Solved Paper 2015

JEE Main

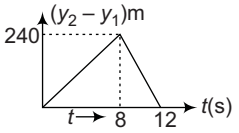
Joint Entrance Examination

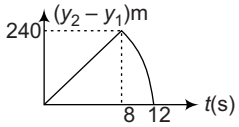
Instructions

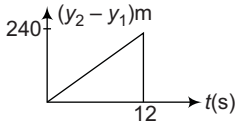
1. This test consists of 90 questions.
2. There are three parts in the question paper A, B, C consisting of Physics, Chemistry & Mathematics having 30 questions in each part of equal weightage. Each question is allotted 4 marks for correct response.
3. Candidates will be awarded marks as stated above in instruction no. 2 for correct response of each question. 1 marks will be deducted for indicating incorrect response of each question. No deduction from the total score will be made if no response is indicated for an item in the answer sheet.
4. There is only one correct response for each question. Filling up more than one response in any question will be treated as wrong response and marks for wrong response will be deducted according as per instructions.

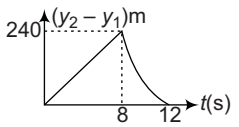
Physics

1. Two stones are thrown up simultaneously from the edge of a cliff 240 m high with initial speed of 10 m/s and 40 m/s respectively. Which of the following graph best represents the time variation of relative position of the second stone with respect to the first? Assume stones do not rebound after hitting the ground and neglect air resistance, take $\mathbf{g = 10\ m/s^2}$)

(a)


(c)


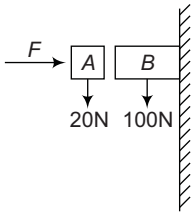
(b)


(d)


2. The period of oscillation of a simple pendulum is $\mathbf{T = 2\pi\sqrt{\frac{L}{g}}}$. Measured value of L is 20.0 cm known to 1 mm accuracy and time for 100 oscillations of the pendulum is found to be 90 s using a wrist watch of 1s resolution. The accuracy in the determination of g is

(a) 2% (b) 3% (c) 1% (d) 5%

3. Given in the figure are two blocks A and B of weight 20 N and 100 N respectively. These are being pressed against a wall by a force F as shown in figure. If the coefficient of friction between the blocks is 0.1 and between block B and the wall is 0.15, the frictional force applied by the wall in block B is



(a) 100 N
(b) 80 N
(c) 120 N
(d) 150 N

4. A particle of mass m moving in the x-direction with speed $\mathbf{2v}$ is hit by another particle of mass $2m$ moving in the y-direction with speed v. If the collision is perfectly inelastic, the percentage loss in the energy during the collision is close to

(a) 44% (b) 50% (c) 56% (d) 62%

5. Distance of the centre of mass of a solid uniform cone from its vertex is $\mathbf{z_0}$. If the radius of its base is R and its height is h, then $\mathbf{z_0}$ is equal to

(a) $\frac{h^2}{4R}$ (b) $\frac{3h}{4}$

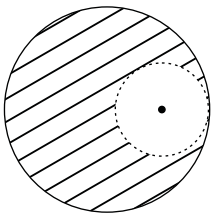
(c) $\frac{5h}{8}$ (d) $\frac{3h^2}{8R}$

6. From a solid sphere of mass M and radius R, a cube of maximum possible volume is cut. Moment of inertia of cube about an axis passing through its centre and perpendicular to one of its faces is

(a) $\frac{MR^2}{32\sqrt{2}\pi}$ (b) $\frac{MR^2}{16\sqrt{2}\pi}$

(c) $\frac{4MR^2}{9\sqrt{3}\pi}$ (d) $\frac{4MR^2}{3\sqrt{3}\pi}$

7. From a solid sphere of mass M and radius R, a spherical portion of radius $\left(\frac{\mathbf{R}}{\mathbf{2}}\right)$ is removed as shown in the figure. Taking gravitational potential $\mathbf{V = 0}$ at $\mathbf{r} = \infty$, the potential at the centre of the cavity thus formed is (G = gravitational constant)



(a) $\frac{-GM}{2R}$ (b) $\frac{-GM}{R}$

(c) $\frac{-2GM}{3R}$ (d) $\frac{-2GM}{R}$

8. A pendulum made of a uniform wire of cross-sectional area A has time period T. When an additional mass M is added to its bob, the time period changes $\mathbf{T_M}$. If the Young's modulus of the material of the wire is Y, then $\frac{\mathbf{1}}{\mathbf{Y}}$ is equal to ($\mathbf{g}$ = gravitational acceleration)

(a) $\left[\left(\frac{T_M}{T}\right)^2 - 1\right]\frac{A}{Mg}$

(b) $\left[\left(\frac{T_M}{T}\right)^2 - 1\right]\frac{Mg}{A}$

(c) $\left[1 - \left(\frac{T_M}{T}\right)^2\right]\frac{A}{Mg}$

(d) $\left[1 - \left(\frac{T}{T_M}\right)^2\right]\frac{A}{Mg}$

9. Consider a spherical shell of radius R at temperature T. The black body radiation inside it can be considered as an ideal gas of photons with internal energy per unit volume $\mathbf{u} = \frac{\mathbf{U}}{\mathbf{V}} \propto \mathbf{T^4}$ and pressure $\mathbf{p} = \frac{\mathbf{1}}{\mathbf{3}}\left(\frac{\mathbf{U}}{\mathbf{V}}\right)$. If the shell now undergoes an adiabatic expansion, the relation between T and R is

(a) $T \propto e^{-R}$

(b) $T \propto e^{-3R}$

(c) $T \propto \frac{1}{R}$

(d) $T \propto \frac{1}{R^3}$

10. A solid body of constant heat capacity 1 J/°C is being heated by keeping it in contact with reservoirs in two ways

(i) Sequentially keeping in contact with 2 reservoirs such that each reservoir supplies same amount of heat.

(ii) Sequentially keeping in contact with 8 reservoirs such that each reservoir supplies same amount of heat.

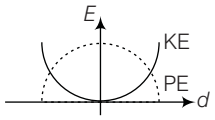
In both the cases, body is brought from initial temperature 100°C to final temperature 200°C. Entropy change of the body in the two cases respectively, is

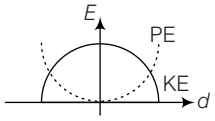
(a) ln2, 4ln2 (b) ln2, ln2
(c) ln2, 2ln2 (d) 2ln2, 8ln2

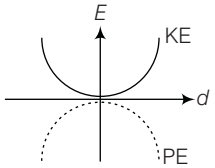
11. Consider an ideal gas confined in an isolated closed chamber. As the gas undergoes an adiabatic expansion, the average time of collision between molecules increases as $\mathbf{V^q}$, where V is the volume of the gas. The value of q is $\left(\gamma = \frac{\mathbf{C_p}}{\mathbf{C_V}}\right)$

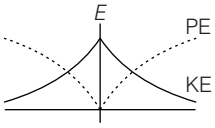
(a) $\frac{3\gamma + 5}{6}$
(b) $\frac{3\gamma - 5}{6}$
(c) $\frac{\gamma + 1}{2}$
(d) $\frac{\gamma - 1}{2}$

12. For a simple pendulum, a graph is plotted between its Kinetic Energy (KE) and Potential Energy (PE) against its displacement d. Which one of the following represents these correctly? (graphs are schematic and not drawn to scale)

(a) 

(b) 

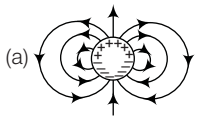
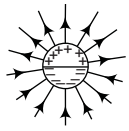
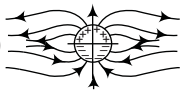
(c) 

(d) 

13. A train is moving on a straight track with speed $\mathbf{20\ ms^{-1}}$. It is blowing its whistle at the frequency of 1000 Hz. The percentage change in the frequency heard by a person standing near the track as the train passes him is close to (speed of sound = $\mathbf{320\ ms^{-1}}$)

(a) 6% (b) 12% (c) 18% (d) 24%

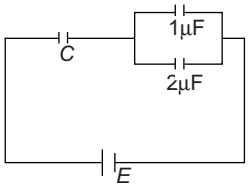
14. A long cylindrical shell carries positive surface charge σ in the upper half and negative surface charge $-\sigma$ in the lower half. The electric field lines around the cylinder will look like figure given in (figures are schematic and not drawn to scale)

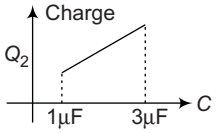
(a) 
(b) 
(c) 
(d) 

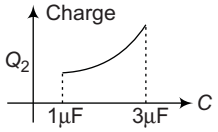
15. A uniformly charged solid sphere of radius R has potential $\mathbf{V_0}$ (measured with respect to ∞) on its surface. For this sphere, the equipotential surfaces with potentials $\frac{\mathbf{3V_0}}{\mathbf{2}}$, $\frac{\mathbf{5V_0}}{\mathbf{4}}$, $\frac{\mathbf{3V_0}}{\mathbf{4}}$ and $\frac{\mathbf{V_0}}{\mathbf{4}}$ have radius $\mathbf{R_1}$, $\mathbf{R_2}$, $\mathbf{R_3}$, and $\mathbf{R_4}$ respectively. Then,

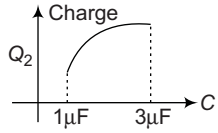
(a) $R_1 = 0$ and $R_2 > (R_4 - R_3)$
(b) $R_1 \neq 0$ and $(R_2 - R_1) > (R_4 - R_3)$
(c) $R_1 = 0$ and $R_2 < (R_4 - R_3)$
(d) $2R < R_4$

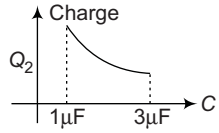
16. In the given circuit, charge $\mathbf{Q_2}$ on the $\mathbf{2\ \mu F}$ capacitor changes as C is varied from $\mathbf{1\ \mu F}$ to $\mathbf{3\mu F}$. $\mathbf{Q_2}$ as a function of C is given properly by (figures are drawn schematically and are not to scale)



(a) 

(c) 

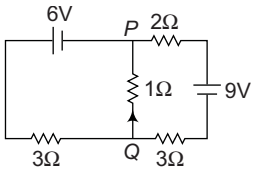
(b) 

(d) 

17. When 5V potential difference is applied across a wire of length 0.1m, the drift speed of electrons is $\mathbf{2.5 \times 10^{-4} ms^{-1}}$. If the electron density in the wire is $\mathbf{8 \times 10^{28}\ m^{-3}}$ the resistivity of the material is close to

(a) $1.6 \times 10^{-8}\ \Omega$m (b) $1.6 \times 10^{-7}\ \Omega$m
(c) $1.6 \times 10^{-6}\ \Omega$m (d) $1.6 \times 10^{-5}\ \Omega$m

18. In the circuit shown below, the current in the $\mathbf{1\Omega}$ resistor is

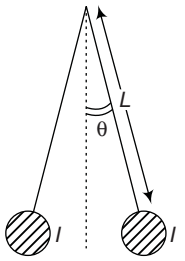


(a) 1.3 A, from *P* to *Q*
(b) 0 A
(c) 0.13 A, from *Q* to *P*
(d) 0.13 A, from *P* to *Q*

