coordinate of the intersection point in fourth quadrant be $(\alpha, -\alpha)$.
Since, $(\alpha, -\alpha)$ lies on both lines $4ax + 2ay + c = 0$ and $5bx + 2by + d = 0$.

$\therefore \quad 4a\alpha - 2a\alpha + c = 0 \Rightarrow \alpha = \frac{-c}{2a}$ ...(i)

and $\quad 5b\alpha - 2b\alpha + d = 0 \Rightarrow \alpha = \frac{-d}{3b}$ ...(ii)

From Eqs. (i) and (ii), we get

$$\frac{-c}{2a} = \frac{-d}{3b}$$

$\Rightarrow \quad 3bc = 2ad \quad \Rightarrow \quad 2ad - 3bc = 0$

81. Equation of ellipse is $x^2 + 3y^2 = 6$ or $\frac{x^2}{6} + \frac{y^2}{2} = 1$.

Equation of the tangent is

$$\frac{x\cos\theta}{a} + \frac{y\sin\theta}{b} = 1$$

Let (h, k) be any point on the locus.

$\therefore \quad \frac{h}{a}\cos\theta + \frac{k}{b}\sin\theta = 1$ ...(i)

Slope of the tangent line is $\frac{-b}{a}\cot\theta$.

Slope of perpendicular drawn from centre (0,0) to (h, k) is $\frac{k}{h}$.

Since, both the lines are perpendicular.

$\therefore \quad \left(\frac{k}{h}\right) \times \left(-\frac{b}{a}\cot\theta\right) = -1$

$\Rightarrow \quad \frac{\cos\theta}{ha} = \frac{\sin\theta}{kb} = \alpha$ [Say]

$\Rightarrow \quad \cos\theta = \alpha ha, \ \sin\theta = \alpha kb$

From Eq. (i), we get

$$\frac{h}{a}(\alpha ha) + \frac{k}{b}(\alpha kb) = 1$$

$\Rightarrow \quad h^2\alpha + k^2\alpha = 1$

$\Rightarrow \quad \alpha = \frac{1}{h^2 + k^2}$

Also, $\quad \sin^2\theta + \cos^2\theta = 1$

$\Rightarrow \quad (\alpha kb)^2 + (\alpha ha)^2 = 1$

$\Rightarrow \quad \alpha^2 k^2 b^2 + \alpha^2 h^2 a^2 = 1$

$\Rightarrow \quad \frac{k^2 b^2}{(h^2 + k^2)^2} + \frac{h^2 a^2}{(h^2 + k^2)^2} = 1$

$\Rightarrow \quad \frac{2k^2}{(h^2 + k^2)^2} + \frac{6h^2}{(h^2 + k^2)^2} = 1$

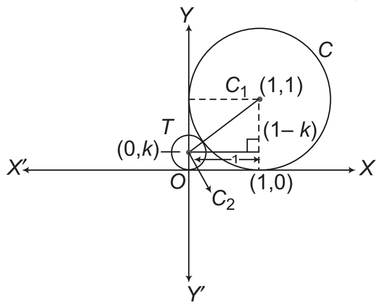
[$\because a^2 = 6, b^2 = 2$]

$\Rightarrow \quad 6x^2 + 2y^2 = (x^2 + y^2)^2$

[Replacing k by y and h by x]

82. **Use the property, when two circles touch each other externally, then distance between the centre is equal to sum of their radii, to get required radius.**

Let the coordinate of the centre of T be $(0, k)$.



Distance between their centre

$k + 1 = \sqrt{1 + (k - 1)^2}$ [$\because C_1C_2 = k + 1$]

$\Rightarrow \quad k + 1 = \sqrt{1 + k^2 + 1 - 2k}$

$\Rightarrow \quad k + 1 = \sqrt{k^2 + 2 - 2k}$

$\Rightarrow \quad k^2 + 1 + 2k = k^2 + 2 - 2k \Rightarrow k = \frac{1}{4}$

So, the radius of circle T is k *i.e.*, $\frac{1}{4}$.

83. **Let the tangent to parabola be $y = mx + a/m$, if it touches the other curve, then $D = 0$, to get the value of m.**

For parabola, $y^2 = 4x$

Let $y = mx + \frac{1}{m}$ be tangent line and it touches the parabola $x^2 = -32y$.