19. Two coaxial solenoids of different radii carry current *I* in the same direction. Let $\mathbf{F_1}$ be the magnetic force on the inner solenoid due to the outer one and $\mathbf{F_2}$ be the magnetic force on the outer solenoid due to the inner one. Then,

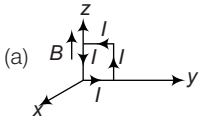
(a) $F_1 = F_2 = 0$
(b) F_1 is radially inwards and F_2 is radially outwards
(c) F_1 is radially inwards and $F_2 = 0$
(d) F_1 is radially outwards and $F_2 = 0$

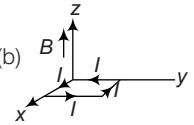
20. Two long current carrying thin wires, both with current *I*, are held by insulating threads of length *L* and are in equilibrium as shown in the figure, with threads making an angle θ with the vertical. If wires have mass λ per unit length then, the value of *I* is (**g** = gravitational acceleration)

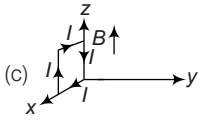


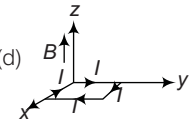
(a) $\sin\theta \sqrt{\frac{\pi\lambda g L}{\mu_0 \cos\theta}}$ (b) $2\sin\theta \sqrt{\frac{\pi\lambda g L}{\mu_0 \cos\theta}}$
(c) $2\sqrt{\frac{\pi g L}{\mu_0}\tan\theta}$ (d) $\sqrt{\frac{\pi\lambda g L}{\mu_0}\tan\theta}$

21. A rectangular loop of sides 10 cm and 5 cm carrying a current *I* of 12 A is placed in different orientations as shown in the figures below.

(a) 

(b) 

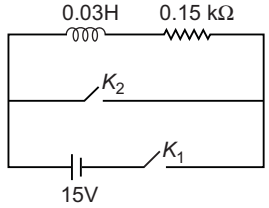
(c) 

(d) 

If there is a uniform magnetic field of 0.3 T in the positive *z*-direction in which orientations the loop would be in (i) stable equilibrium and (ii) unstable equilibrium?

(a) (a) and (b) respectively
(b) (a) and (c) respectively
(c) (b) and (d) respectively
(d) (b) and (c) respectively

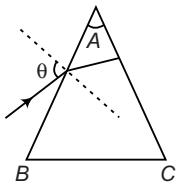
22. An inductor **(L = 0.03 H)** and a resistor **(R = 0.15 kΩ)** are connected in series to a battery of 15V EMF in a circuit shown below. The key $\mathbf{K_1}$ has been kept closed for a long time. Then at **t = 0**, $\mathbf{K_1}$ is opened and key $\mathbf{K_2}$ is closed simultaneously. At **t = 1 ms**, the current in the circuit will be **($e^5 \cong 150$)**



(a) 100 mA (b) 67 mA
(c) 6.7 mA (d) 0.67mA

23. A red LED emits light at 0.1 W uniformly around it. The amplitude of the electric field of the light at a distance of 1 m from the diode is
(a) 1.73 V/m
(b) 2.45 V/m
(c) 5.48 V/m
(d) 7.75 V/m

24. Monochromatic light is incident on a glass prism of angle *A*. If the refractive index of the material of the prism is μ, a ray incident at an angle θ, on the face *AB* would get transmitted through the face *AC* of the prism provided



(a) $\theta > \sin^{-1}\left[\mu\sin\left(A - \sin^{-1}\left(\frac{1}{\mu}\right)\right)\right]$

(b) $\theta < \sin^{-1}\left[\mu\sin\left(A - \sin^{-1}\left(\frac{1}{\mu}\right)\right)\right]$

(c) $\theta > \cos^{-1}\left[\mu\sin\left(A + \sin^{-1}\left(\frac{1}{\mu}\right)\right)\right]$

(d) $\theta < \cos^{-1}\left[\mu\sin\left(A + \sin^{-1}\left(\frac{1}{\mu}\right)\right)\right]$

25. On a hot summer night, the refractive index of air is smallest near the ground and increases with height from the ground. When a light beam is directed horizontally, the Huygens principle leads us to conclude that as it travels, the light beam
(a) becomes narrower
(b) goes horizontally without any deflection
(c) bends downwards
(d) bends upwards

26. Assuming human pupil to have a radius of 0.25 cm and a comfortable viewing distance of 25 cm, the minimum separation between two objects that human eye can resolve at 500 nm wavelength is
(a) 1 μm (b) 30 μm (c) 100 μm (d) 300 μm

27. As an electron makes a transition from an excited state to the ground state of a hydrogen like atom/ion
(a) its kinetic energy increases but potential energy and total energy decrease
(b) kinetic energy, potential energy and total energy decrease
(c) kinetic energy decreases, potential energy increases but total energy remains same
(d) kinetic energy and total energy decrease but potential energy increases

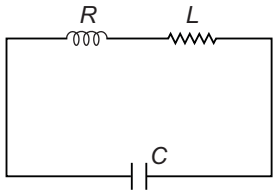
28. Match List I (fundamental experiment) with List II (its conclusion) and select the correct option from the choices given below the list.

List I	List II
A. Franck-Hertz experiment	1. Particle nature of light
B. Photo-electric experiment	2. Discrete energy levels of atom
C. Davisson-Germer experiment	3. Wave nature of electron
	4. Structure of atom

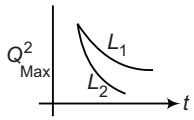
	A	B	C		A	B	C
(a)	1	4	3	(b)	2	4	3
(c)	2	1	3	(d)	4	3	2

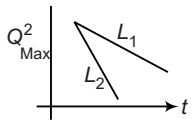
29. A signal of 5 kHz frequency is amplitude modulated on a carrier wave of frequency 2MHz. The frequencies of the resultant signal is/are
(a) 2 MHz only
(b) 2005 kHz and 1995 kHz
(c) 2005 kHz 2000 kHz and 1995 kHz
(d) 2000 kHz and 1995 kHz

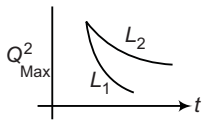
30. An *LCR* circuit is equivalent to a damped pendulum. In an *LCR* circuit, the capacitor is charged to $\mathbf{Q_0}$ and then connected to the *L* and *R* as shown below.

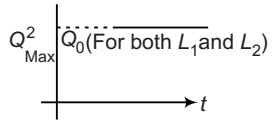


If a student plots graphs of the square of maximum charge $(\mathbf{Q^2_{Max}})$ on the capacitor with time (*t*) for two different values $\mathbf{L_1}$ and $\mathbf{L_2}$ $(\mathbf{L_1 > L_2})$ of *L*, then which of the following represents this graph correctly? (plots are schematic and not drawn to scale)

(a) 

(c) 

(b) 

(d) 

Chemistry

1. The molecular formula of a commercial resin used for exchanging ions in water softening is $\mathbf{C_8H_7SO_3Na}$ (mol. wt. = 206). What would be the maximum uptake of $\mathbf{Ca^{2+}}$ ions by the resin when expressed in mole per gram resin?
(a) $\frac{1}{103}$ (b) $\frac{1}{206}$
(c) $\frac{2}{309}$ (d) $\frac{1}{412}$

2. Sodium metal crystallises in a body centred cubic lattice with a unit cell edge of 4.29 **Å**. The radius of sodium atom is approximately
(a) 1.86 Å (b) 3.22 Å (c) 5.72 Å (d) 0.93 Å

3. Which of the following is the energy of a possible excited state of hydrogen?
(a) +13.6 eV (b) −6.8 eV
(c) −3.4 eV (d) +6.8 eV

4. The intermolecular interaction that is dependent on the inverse cube of distance between the molecules is
(a) ion-ion interaction
(b) ion-dipole interaction
(c) London force
(d) hydrogen bond

5. The following reaction is performed at 298K

$$\mathbf{2NO(g) + O_2(g) \rightleftharpoons 2NO_2(g)}$$

The standard free energy of formation of NO (*g*) is 86.6 kJ/mol at 298 K. What is the standard free energy of formation of $\mathbf{NO_2(g)}$ at 298 K? $(\mathbf{K_p = 1.6 \times 10^{12}})$
(a) $R(298)\ \ln(1.6\times 10^{12}) - 86600$
(b) $86600 + R(298)\ \ln(1.6\times 10^{12})$
(c) $86600 - \frac{\ln(1.6\times 10^{12})}{R(298)}$
(d) $0.5\ [2\times 86600 - R(298)\ \ln(1.6\times 10^{12})]$

6. The vapour pressure of acetone at 20°C is 185 torr. When 1.2 g of a non-volatile substance was dissolved in 100 g of acetone at 20°C, its vapour pressure was 183 torr. The molar mass $(\mathbf{g\ mol^{-1}})$ of the substance is
(a) 32 (b) 64 (c) 128 (d) 488

7. The standard Gibbs energy change at 300K for the reaction, $\mathbf{2A \rightleftharpoons B + C}$ is 2494. 2 J. At a given time, the composition of the reaction mixture is $\mathbf{[A] = \frac{1}{2}, [B] = 2}$ and $\mathbf{[C] = \frac{1}{2}}$. The reaction proceeds in the $\mathbf{[R = 8.314\ JK/mol, e = 2.718]}$
(a) forward direction because $Q > K_c$
(b) reverse direction because $Q > K_c$
(c) forward direction because $Q < K_c$
(d) reverse direction because $Q < K_c$

8. Two Faraday of electricity is passed through a solution of $\mathbf{CuSO_4}$. The mass of copper deposited at the cathode is (at. mass of Cu = 63.5 u)
(a) 0g (b) 63.5g (c) 2g (d) 127g

9. Higher order (>3) reactions are rare due to
(a) low probability of simultaneous collision of all the reacting species
(b) increase in entropy and activation energy as more molecules are involved
(c) shifting of equilibrium towards reactants due to elastic collisions
(d) loss of active species on collision

10. 3 g of activated charcoal was added to 50 mL of acetic acid solution (0.06 N) in a flask. After an hour it was filtered and the strength of the filtrate was found to be 0.042 N. The amount of acetic acid adsorbed (per gram of charcoal) is
(a) 18 mg (b) 36 mg
(c) 42 mg (d) 54 mg

11. The ionic radii (in Å) of $\mathbf{N^{3-}}$, $\mathbf{O^{2-}}$ and F^- respectively are
(a) 1.36, 1.40 and 1.71
(b) 1.36, 1.71 and 1.40
(c) 1.71, 1.40 and 1.36
(d) 1.71, 1.36 and 1.40

12. In the context of the Hall-Heroult process for the extraction of Al, which of the following statements is false?
(a) CO and CO_2 are produced in this process
(b) Al_2O_3 is mixed with CaF_2 which lowers the melting point of the mixture and brings conductivity
(c) Al^{3+} is reduced at the cathode to form Al
(d) Na_3AlF_6 serves as the electrolyte

13. From the following statements regarding $\mathbf{H_2O_2}$, choose the incorrect statement.
(a) It can act only as an oxidising agent
(b) It decomposed on exposure to light
(c) It has to be stored in plastic or wax lined glass bottles in dark
(d) It has to be kept away from dust

14. Which one of the following alkaline earth metal sulphates has its hydration enthalpy greater than its lattice enthalpy?
(a) $CaSO_4$ (b) $BeSO_4$ (c) $BaSO_4$ (d) $SrSO_4$

15. Which among the following is the most reactive?
(a) Cl_2 (b) Br_2 (c) I_2 (d) ICl

16. Match the catalysts to the correct processes.

Catalyst	Process
(A) $TiCl_3$	(i) Wacker process
(B) $PdCl_2$	(ii) Ziegler- Natta polymerisation
(C) $CuCl_2$	(iii) Contact process
(D) V_2O_5	(iv) Deacon's process

(a) (A)- (iii), (B) - (ii), (C) - (iv), (D) - (i)
(b) (A)- (ii), (B) - (i), (C) - (iv), (D) - (iii)
(c) (A)- (ii), (B) - (iii), (C) - (iv), (D) - (i)
(d) (A)- (iii), (B) - (i), (C) - (ii), (D) - (iv)

17. Which one has the highest boiling point?
(a) He (b) Ne (c) Kr (d) Xe

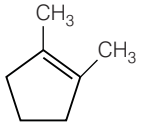
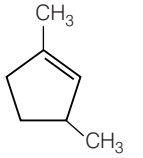
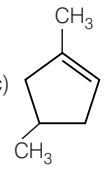
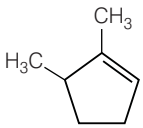
18. The number of geometric isomers that can exist for square planar [Pt (Cl) (py) $(\mathbf{NH_3})$ $\mathbf{(NH_2OH)]^+}$ is (py = pyridine)
(a) 2 (b) 3 (c) 4 (d) 6

19. The colour of $KMnO_4$ is due to
(a) $M \rightarrow L$ charge transfer transition
(b) $d - d$ transition
(c) $L \rightarrow M$ charge transfer transition
(d) $\sigma - \breve{\sigma}$ transition

20. **Assertion (A)** Nitrogen and oxygen are the main components in the atmosphere but these do not react to form oxides of nitrogen.
Reason (R) The reaction between nitrogen and oxygen requires high temperature.
(a) Both Assertion and Reason are correct and the reason is the correct explanation for the Assertion.
(b) Both Assertion and Reason are correct but the reason is not the correct explanation for the Assertion.
(c) The Assertion is incorrect but the Reason is correct.
(d) Both the Assertion and Reason are incorrect.

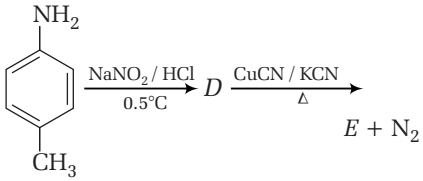
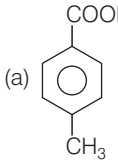
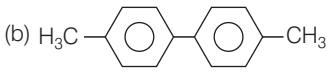
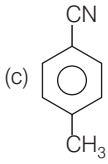
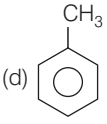
21. In Carius method of estimation of halogens, 250 mg of an organic compound gave 141 mg of AgBr. The percentage of bromine in the compound is
(at. mass Ag = 108, Br = 80)
(a) 24 (b) 36 (c) 48 (d) 60

22. Which of the following compound will exhibit geometrical isomerism?
(a) 1-phenyl-2-butene
(b) 3-phenyl-1-butene
(c) 2-phenyl-1-butene
(d) 1, 1-diphenyl-1-propane

23. Which compound would give 5-keto-2-methyl hexanal upon ozonolysis?
(a) 
(b) 
(c) 
(d) 

24. The synthesis of alkyl fluorides is best accomplished by
(a) free radical fluorination
(b) Sandmeyer's reaction
(c) Finkelstein reaction
(d) Swarts reaction

25. In the following sequence of reaction,
$$\text{Toluene} \xrightarrow{KMnO_4} \mathbf{A} \xrightarrow{SOCl_2} \mathbf{B} \xrightarrow[BaSO_4]{H_2/Pd} \mathbf{C}$$
The product C is
(a) C_6H_5COOH
(b) $C_6H_5CH_3$
(c) $C_6H_5CH_2OH$
(d) C_6H_5CHO

26. In the reaction,

The product E is
(a) 
(b) 
(c) 
(d) 

27. Which polymer is used in the manufacture of paints and lacquers?
(a) Bakelite
(b) Glyptal
(c) Polypropene
(d) Polyvinyl chloride

28. Which of the vitamins given below is water soluble?
(a) Vitamin C (b) Vitamin D
(c) Vitamin E (d) Vitamin K

29. Which of the following compounds is not an antacid?
(a) Aluminium hydroxide
(b) Cimetidine
(c) Phenelzine
(d) Ranitidine

30. Which of the following compounds is not coloured yellow?
(a) $Zn_2[Fe(CN)_6]$
(b) $K_3[Co(NO_2)_6]$
(c) $(NH_4)_3[As(Mo_3O_{10})_4]$
(d) $BaCrO_4$

Mathematics

1. Let **A** and **B** be two sets containing four and two elements respectively. Then, the number of subsets of the set $\mathbf{A \times B}$, each having atleast three elements are
(a) 219
(b) 256
(c) 275
(d) 510

2. A complex number $\mathbf{z}$ is said to be unimodular, if $\mathbf{|z| = 1}$. Suppose $\mathbf{z_1}$ and $\mathbf{z_2}$ are complex numbers such that $\frac{\mathbf{z_1 - 2z_2}}{\mathbf{2 - z_1\bar{z}_2}}$ is unimodular and $\mathbf{z_2}$ is not unimodular. Then, the point $\mathbf{z_1}$ lies on a
(a) straight line parallel to *X*-axis
(b) straight line parallel to *Y*-axis
(c) circle of radius 2
(d) circle of radius $\sqrt{2}$

3. Let α and β be the roots of equation $\mathbf{x^2 - 6x - 2 = 0}$. If $\mathbf{a_n = \alpha^n - \beta^n}$, for $\mathbf{n \geq 1}$, then the value of $\frac{\mathbf{a_{10} - 2a_8}}{\mathbf{2a_9}}$ is equal to
(a) 6 (b) –6 (c) 3 (d) –3

4. If $\mathbf{A} = \begin{bmatrix} \mathbf{1} & \mathbf{2} & \mathbf{2} \\ \mathbf{2} & \mathbf{1} & \mathbf{-2} \\ \mathbf{a} & \mathbf{2} & \mathbf{b} \end{bmatrix}$ is a matrix satisfying the equation $\mathbf{AA^T = 9I}$, where *I* is $\mathbf{3 \times 3}$ identity matrix, then the ordered pair $\mathbf{(a, b)}$ is equal to
(a) (2, –1) (b) (–2, 1)
(c) (2, 1) (d) (–2, –1)

5. The set of all values of λ for which the system of linear equations $\mathbf{2x_1 - 2x_2 + x_3 = \lambda x_1}$, $\mathbf{2x_1 - 3x_2 + 2x_3 = \lambda x_2}$ and $\mathbf{-x_1 + 2x_2 = \lambda x_3}$ has a non-trivial solution,
(a) is an empty set
(b) is a singleton set
(c) contains two elements
(d) contains more than two elements

6. The number of integers greater than 6000 that can be formed, using the digits 3, 5, 6, 7 and 8 without repetition, is
(a) 216 (b) 192 (c) 120 (d) 72

7. The sum of coefficients of integral powers of $\mathbf{x}$ in the binomial expansion of $\mathbf{(1 - 2\sqrt{x})^{50}}$ is
(a) $\frac{1}{2}(3^{50} + 1)$
(b) $\frac{1}{2}(3^{50})$
(c) $\frac{1}{2}(3^{50} - 1)$
(d) $\frac{1}{2}(2^{50} + 1)$

8. If $\mathbf{m}$ is the AM of two distinct real numbers $\mathbf{l}$ and $\mathbf{n(l, n > 1)}$ and $\mathbf{G_1, G_2}$ and $\mathbf{G_3}$ are three geometric means between $\mathbf{l}$ and $\mathbf{n}$, then $\mathbf{G_1^4 + 2G_2^4 + G_3^4}$ equals
(a) $4\,l^2mn$
(b) $4\,lm^2n$
(c) $4\,lmn^2$
(d) $4l^2m^2n^2$

9. The sum of first 9 terms of the series $\frac{1^3}{1}+\frac{1^3+2^3}{1+3}+\frac{1^3+2^3+3^3}{1+3+5}+\ldots$ is

(a) 71 (b) 96
(c) 142 (d) 192

10. $\lim\limits_{x\to 0}\frac{(1-\cos 2x)(3+\cos x)}{x\tan 4x}$ is equal to

(a) 4 (b) 3
(c) 2 (d) $\frac{1}{2}$

11. If the function $g(x)=\begin{cases} k\sqrt{x+1}, & 0\le x\le 3 \\ mx+2, & 3<x\le 5 \end{cases}$ is differentiable, then the value of $k+m$ is

(a) 2 (b) $\frac{16}{5}$
(c) $\frac{10}{3}$ (d) 4

12. The normal to the curve $x^2+2xy-3y^2=0$ at $(1,1)$

(a) does not meet the curve again
(b) meets the curve again in the second quadrant
(c) meets the curve again in the third quadrant
(d) meets the curve again in the fourth quadrant

13. Let $f(x)$ be a polynomial of degree four having extreme values at $x=1$ and $x=2$. If $\lim\limits_{x\to 0}\left[1+\frac{f(x)}{x^2}\right]=3$, then $f(2)$ is equal to

(a) –8
(b) –4
(c) 0
(d) 4

14. The integral $\int\frac{dx}{x^2(x^4+1)^{\frac{3}{4}}}$ equals

(a) $\left(\frac{x^4+1}{x^4}\right)^{\frac{1}{4}}+c$ (b) $(x^4+1)^{\frac{1}{4}}+c$
(c) $-(x^4+1)^{\frac{1}{4}}+c$ (d) $-\left(\frac{x^4+1}{x^4}\right)^{\frac{1}{4}}+c$

15. The integral $\int_2^4\frac{\log x^2}{\log x^2+\log(36-12x+x^2)}dx$ is equal to

(a) 2 (b) 4
(c) 1 (d) 6

16. The area (in sq units) of the region described by $\{(x,y): y^2\le 2x$ and $y\ge 4x-1\}$ is

(a) $\frac{7}{32}$ (b) $\frac{5}{64}$ (c) $\frac{15}{64}$ (d) $\frac{9}{32}$

17. Let $y(x)$ be the solution of the differential equation $(x\log x)\frac{dy}{dx}+y=2x\log x,\quad (x\ge 1)$. Then, $y(e)$ is equal to