$\therefore \quad x^2 = -32\left(mx + \frac{1}{m}\right)$

$\Rightarrow \quad x^2 + 32mx + \frac{32}{m} = 0$

$\because \quad D = 0$

$\therefore \quad (32m)^2 - 4 \cdot \left(\frac{32}{m}\right) = 0 \Rightarrow m^3 = \frac{1}{8}$

$\therefore \quad m = \frac{1}{2}$

84. Here, plane, line and its image are parallel to each other. So, find any point on the normal to the plane from which the image line will be passed and then find equation of image line.

Here, plane and line are parallel to each other. Equation of normal to the plane through the point (1, 3, 4) is

$$\frac{x-1}{2}=\frac{y-3}{-1}=\frac{z-4}{1}=k \quad \text{[Say]}$$

Any point in this normal is $(2k+1, -k+3, 4+k)$.

Then, $\left(\frac{2k+1+1}{2}, \frac{3-k+3}{2}, \frac{4+k+4}{2}\right)$ lies on plane.

$$\Rightarrow \quad 2(k+1)-\left(\frac{6-k}{2}\right)+\left(\frac{8+k}{2}\right)+3=0$$

$$\Rightarrow \quad k=-2$$

Hence, point through which this image pass is

$$(2k+1, 3-k, 4+k)$$

i.e., $(2(-2)+1, 3+2, 4-2)=(-3, 5, 2)$

Hence, equation of image line is

$$\frac{x+3}{3}=\frac{y-5}{1}=\frac{z-2}{-5}$$

85. Use the following formula to simplify

$$\mathbf{a}\times(\mathbf{b}\times\mathbf{c})=(\mathbf{a}\cdot\mathbf{c})\,\mathbf{b}-(\mathbf{a}\cdot\mathbf{b})\,\mathbf{c}$$

$$[\mathbf{a}\ \mathbf{b}\ \mathbf{c}]=[\mathbf{b}\ \mathbf{c}\ \mathbf{a}]=[\mathbf{c}\ \mathbf{a}\ \mathbf{b}]$$

and $[\mathbf{a}\ \mathbf{a}\ \mathbf{b}]=[\mathbf{a}\ \mathbf{b}\ \mathbf{b}]=[\mathbf{a}\ \mathbf{c}\ \mathbf{c}]=0.$

Now, $[\mathbf{a}\times\mathbf{b}\ \mathbf{b}\times\mathbf{c}\ \mathbf{c}\times\mathbf{a}]$

$$=\mathbf{a}\times\mathbf{b}\cdot[(\mathbf{b}\times\mathbf{c})\times(\mathbf{c}\times\mathbf{a})]$$

$$=\mathbf{a}\times\mathbf{b}\cdot[\mathbf{k}\times(\mathbf{c}\times\mathbf{a})] \quad [\text{Let } \mathbf{k}=\mathbf{b}\times\mathbf{c}]$$

$$=\mathbf{a}\times\mathbf{b}\cdot[(\mathbf{k}\cdot\mathbf{a})\mathbf{c}-(\mathbf{k}\cdot\mathbf{c})\mathbf{a}]$$

$$=(\mathbf{a}\times\mathbf{b})\cdot[(\mathbf{b}\times\mathbf{c}\cdot\mathbf{a})\mathbf{c}-(\mathbf{b}\times\mathbf{c}\cdot\mathbf{c})\mathbf{a}]$$

$$=(\mathbf{a}\times\mathbf{b})\cdot([\mathbf{b}\ \mathbf{c}\ \mathbf{a}]\,\mathbf{c})-0 \quad \{\because [\mathbf{b}\times\mathbf{c}\cdot\mathbf{c}]=0\}$$

$$=(\mathbf{a}\times\mathbf{b})\cdot\mathbf{c}\,[\mathbf{b}\ \mathbf{c}\ \mathbf{a}]=[\mathbf{a}\ \mathbf{b}\ \mathbf{c}][\mathbf{b}\ \mathbf{c}\ \mathbf{a}]$$

$$=[\mathbf{a}\ \mathbf{b}\ \mathbf{c}]^2 \quad \{\because [\mathbf{a}\ \mathbf{b}\ \mathbf{c}]=[\mathbf{b}\ \mathbf{c}\ \mathbf{a}]\}$$