(a) e (b) 0
(c) 2 (d) $2e$

18. The number of points having both coordinates as integers that lie in the interior of the triangle with vertices (0, 0), (0, 41) and (41, 0) is

(a) 901 (b) 861
(c) 820 (d) 780

19. Locus of the image of the point (2, 3) in the line $(2x-3y+4)+k(x-2y+3)=0$, $k\in R$, is a

(a) straight line parallel to X-axis
(b) straight line parallel to Y-axis
(c) circle of radius $\sqrt{2}$
(d) circle of radius $\sqrt{3}$

20. The number of common tangents to the circles $x^2+y^2-4x-6y-12=0$ and $x^2+y^2+6x+18y+26=0$ is

(a) 1 (b) 2 (c) 3 (d) 4

21. The area (in sq units) of the quadrilateral formed by the tangents at the end points of the latusrectum to the ellipse $\frac{x^2}{9}+\frac{y^2}{5}=1$ is

(a) $\frac{27}{4}$ (b) 18

(c) $\frac{27}{2}$ (d) 27

22. Let **O** be the vertex and **Q** be any point on the parabola $\mathbf{x^2 = 8y}$. If the point **P** divides the line segment **OQ** internally in the ratio **1 : 3**, then the locus of **P** is

(a) $x^2 = y$

(b) $y^2 = x$

(c) $y^2 = 2x$

(d) $x^2 = 2y$

23. The distance of the point (1, 0, 2) from the point of intersection of the line $\frac{\mathbf{x-2}}{\mathbf{3}} = \frac{\mathbf{y+1}}{\mathbf{4}} = \frac{\mathbf{z-2}}{\mathbf{12}}$ and the plane $\mathbf{x - y + z = 16}$ is

(a) $2\sqrt{14}$

(b) 8

(c) $3\sqrt{21}$

(d) 13

24. The equation of the plane containing the line $\mathbf{2x - 5y + z = 3}$, $\mathbf{x + y + 4z = 5}$ and parallel to the plane $\mathbf{x + 3y + 6z = 1}$ is

(a) $2x + 6y + 12z = 13$

(b) $x + 3y + 6z = -7$

(c) $x + 3y + 6z = 7$

(d) $2x + 6y + 12z = -13$

25. Let **a**, **b** and **c** be three non-zero vectors such that no two of them are collinear and $\mathbf{(a \times b) \times c = \frac{1}{3}|b||c|a}$. If θ is the angle between vectors **b** and **c**, then a value of $\mathbf{\sin\theta}$ is

(a) $\frac{2\sqrt{2}}{3}$

(b) $\frac{-\sqrt{2}}{3}$

(c) $\frac{2}{3}$

(d) $\frac{-2\sqrt{3}}{3}$

26. If 12 identical balls are to be placed in 3 identical boxes, then the probability that one of the boxes contains exactly 3 balls, is

(a) $\frac{55}{3}\left(\frac{2}{3}\right)^{11}$ (b) $55\left(\frac{2}{3}\right)^{10}$

(c) $220\left(\frac{1}{3}\right)^{12}$ (d) $22\left(\frac{1}{3}\right)^{11}$

27. The mean of the data set comprising of 16 observations is 16. If one of the observation valued 16 is deleted and three new observations valued 3, 4 and 5 are added to the data, then the mean of the resultant data is

(a) 16.8 (b) 16.0 (c) 15.8 (d) 14.0

28. If the angles of elevation of the top of a tower from three collinear points **A**, **B** and **C** on a line leading to the foot of the tower are 30°, 45° and 60° respectively, then the ratio **AB : BC** is

(a) $\sqrt{3} : 1$

(b) $\sqrt{3} : \sqrt{2}$

(c) $1 : \sqrt{3}$

(d) 2 : 3

29. Let

$$\mathbf{\tan^{-1} y = \tan^{-1} x + \tan^{-1}\left(\frac{2x}{1 - x^2}\right)},$$

where $\mathbf{|x| < \frac{1}{\sqrt{3}}}$. Then, the value of **y** is

(a) $\frac{3x - x^3}{1 - 3x^2}$ (b) $\frac{3x + x^3}{1 - 3x^2}$

(c) $\frac{3x - x^3}{1 + 3x^2}$ (d) $\frac{3x + x^3}{1 + 3x^2}$

30. The negation of $\mathbf{\sim s \vee (\sim r \wedge s)}$ is equivalent to

(a) $s \wedge \sim r$ (b) $s \wedge (r \wedge \sim s)$

(c) $s \vee (r \vee \sim s)$ (d) $s \wedge r$

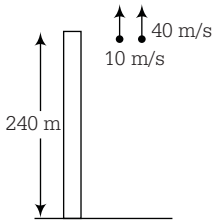
Answer *with* Explanations

Physics

1. *(c)* ***Central Idea*** *Concept of relative motion can be applied to predict the nature of motion of one particle with respect to the other.*

Considering motion of the second particle with respect to the first we have elative acceleration

$|a_{21}| = |a_2 - a_1| = g - g = 0$



Thus, motion of first particle is straight line with respect to second particle till the first particle strikes ground at a time given by

$$-240 = 10\,t - \frac{1}{2} \times 10 \times t^2$$

or $t^2 - 2t - 48 = 0$

or $t^2 - 8t + 6t - 48 = 0$

or $t = 8, -6$ (not possible)

Thus, distance covered by second particle with respect to first particle in 8 s is

$$s_{12} = (v_{21})\,t = (40 - 10)\,(8\text{s}) = 30 \times 8 = 240 \text{ m}$$

Similarly, time taken by second particle to strike the ground is given by

$$-240 = 40t - \frac{1}{2} \times 10 \times t^2$$

or $-240 = 40t - 5t^2$

or $5t^2 - 40t - 240 = 0$

or $t^2 - 8t - 48 = 0$

$t^2 - 12t + 4t - 48 = 0$

or $t\,(t - 12) + 4\,(t - 12) = 0$

or $t = 12, -4$ (not possible)

Thus, after 8 s, magnitude of relative velocity will increase upto 12 s when second particle strikes the ground.

2. *(b)* ***Central Idea*** *Given time period* $T = 2\pi\sqrt{L/g}$

Thus, changes can be expressed as

$$\pm\frac{2\Delta T}{T} = \pm\frac{\Delta L}{L} \pm \frac{\Delta g}{g}$$

$$\frac{\Delta L}{L} = \frac{0.1\text{cm}}{20.0\text{cm}} = \frac{1}{200}$$

Again time period

$$T = \frac{90}{100}\text{ s}$$

and $\Delta T = \frac{1}{100}\text{s}$

$\Rightarrow \frac{\Delta T}{T} = \frac{1}{90}$

Now,

$\because T = 2\pi\sqrt{\frac{L}{g}}$

$\because g = 4\pi^2\frac{L}{T^2}$

$\therefore \frac{\Delta g}{g} = \frac{\Delta L}{L} + \frac{2\Delta T}{T}$

or $\frac{\Delta g}{g} \times 100\% = \left(\frac{\Delta L}{L}\right) \times 100\% + \left(\frac{2\Delta T}{T}\right) \times 100\%$

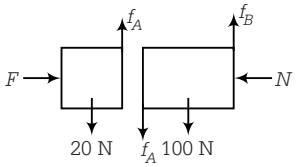
$= \left(\frac{1}{200} \times 100\right)\% + 2 \times \frac{1}{90} \times 100\%$

$\simeq 2.72\% \simeq 3\%$

Thus, accuracy in the determination of g is approx 3%.

3. *(c)* ***Central Idea*** *In vertical direction, weights are balanced by frictional forces.*

Consider FBD of block A and B as shown in diagram below.



As the blocks are in equilibrium, balance forces are in horizontal and vertical direction.

For the system of blocks $(A + B)$.

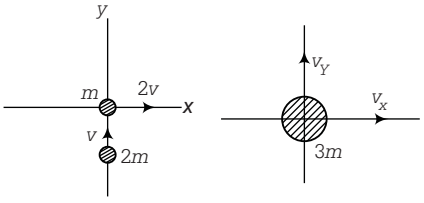
$$F = N$$

For block A, $f_A = 20\,\text{N}$ and for block B,

$$f_B = f_A + 100$$
$$= 120\ \text{N}$$

4. *(c)* ***Central Idea*** *Conservation of linear momentum can be applied but energy is not conserved.*

Consider the movement of two particles as shown below.



(Just before collision) (Just after collision)

Conserving linear momentum in x-direction

$$(p_i)x = (p_f)x$$

or $$2mv = (2m + m)\, v_x$$

or $$v_x = \frac{2}{3}v$$

Conserving linear momentum in y-direction

$(p_i)y = (p_f)y$ or $2mv = (2m + m)\, v_y$

or $$v_y = \frac{2}{3}\, v$$

Initial kinetic energy of the two particles system is

$$E_i = \frac{1}{2}\, m\,(2v)^2 + \frac{1}{2}\,(2m)\,(v)^2$$
$$= \frac{1}{2} \times 4mv^2 + \frac{1}{2} \times 2mv^2$$
$$= 2mv^2 + mv^2 = 3mv^2$$

Final energy of the combined two particles system is

$$E_f = \frac{1}{2}\,(3m)\,(v_x^2 + v_y^2)$$
$$= \frac{1}{2}\,(3m)\left[\frac{4v^2}{9} + \frac{4v^2}{9}\right]$$
$$= \frac{3m}{2}\left[\frac{8\,v^2}{9}\right] = \frac{4mv^2}{3}$$

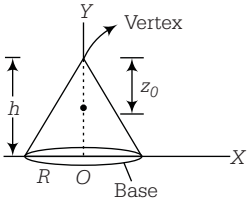
loss in the energy $\Delta E = E_i - E_f$

$$= mv^2\left[3 - \frac{4}{3}\right] = \frac{5}{3}\, mv^2$$

Percentage loss in the energy during the collision

$$\frac{\Delta E}{E_i} \times 100 = \frac{\frac{5}{3}\, mv^2}{3mv^2} \times 100 = \frac{5}{9} \times 100$$
$$\simeq 56\%$$

5. *(b)* We know that centre of mass of a uniform solid cone of height h is at height $h/4$ from base, therefore

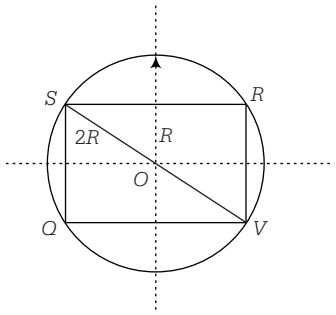


$$h - z_0 = \frac{h}{4}$$

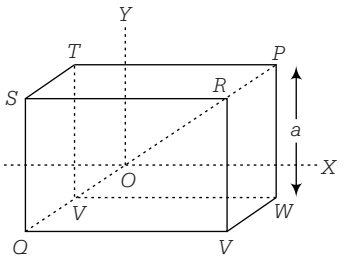
or $$z_0 = h - \frac{h}{4} = \frac{3h}{4}$$

6. *(c)* ***Central Idea*** *Use geometry of the figure to calculate mass and side length of the cube interms of M and R respectively.*

Consider the cross-sectional view of a diametric plane as shown in the adjacent diagram.



Cross-sectional view of the cube and sphere



Using geometry of the cube

$$PQ = 2R = (\sqrt{3})\, a \quad \text{or} \quad a = \frac{2R}{\sqrt{3}}$$

Volume density of the solid sphere $\rho = \dfrac{M}{\frac{4}{3}\pi R^3} = \dfrac{3}{4\pi}\left(\dfrac{M}{R^3}\right)$

Mass of cube $(m) = (\rho)(a)^3$

$$= \left(\frac{3}{4\pi} \times \frac{M}{R^3}\right)\left[\frac{2R}{\sqrt{3}}\right]^3$$

$$= \frac{3M}{4\pi R^3} \times \frac{8R^3}{3\sqrt{3}} = \frac{2M}{\sqrt{3}\pi}$$

Moment of inertia of the cube about given axis is

$$I_Y = \frac{ma^2}{12}(a^2 + a^2) = \frac{ma^2}{6}$$

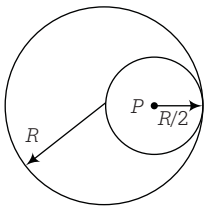
$$\Rightarrow \quad I_Y = \frac{ma^2}{6} = \frac{2M}{\sqrt{3}\pi} \times \frac{1}{6} \times \frac{4R^2}{3}$$

$$= \frac{4MR^2}{9\sqrt{3}\pi}$$

7. *(b)* ***Central Idea*** *Consider cavity as negative mass and apply superposition of gravitational potential.*

Consider the cavity formed in a solid sphere as shown in figure.

$V(\infty) = 0$



According to the question,we can write potential at an internal point P due to complete solid sphere,

$$V_s = -\frac{GM}{2R^3}\left[3R^2 - \left(\frac{R}{2}\right)^2\right]$$

$$= \frac{-GM}{2R^3}\left[3R^2 - \frac{R^2}{4}\right]$$

$$= \frac{-GM}{2R^3}\left[\frac{11R^2}{4}\right] = \frac{-11GM}{8R}$$

Mass of removed part

$$= \frac{M}{\frac{4}{3} \times \pi R^3} \times \frac{4}{3}\pi\left(\frac{R}{2}\right)^3 = \frac{M}{8}$$

Potential at point P due to removed part

$$V_c = \frac{-3}{2} \times \frac{GM/8}{\frac{R}{2}} = \frac{-3GM}{8R}$$

Thus, potential due to remaining part at point P,

$$V_P = V_s - V_c = \frac{-11GM}{8R} - \left(-\frac{3GM}{8R}\right)$$

$$= \frac{(-11+3)GM}{8R} = \frac{-GM}{R}$$

8. *(a)* We know that time period, $T = 2\pi\sqrt{\dfrac{L}{g}}$

When additional mass M is added to its bob

$$T_M = 2\pi\sqrt{\frac{L + \Delta L}{g}}$$

where, ΔL is increase in length.

We know that

$$Y = \frac{Mg/A}{\Delta L/L} = \frac{MgL}{A\Delta L}$$

$$\Rightarrow \quad \Delta L = \frac{MgL}{AY}$$

$$\Rightarrow \therefore \quad T_M = 2\pi\sqrt{\frac{L + \frac{MgL}{AY}}{g}}$$

$$\Rightarrow \quad \left(\frac{T_M}{T}\right)^2 = 1 + \frac{Mg}{AY}$$

or $$\frac{Mg}{AY} = \left(\frac{T_M}{T}\right)^2 - 1$$

or $$\frac{1}{Y} = \frac{A}{Mg}\left[\left(\frac{T_M}{T}\right)^2 - 1\right]$$

9. *(c)* According to question,

$$p = \frac{1}{3}\left(\frac{U}{V}\right)$$

$$\Rightarrow \quad \frac{nRT}{V} = \frac{1}{3}\left(\frac{U}{V}\right) \qquad [\because pV = nRT]$$

or $$\frac{nRT}{V} \propto \frac{1}{3}T^4$$

or $$VT^3 = \text{constant}$$

or $$\frac{4}{3}\pi R^3 T^3 = \text{constant}$$

or $$TR = \text{constant}$$

$$\Rightarrow \quad T \propto \frac{1}{R}$$

10. *(b)* Since, entropy is a state function, therefore change in entropy in both the processes must be same.Therefore, correct option should be (b).

11. *(c)* ***Central Idea*** *For an adiabatic process* $TV^{\gamma-1} = \text{constant}$.

We know that average time of collision between molecules

$$\tau = \frac{1}{n\pi\sqrt{2}\, v_{rms}\, d^2}$$

where, n = number of molecules per unit volume

v_{rms} = rms velocity of molecules

As $n \propto \frac{1}{V}$ and $v_{rms} \propto \sqrt{T}$

$$\tau \propto \frac{V}{\sqrt{T}}$$

Thus, we can write

$$n = K_1 V^{-1} \quad \text{and} \quad v_{rms} = K_2\, T^{1/2}$$

where, K_1 and K_2 are constants.

For adiabatic process, $TV^{\gamma-1}$ = constant.

Thus, we can write

$$\tau \propto VT^{-1/2} \propto V\,(V^{1-\gamma})^{-1/2}$$

or $$\tau \propto V^{\frac{\gamma+1}{2}}$$

12. ***(b)*** During oscillation, motion of a simple pendulum KE is maximum of mean position where PE is minimum. At extreme position, KE is minimum and PE is maximum. Thus, correct graph is depicted in option (b).

13. ***(b)*** Apparent frequency heard by the person before crossing the train.

$$f_1 = \left(\frac{c}{c - v_s}\right) f_0 = \left(\frac{320}{320 - 20}\right) 1000$$

Similarly, apparent frequency heard, after crossing the trains

$$f_2 = \left(\frac{c}{c + v_s}\right) f_0 = \left(\frac{320}{320 + 20}\right) 1000$$

[c = speed of sound]

$$\Delta f = f_1 - f_2 = \left(\frac{2\, cv_s}{c^2 - v_s^2}\right) f_0$$

or $$\frac{\Delta f}{f_0} \times 100 = \left(\frac{2cv_s}{c^2 - v_s^2}\right) \times 100$$

$$= \frac{2 \times 320 \times 20}{300 \times 340} \times 100$$

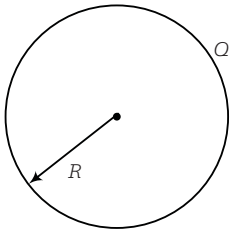
$$= \frac{2 \times 32 \times 20}{3 \times 34} = 12.54\% = 12\%$$

14. ***(a)*** Field lines should originate from positive charge and terminate to negative charge. Thus, (b) and (c) are not possible.

Electric field lines cannot form corners as shown in (d).

Thus, correct option is (a).

15. ***(c, d)*** Potential at the surface of the charged sphere



Charged sphere

$$V_0 = \frac{KQ}{R}$$

$$V = \frac{KQ}{r}, r \geq R$$

$$= \frac{KQ}{2R^3}(3R^2 - r^2); r \leq R$$

$$V_{centre} = V_c = \frac{KQ}{2R^3} \times 3R^2$$

$$= \frac{3KQ}{2R} = \frac{3V_0}{2}$$

$\Rightarrow$ $R_1 = 0$

As potential decreases for outside points.

Thus, according to the question, we can write

$$V_{R_2} = \frac{5V_0}{4} = \frac{KQ}{2R^3}(3R^2 - R_2^2)$$

$$\frac{5V_0}{4} = \frac{V_0}{2R^2}(3R^2 - R_2^2)$$

or $$\frac{5}{2} = 3 - \left(\frac{R_2}{R}\right)^2$$

or $$\left(\frac{R_2}{R}\right)^2 = 3 - \frac{5}{2} = \frac{1}{2}$$

or $$R_2 = \frac{R}{\sqrt{2}}$$

Similarly,

$$V_{R_3} = \frac{3V_0}{4}$$

$\Rightarrow$ $$\frac{KQ}{R_3} = \frac{3}{4} \times \frac{KQ}{R}$$

or $$R_3 = \frac{4}{3} R$$

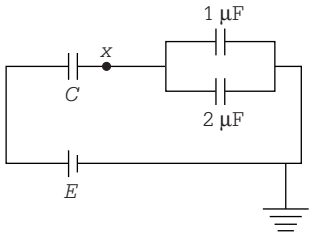
$$V_{R4} = \frac{KQ}{R_4} = \frac{V_0}{4}$$

$\Rightarrow$ $$\frac{KQ}{R_4} = \frac{1}{4} \times \frac{KQ}{R}$$

or $$R_4 = 4R$$

16. *(b)* ***Central Idea*** *Assume negative terminal of the battery as grounded* (0 V).

Suppose, potential of point x is V.



From the circuit diagram, we can write

$$Q_C = Q_1 + Q_2$$

or $$C(E - V) = 1 \times V + 2 \times V$$

or $$V[C + 3] = CE$$

or $$V = \frac{CE}{3 + C}$$

$\therefore$ $$Q_2 = C_2 (V)$$

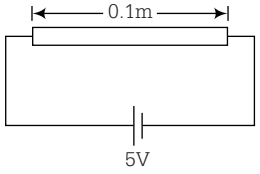
$$= \frac{2CE}{3 + C}$$

$$= \frac{2E}{1 + 3/C}$$

As C_1 varied from 1 μF to 3 μF, charge increases with decreasing slope.

Note As $C \to \infty$, $Q_2 \to 2E$ = constant

17. *(d)* According to the question



$$v_d = 2.5 \times 10^{-4} \text{ m/s}$$

$\Rightarrow$ $$n = 8 \times 10^{28} / \text{m}^3$$

we know that

$$J = nev_d \text{ or } I = nev_d A$$

where, symbols have their usual meaning.

$\Rightarrow$ $$\frac{V}{R} = nev_d A$$

or $$\frac{V}{\frac{\rho L}{A}} = nev_d A$$

or $$\frac{V}{\rho L} = nev_d$$

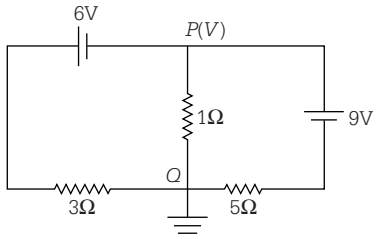
or $$\rho = \frac{V}{nev_d L}$$

$$= \frac{5}{8 \times 10^{28} \times 1.6 \times 10^{-19} \times 2.5 \times 10^{-4} \times 0.1}$$

or $$\rho = 1.6 \times 10^{-5} \ \Omega\text{m}$$

18. *(c)* ***Central Idea*** *Connect point Q to ground and apply KCL.*

Consider the grounded circuit as shown below.