But given, $[\mathbf{a}\times\mathbf{b}\ \mathbf{b}\times\mathbf{c}\ \mathbf{c}\times\mathbf{a}]=\lambda[\mathrm{a\ b\ c}]^2$

So, $[\mathbf{a}\mathbf{b}\mathbf{c}]^2=\lambda[\mathbf{a}\mathbf{b}\mathbf{c}]^2 \Rightarrow \lambda=1$

86. Given, $P(\overline{A\cup B})=\frac{1}{6}$, $P(A\cap B)=\frac{1}{4}$, $P(\overline{A})=\frac{1}{4}$

$\therefore P(A\cup B)=1-P(\overline{A\cup B})=1-\frac{1}{6}=\frac{5}{6}$

and $\quad P(A)=1-P(\overline{A})=1-\frac{1}{4}=\frac{3}{4}$

$P(A\cup B)=P(A)+P(B)-P(A\cap B)$

$$\Rightarrow \quad \frac{5}{6}=\frac{3}{4}+P(B)-\frac{1}{4}$$

$P(B)=\frac{1}{3}\Rightarrow A$ and B are not equally likely.

$P(A\cap B)=P(A)\cdot P(B)=\frac{1}{4}$

So, events are independent.

87. Here, $\overline{X}=\frac{\Sigma X_i}{n}=\frac{2+4+6+8+\ldots+100}{50}$

$$=\frac{50\times 51}{50}=51$$

$[\because \Sigma 2n=n(n+1), \text{ here } n=50]$

Variance, $\sigma^2=\frac{1}{n}\Sigma Xi^2-(\overline{x})^{-2}$

$$\sigma^2=\frac{1}{50}(2^2+4^2+\ldots+100^2)-(51)^2$$

$$=833$$

88. Given, $f_k(x)=\frac{1}{k}(\sin^k x+\cos^k x)$,

where $x\in R$ and $k>1$

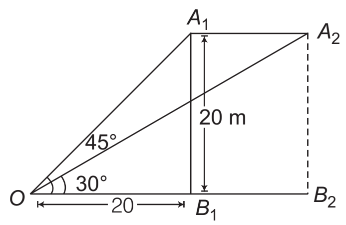
$$f_4(x)-f_6(x)=\frac{1}{4}(\sin^4 x+\cos^4 x)-\frac{1}{6}(\sin^6 x+\cos^6 x)$$

$$=\frac{1}{4}(1-2\sin^2 x\cdot\cos^2 x)-\frac{1}{6}(1-3\sin^2 x\cdot\cos^2 x)=\frac{1}{4}-\frac{1}{6}=\frac{1}{12}$$

89. In ΔOA_1B_1,

$$\tan 45°=\frac{A_1B_1}{OB_1} \quad \Rightarrow \quad \frac{20}{OB_1}=1$$

$$\Rightarrow \quad OB_1=20$$



In ΔOA_2B_2,

$$\tan 30°=\frac{20}{OB_2} \Rightarrow OB_2=20\sqrt{3}$$

$$\Rightarrow \quad B_1B_2+OB_1=20\sqrt{3}$$

$$\Rightarrow \quad B_1B_2=20\sqrt{3}-20$$

$$\Rightarrow \quad B_1B_2=20(\sqrt{3}-1)\,\text{m}$$

Now, speed $=\frac{\text{Distance}}{\text{Time}}=\frac{20(\sqrt{3}-1)}{1}$

$$=20(\sqrt{3}-1)\,\text{m/s}$$

90.

p	q	$\sim p$	$\sim q$	$p\leftrightarrow q$	$p\leftrightarrow\sim q$	$\sim p\leftrightarrow q$	$\sim(p\leftrightarrow\sim q)$
T	F	F	T	F	T	T	F
F	T	T	F	F	T	T	F
T	T	F	F	T	F	F	T
F	F	T	T	T	F	F	T

$\sim(p\leftrightarrow\sim q)$ is equivalent to $(p\leftrightarrow q)$.