Applying KCL of point Q we can write

Incoming current at Q = outgoing current from Q

$\Rightarrow$ $$\frac{V + 6}{3} + \frac{V}{1} = \frac{9 - V}{5}$$

or $$V\left[\frac{1}{3} + \frac{1}{5} + 1\right] = \frac{9}{5} - 2$$

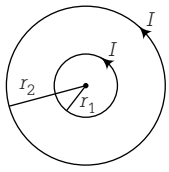
or $$V\left[\frac{5 + 3 + 15}{15}\right] = \frac{9 - 10}{5}$$

or $$V\left[\frac{23}{15}\right] = \frac{-1}{5}$$

or $$V = \frac{-3}{23} = -0.13 \text{ V}$$

Thus, current in the 1 Ω resistance is 0.13 A, from Q to P.

19. *(a)* Consider the two coaxial solenoids. Due to one of the solenoids magnetic field at the centre of the other can be assumed to be constant.

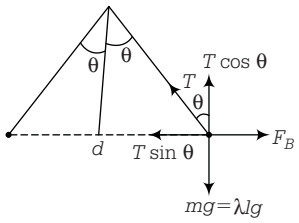


Due to symmetry, forces on upper and lower part of a solenoid will be equal and opposite and hence resultant is zero.

Therefore $F_1 = F_2 = 0$

20. *(b)* Consider free body diagram of the wire.

As the wires are in equilibrium, they must carry current in opposite direction .



Here, $F_B = \frac{\mu_0 I^2 l}{2\pi d}$, where l is length of each wire and d is separation between wires.

From figure, $d = 2L\sin\theta$

$$T = \cos\theta = mg = \lambda lg$$

(in vertical direction)…(i)

$$T\sin\theta = F_B = \frac{\mu_0 I^2 l}{4\pi L\sin\theta}$$

(in horizontal direction)…(ii)

From Eqs. (i) and (ii),

$$\frac{T\sin\theta}{T\cos\theta} = \frac{\mu_0 I^2 l}{4\pi L\sin\theta \times \lambda lg}$$

$$\therefore \quad I = \sqrt{\frac{4\pi\lambda Lg\sin^2\theta}{\mu_0\cos\theta}}$$

$$= 2\sin\theta\sqrt{\frac{\pi\lambda Lg}{\mu_0\cos\theta}}$$

21. *(c) Since* B *is uniform only torque acts on a current carrying loop.*

As, $\tau = M \times B$

$\Rightarrow$ $|\tau| = |M||B|\sin\theta$

For orientation shown in (b) $\theta = 0^0$, $\tau = 0$ (stable equilibrium)

and for (d) $\theta = \pi$, $\tau = 0$ (unstable equilibrium)

22. *(d)* ***Central Idea*** *After long time inductor behave as short-circuit.*

At $t = 0$, the inductor behaves as short-circuited. The current

$$I_0 = \frac{E_0}{R} = \frac{15\,\text{V}}{0.15\,\text{k}\Omega} = 100\ \text{mA}$$

As K_2 is closed, current through the inductor starts decay, which is given at any time t as

$$I = I_0\, e^{\frac{-tR}{L}} = (100\ \text{mA})\, e^{\frac{-t\times 15000}{3}}$$

At $t = 1\,\text{ms}$

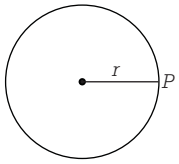
$$I = (100\,\text{mA})e^{-\frac{1\times10^{-3}\times15\times10^3}{3}}$$

$$I = (100\ \text{mA})\, e^{-5} = 0.6737\ \ \text{mA}$$

or $I = 0.67$ mA

23. *(b)* Consider the LED as a point source of light.

Let power of the LED is P.



Intensity at r from the source

$$I = \frac{P}{4\pi r^2} \qquad \text{...(i)}$$

As we know that $I = \frac{1}{2}\varepsilon_0 E_0^2\ c$ ...(ii)

From Eqs. (i) and (ii), we can write

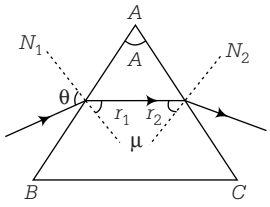
$$\frac{P}{4\pi r^2} = \frac{1}{2}\varepsilon_0 E_0^2\ c$$

or $E_0^2 = \frac{2P}{4\pi\varepsilon_0 r^2 c} = \frac{2\times0.1\times9\times10^9}{1\times3\times10^8}$

or $E_0^2 = 6 \Rightarrow E_0 = \sqrt{6} = 2.45\ \text{V/m}$

24. *(a)* ***Central Idea*** *The ray will get transmitted through face AC if* $i_{AC} < i_C$

Consider the ray diagram is shown below.



A ray of light incident on face AB at an angle θ.

r_1 = Angle of refraction on face AB

r_2 = Angle of incidence at face AC

For transmission of light through face AC

$$i_{AC} < i_C \quad \text{or} \quad A - r_1 < i_C$$

or $\sin(A - r_1) < \sin i_C$

or $\sin(A - r_1) < \frac{1}{\mu}$

$$A - r_1 < \sin^{-1}\left(\frac{1}{\mu}\right)$$

or $\sin r_1 > \sin\left[A - \sin^{-1}\left(\frac{1}{\mu}\right)\right]$

Now, applying Snell's law at the face AB

$$1 \times \sin\theta = \mu \sin r_1 \quad \text{or} \quad \sin r_1 = \frac{\sin\theta}{\mu}$$

$$\Rightarrow \frac{\sin\theta}{\mu} > \sin\left[A - \sin^{-1}\left(\frac{1}{\mu}\right)\right]$$

$$\text{or} \quad \theta > \sin^{-1}\left[\mu \sin\left\{A - \sin^{-1}\left(\frac{1}{\mu}\right)\right\}\right]$$

25. *(d)* According to Snell's law,

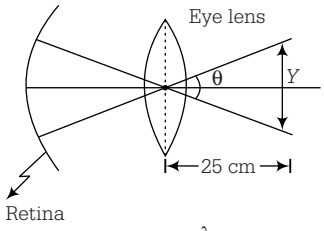
$$\mu \sin\theta = \text{Constant}$$

$$\therefore \quad \sin\theta \propto \frac{1}{\mu}$$

As μ increases, θ decreases.

Hence, beam will bend upward.

26. *(b)* We can write resolving angle of necked eye as



$$\theta = 1.22\,\frac{\lambda}{D}$$

Where, D is the diameter of eye lens.

$$\text{or} \quad \frac{Y}{25 \times 10^{-2}} = \frac{1.22 \times 500 \times 10^{-9}}{0.25 \times 2 \times 10^{-2}}$$

$$Y = 30 \times 10^{-6}\ \text{m} = 30\ \mu\text{m}$$

27. *(a)* As we know that kinetic energy of an electron is

$$\text{KE} \propto \left(\frac{Z}{n}\right)^2$$

When the electron makes transition from an excited state to the ground state, then n decreases and KE increases. We know that PE is lowest for ground state. As TE = – KE. TE also decreases.

28. *(c)* (A) Franck-Hertz experiments is associated with discrete energy levels of atom.

(B) Photo-electric experiment is associated with particle nature of light.

(C) Davisson-Germer experiment is associated with wave nature of electron.

29. *(c)* ***Central Idea*** *Frequency associated with AM are* $f_c - f_m, f, f_c + f_m$

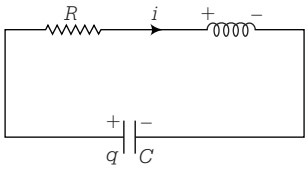
According to the question

$$f_c = 2\,\text{MHz} = 2000\ \text{kHz}$$

$$f_m = 5\ \text{kHz}$$

Thus, frequency of the resultant signal is/are carrier frequency $f_c = 2000$ kHz, LSB frequency $f_c - f_m = 2000\,\text{kHz} - 5\,\text{kHz} = 1995$ kHz and USB frequency $f_c + f_m = 2005$ kHz

30. *(a)* Considert the LCR circuit at any time t



Now, applying KVL

We have $\frac{q}{C} - iR - \frac{Ldi}{dt} = 0$

As current is decreasing with time we can write $i = -\frac{dq}{dt}$

$$\Rightarrow \quad \frac{q}{C} + \frac{dq}{dt}R + \frac{Ld^2q}{dt^2} = 0$$

$$\text{or} \quad \frac{d^2q}{dt^2} + \frac{R}{L}\frac{dq}{dt} + \frac{q}{LC} = 0$$

This equation is equivalent to that of a damped oscillator

Thus, we can write the solution as

$$Q_{\max}(t) = Q_0 \cdot e^{-Rt/2L}$$

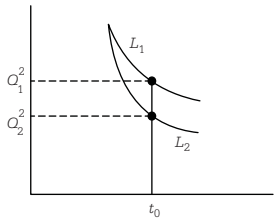
$$\text{or} \quad Q^2_{\max} = Q_0^2\, e^{-\frac{Rt}{L}}$$

As $L_1 > L_2$ damping is faster for L_2

Aliter

Inductance is inertia of circuit. It means inductance opposes the flow of charge, more inductance means decay of charge is slow.

In option (a), in a given time to, $Q_1^2 > Q_2^2$.



So, $L_1 > L_2$.

Hence option (a) is correct.

Chemistry

1. *(d)* We know the molecular weight of $C_8H_7SO_3Na$

$= 12 \times 8 + 1 \times 7 + 32 + 16 \times 3 + 23 = 206$

We have to find, mole per gram of resin.

$\therefore$ 1 g of $C_8H_7SO_3Na$ has number of mole

$$= \frac{\text{Weight of given resin}}{\text{Molecular weight of resin}}$$

$$= \frac{1}{206} \text{ mol}$$

Now, reaction looks like

$$2C_8H_7SO_3Na + Ca^{2+} \longrightarrow (C_8H_7SO_3)_2Ca + 2Na^+$$

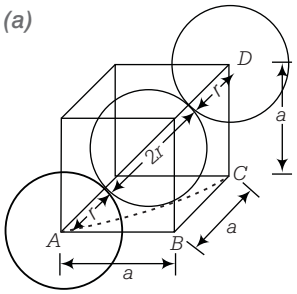
$\because$ 2 moles of $C_8H_7SO_3Na$ combines with 1 mol Ca^{2+}

$\therefore$ 1 mole of $C_8H_7SO_3Na$ will combine with $\frac{1}{2}$ mol Ca^{2+}

$\therefore \frac{1}{206}$ mole of $C_8H_7SO_3Na$ will combine with

$$\frac{1}{2} \times \frac{1}{206} \text{ mol } Ca^{2+} = \frac{1}{412} \text{ mol } Ca^{2+}$$

2. *(a)*



From this figure ,

$$(AC)^2 = (AB)^2 + (BC)^2$$

$$(AC)^2 = a^2 + a^2 = 2a^2$$

Also, $(AD)^2 = (AC)^2 + (DC)^2$

$$(4r)^2 = 2a^2 + a^2$$

$$16r^2 = 3a^2$$

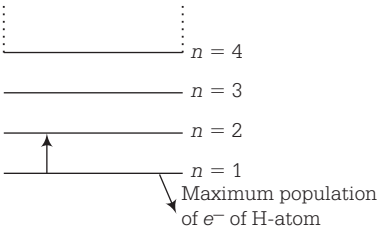
$$r = \frac{\sqrt{3}}{4} a$$

Now, when Na metal crystallises in bcc unit cell with unit cell edge, $a = 4.29$Å

We have the formula for radius,

i.e. $r = \frac{\sqrt{3}}{4} \times 4.29 \text{ Å} = 1.86$Å

3. *(c)*



Since, at $n = 1$, the population of electrons is maximum *i.e.* at ground state. So, maximum excitation will take place from $n = 1$ to $n = 2$.

Hence, $n = 2$ is the possible excited state.

Now, we have the formula for energy of H-atom

$$(E_n)_H = -13.6 \frac{Z^2}{n^2} \text{eV}$$

where, Z = atomic number

Z for H-atom = 1

$$\therefore \quad (E_n)_H = -13.6 \times \frac{1}{2^2} \text{eV}$$

$$= -\frac{13.6}{4} \text{eV}$$

$$= -3.4 \text{ eV}$$

4. *(b)* Ion-ion interaction is dependent on the square of distance,

i.e. ion-ion interaction $\propto \frac{1}{r^2}$

Similarly,

ion-dipole interaction $\propto \frac{1}{r^3}$

London forces $\propto \frac{1}{r^6}$

and dipole-dipole interactions $\propto \frac{1}{r^3}$

Superficially it seems as both ion dipole interaction and hydrogen bonding vary with the inverse cube of distance between the molecules but when we look at the exact expressions of field (force) created in two situations it comes as

$$|\mathbf{E}| \text{ or } |\mathbf{F}| = \frac{2|\mathbf{P}|}{4\pi\varepsilon_0 r^3}$$

(In case of ion-dipole interaction)

and $F = \frac{2q^2r - 4q^2a}{4\pi\varepsilon_0 r^3}$

(In case of dipole-dipole interaction)

From the above it is very clear, the ion-dipole interaction is the better answer as compared to dipole-dipole interaction i.e. hydrogen bonding.

5. ***(d)*** For the given reaction,

$$2NO(g) + O_2(g) \rightleftharpoons 2NO_2(g)$$

Given, ΔG_f° (NO) = 86.6 kJ / mol

ΔG_f° (NO_2) = ?

$K_p = 1.6 \times 10^{12}$

Now, we have,

$\Delta G_f^\circ = 2\Delta G_{f(NO_2)}^\circ - [2\Delta G_{f(NO)}^\circ + \Delta G_{f(O_2)}^\circ]$

$= -RT \ln K_p = 2\Delta G_{f(NO_2)}^\circ - [2 \times 86{,}600 + 0]$

$\Delta G_{f(NO_2)}^\circ = \frac{1}{2}$

$[2 \times 86{,}600 - R \times 298 \ln (1.6 \times 10^{12})]$

$\Delta G_{f(NO_2)}^\circ = 0.5$

$[2 \times 86{,}600 - R \times (298) \ln (1.6 \times 10^{12})]$

6. ***(b)*** Given,

p_o = 185 torr at 20°C

p_s = 183 torr at 20°C

Mass of non-volatile substance,

m = 1.2 g

Mass of acetone taken = 100 g

M = ?

As , we have

$$\frac{p_o - p_s}{p_s} = \frac{n}{N}$$

Putting the values, we get,

$$\frac{185 - 183}{183} = \frac{\frac{1.2}{M}}{\frac{100}{58}} \Rightarrow \frac{2}{183} = \frac{1.2 \times 58}{100 \times M}$$

$$\therefore \quad M = \frac{183 \times 1.2 \times 58}{2 \times 100}$$

$M = 63.684 \approx 64$ g / mol

7. ***(b)*** Given, $\Delta G^\circ = 2494.2$J

$$Q = \frac{[B][C]}{[A]^2} = \frac{2 \times \frac{1}{2}}{\left(\frac{1}{2}\right)^2} = 4$$

$\therefore$ We know,

$\Delta G = \Delta G^\circ + RT \ln Q$

$= 2494.2 + 8.314 \times 300 \ln 4$

$= 28747.27$J

= positive value

Also, we have

$$\Delta G = RT \ln \frac{Q}{K}$$

if ΔG is positive, $Q > K$.

Therefore, reaction shifts in reverse direction.

8. ***(b)*** Given, $Q = 2F$

Atomic mass of Cu = 63.5 u

Valency of the metal $Z = 2$

We have, $CuSO_4 \longrightarrow Cu^{2+} + SO_4^{2-}$

$$\underset{1\text{mol}}{Cu^{2+}} + \underset{2F}{\underset{2\text{ mol}}{2e^-}} \longrightarrow \underset{1\text{mol} = 63.5\text{g}}{Cu}$$

Alternatively, $W = ZQ$

$$= \frac{E}{F} \cdot 2F = 2E$$

$$= \frac{2 \times 63.5}{2} = 63.5$$

9. ***(a)*** The main conditions for the occurrence of a reaction is proper orientation and effective collision of the reactants.

Since the chances of simultaneous collision with proper orientation between more than 3 species is very rare, so reaction with order greater than 3 are rare.

10. ***(a)*** Given, initial strength of acetic acid = 0.06 N

Final strength = 0.042 N

Volume given = 50 mL

$\therefore$ Initial m moles of CH_3COOH

$= 0.06 \times 50 = 3$

Final m moles of CH_3COOH

$= 0.042 \times 50 = 2.1$

$\therefore$ m moles of CH_3COOH adsorbed

$= 3 - 2.1$

= 0.9 m mol

Hence, mass of CH_3COOH absorbed per gram of charcoal

$$= \frac{0.9 \times 60}{3}$$

($\because$ molar mass of $CH_3COOH = 60\ gmol^{-1}$)

$$= \frac{54}{3} = 18 \text{ mg.}$$

11. *(c)* Number of electrons in $N^{3-} = 7 + 3 = 10$

Number of electrons in $O^{2-} = 8 + 2 = 10$
Number of electrons in $F^{-} = 9 + 1 = 10$

Since, all the three species have each 10 electrons hence they are isoelectronic species.

It is considered that, in case of isoelectronic species as the negative charge increases, ionic radii increases and therefore the value of ionic radii are

$N^{3-} = 1.71$ (highest among the three)
$O^{2-} = 1.40$
$F^{-} = 1.36$ (lowest among the three)

Time Saving Technique *There is no need to mug up the radius values for different ions. This particular question can be solved through following time saving*.

Trick *The charges on the ions indicate the size as* $N^{3-} > O^{2-} > F^{-}$. *Thus, you have to look for the option in which the above trend is followed. Option (c) is the only one in which this trend is followed. Hence, it is the correct answer.*

12. *(d)* (a) In Hall-Heroult process for extraction of Al, carbon anode is oxidised to CO and CO_2.

(b) When Al_2O_3 is mixed with CaF_2, it lowers the melting point of the mixture and brings conductivity.

(c) Al^{3+} is reduced at cathode to form Al.

(d) Here, Al_2O_3 is an electrolyte, undergoing the redox process. Na_3AlF_6 although is an electrolyte but serves as a solvent, not electrolyte.

13. *(a)* H_2O_2 acts as an oxidising as well as reducing agent, because oxidation number of oxygen in H_2O_2 is –1. So, it can be oxidised to oxidation state 0 or reduced to oxidation state –2.

H_2O_2 decomposes on exposure to light. So, it has to be stored in plastic or wax lined glass bottles in dark for the prevention of exposure. It also has to be kept away from dust.

14. *(b)* As we move down the group, size of metal increases. Be has lower size while SO_4^{2-} has bigger size, that's why $BeSO_4$ breaks easily and lattice energy becomes smaller but due to lower size of Be, water molecules are gathered around and hence hydration energy increases.

On the other hand, rest metals i.e. Ca, Ba, Sr have bigger size and that's why lattice energy is greater than hydration energy.

Time Saving Technique *In the question of finding hydration energy only check the size of atom. Smaller sized atom has more hydration energy.*

Thus, in this question Be *is placed upper most in the group has lesser size and not comparable with the size of sulphates. Hence,* $BeSO_4$ *is the right response.*

15. *(d)* Cl_2, Br_2 and I_2 are homonuclear diatomic molecule in which electronegativity of the combining atoms is same, so they are more stable and less reactive, whereas, I and Cl have different electronegativities and bond between them are polarised and hence, reactive. Therefore, interhalogen compounds are more reactive.

Time Saving Technique *In this type of question of halogen, only go through the polarity of the molecule. As we know, diatomic molecule does not have polarity but molecules with dissimilar sizes have polarity resulting in more reactivity.*

16. *(b)* (a) $TiCl_3$ is used as Ziegler-Natta catalyst for the polymerisation of ethene.

(b) $PdCl_2$ is used in Wacker process, in which alkene changed into aldehyde *via* catalytic cyclic process initiated by $PdCl_2$.

(c) $CuCl_2$ is used in Deacon's process. (for Cl_2)

(d) V_2O_5 is used in contact process of manufacturing sulphuric acid.

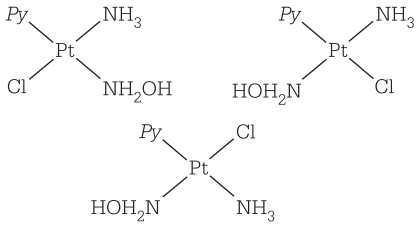
Time Saving Technique *This type of questions can also be solved through elimination technique. There is no need to know all the four matches to select the correct response. Even if you know (b) matches then also you can solve the problem. e.g. suppose you know the usage of* V_2O_5 *in contact process (i.e. D matches with (iii) and* $TiCl_3$ *is connected to Ziegler-Natta catalyst (i.e. A matches with ii). These two combinations are present only in option number (b). Likewise, for this question particularly if you know that* V_2O_5 *is used in contact process then this combination is present in option (b) only out of all the four*

option given. In this way you can eliminate wrong options to get the correct response.

17. *(d)* As we move down the group of noble gases , molecular mass increases by which dipole produced for a moment and hence London forces increases from He to Xe. Therefore more amount of energy is required to break these forces, thus boiling point also increases from He to Xe.

18. *(b)* $[Pt(Cl)(py)(NH_3)(NH_2OH)]^+$ is square planar complex.

The structures are formed by fixing a group and then arranging all the groups.



Hence, this complex shows three geometrical isomers.

19. *(c)* $KMnO_4 \longrightarrow K^+ + MnO_4^-$

$\therefore$ In MnO_4^-, Mn has +7 oxidation state having no electron in *d*-orbitals.

It is considered that higher the oxidation state of metal, greater is the tendency to occur $L \rightarrow M$ charge transfer, because ligand is able to donate the electrons into the vacant *d*- orbital of metal.

Since, charge transfer is Laporte as well as spin allowed, therefore, it shows colour.

Time Saving Technique *There is no need to check all the four options. Just find out the oxidation state of metal ion. If oxidation state is highest and ligand present there is of electron donating nature, gives LMCT, which shows more intense colour.*

20. *(a)* Nitrogen is an inert gas because of the presence of strong bond. That's why although there is 78% N_2 in the atmosphere but nitrogen oxide in not formed under ordinary conditions.

But when temperature is high enough i.e. $\approx$ 2000 K, it reacts with oxygen to form nitrogen oxide.

$$N_2 + O_2 \xrightarrow{\approx 2000\ K} 2NO$$

Thus, Assertion and Reason are true and Reason is the correct explanation of the Assertion.

21. *(a)* Given,

Weight of organic compound = 250 mg

Weight of AgBr = 141 mg

$\therefore$ According to formula of % of bromine by Carius method

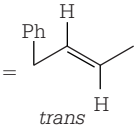
$$\% \text{ of Br} = \frac{\text{Atomic weight of Br}}{\text{Molecular weight of AgBr}} \times \frac{\text{Weight of AgBr}}{\text{Weight of organic bromide}} \times 100$$

$$\therefore \quad \% \text{ of Br} = \frac{80}{188} \times \frac{141}{250} \times 100$$

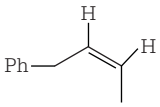
$$= \frac{1128000}{47000} = 24\ \%$$

22. *(a)* Alkene in which different groups are attached with the double bonded carbon atoms, exhibit geometrical isomerism.

1-phenyl-2-butene =



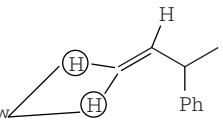
trans



cis

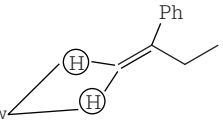
It will show geometrical isomerism.

3-phenyl-1-butene



Same group, does not show geometrical isomerism.

2-phenyl-1-butene

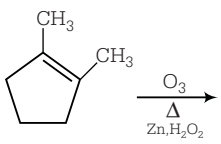


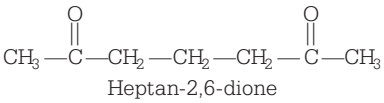
Same group, does not show geometrical isomerism.

1,1-diphenyl-1-propane being an alkane (saturated compound) does not show geometrical isomerism.

Time Saving Technique *We do not need the check all options, but it should remind that double bonded compounds show geometrical isomerism. Thus, (d) is eliminated, (i.e. propane). Now, eliminate the terminal alkene and get the correct response.*

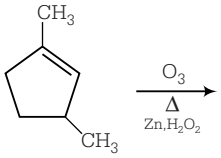
23. *(b)* (a)

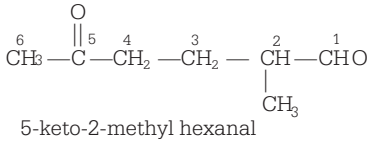




Heptan-2,6-dione

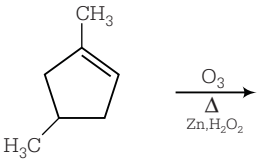
(b)

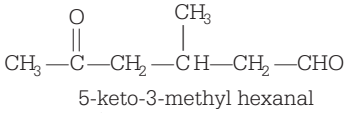




5-keto-2-methyl hexanal

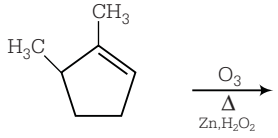
(c)

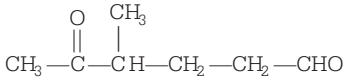




5-keto-3-methyl hexanal

(d)





5-keto-4-methyl hexanal

24. ***(d)*** Alkyl fluorides can be prepared by action of mercurous fluoride or antimony trifluorides (inorganic fluorides) on corresponding alkyl halide. This reaction is known as Swarts reaction.

$$CH_3Br + AgF \longrightarrow \underset{\text{Methyl fluoride}}{CH_3F} + AgBr$$

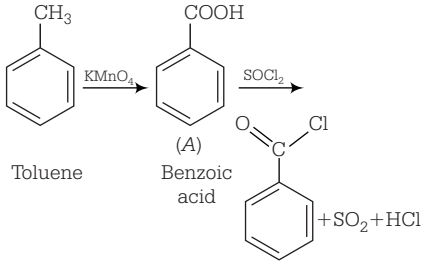
But, when action of NaI/acetone takes place on alkyl chloride or bromide, alkyl iodide forms. This reaction is called 'Finkelstein reaction'.

$$C_2H_5Cl \xrightarrow[\text{acetone}]{NaI} C_2H_5I + NaCl$$

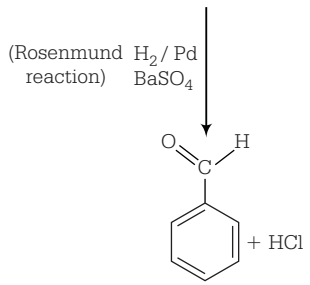
Free radical fluorination is highly explosive reaction. so not preferred for the preparation of fluoride.

25. ***(d)*** Toluene undergoes oxidation with $KMnO_4$, forms benzoic acid. In this conversion, alkyl part of toluene converts into carboxylic group. Further, benzoic acid reacts with thionyl chloride ($SOCl_2$) to give benzoyl chloride which upon reduction with H_2/Pd or $BaSO_4$ forms benzaldehyde (Rosenmund Reduction).

The conversion look like,

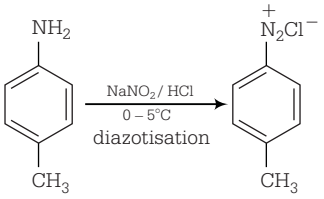


Toluene — (*A*) Benzoic acid — (*B*) Benzoyl chloride

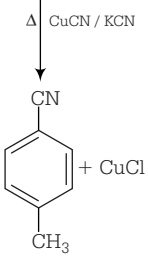


(*C*) Benzaldehyde

26. ***(c)***



4- methyl aniline — (*D*) 4- methyl diazonium chloride



(*E*) 4- methyl benzocyanide

27. ***(b)*** (a) Bakelite is used for making gears, protective coating and electrical fittings.

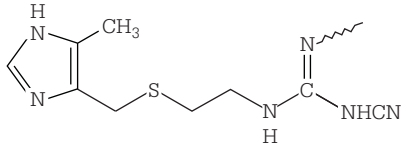
(b) Glyptal is used in the manufacture of paints and lacquers.

(c) PP is used in the manufacture of textile, packaging materials etc.

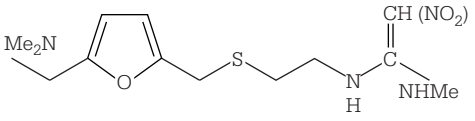
(d) Polyvinyl chloride (PVC) is used in the manufacture of rain coats, hand bags, leather clothes etc.

28. *(a)* Vitamin B and C are water soluble while vitamin A,D,E and K are fat soluble or water insoluble.

29. *(c)* Aluminium hydroxide $Al(OH)_3$, cimetidine and ranitidine are antacids while phenelzine is not.

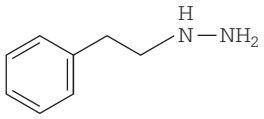


Cimetidine



Ranitidine

Phenelzine is a tranquilizer, not an antacid.



30. *(a)* $Zn_2[Fe(CN)_6]$, $K_3[Co(NO_2)_6]$and $[(NH_4)_3As(Mo_3O_{10})_4]$ show colour due to *d-d* transition while $BaCrO_4$ is coloured due to charge transfer phenomenon.

Further, according to spectrochemical series the strong ligand possessing complex has higher energy and hence lower wavelength. Therefore, complexes containing NO_2, NH_4^+, O^{2-} etc ligands show yellow colour while CN^- forces the complex to impart white colour.

Spectrochemical Series

$$I^- < Br^- < S^{2-} < SCN^- < Cl^- < NO_3^- < N_3^- < F^- < OH^- < C_2O_4^{2-} \approx H_2O < NCS^- < CH_3CN < py < NH_3 < en < bipy < Phen < NO_2^- < PPh_3 < CN^- \approx CO$$

Mathematics

1. *(a)* Given,

$$n(A) = 4,\ n(B) = 2$$

$$\Rightarrow \quad n(A \times B) = 8$$

Total number of subsets of set

$$(A \times B) = 2^8$$

Number of subsets of set $A \times B$ having no element (i.e. ϕ) = 1

Number of subsets of set $A \times B$ having one element = ${}^8C_1$

Number of subsets of set $A \times B$ having two elements = ${}^8C_2$

$\therefore$ Number of subsets having atleast three elements

$$= 2^8 - (1 + {}^8C_1 + {}^8C_2)$$
$$= 2^8 - 1 - 8 - 28$$
$$= 2^8 - 37$$
$$= 256 - 37$$
$$= 219$$

2. *(c)* ***Central Idea*** *If z is unimodular, then* $|z| = 1$. *Also, use property of modulus i.e.* $z\bar{z} = |z|^2$.

Given, z_2 is not unimodular i.e. $|z_2| \neq 1$

and $\dfrac{z_1 - 2z_2}{2 - z_1\bar{z}_2}$ is unimodular

$$\Rightarrow \quad \left|\frac{z_1 - 2z_2}{2 - z_1\bar{z}_2}\right| = 1$$

$$\Rightarrow \quad |z_1 - 2z_2|^2 = |2 - z_1\bar{z}_2|^2$$

$$\Rightarrow (z_1 - 2z_2)(\bar{z}_1 - 2\bar{z}_2) = (2 - z_1\bar{z}_2)(2 - \bar{z}_1 z_2) \quad (\because z\bar{z} = |z|^2)$$

$$\Rightarrow \quad |z_1|^2 + 4|z_2|^2 - 2\bar{z}_1 z_2 - 2z_1\bar{z}_2 = 4 + |z_1|^2|z_2|^2 - 2\bar{z}_1 z_2 - 2z_1\bar{z}_2$$

$$\Rightarrow (|z_2|^2 - 1)(|z_1|^2 - 4) = 0$$

$$\because \quad |z_2| \neq 1$$

$$\therefore \quad |z_1| = 2$$

Let $z_1 = x + iy \Rightarrow x^2 + y^2 = (2)^2$

$\therefore$ Point z_1 lies on a circle of radius 2.

3. ***(c)*** Given, α and β are the roots of the equation $x^2 - 6x - 2 = 0$.

$\because a_n = \alpha^n - \beta^n$ for $n \geq 1$

$\therefore \quad a_{10} = \alpha^{10} - \beta^{10}$

$a_8 = \alpha^8 - \beta^8$

$a_9 = \alpha^9 - \beta^9$

Now, consider

$$\frac{a_{10} - 2a_8}{2a_9} = \frac{\alpha^{10} - \beta^{10} - 2(\alpha^8 - \beta^8)}{2(\alpha^9 - \beta^9)}$$

$$= \frac{\alpha^8(\alpha^2 - 2) - \beta^8(\beta^2 - 2)}{2(\alpha^9 - \beta^9)}$$

$$= \frac{\alpha^8 \cdot 6\alpha - \beta^8 \cdot 6\beta}{2(\alpha^9 - \beta^9)}$$

$$= \frac{6\alpha^9 - 6\beta^9}{2(\alpha^9 - \beta^9)} = \frac{6}{2} = 3$$

Aliter

Since, α and β are the roots of the equation

$x^2 - 6x - 2 = 0$

or $\quad x^2 = 6x + 2$

$\therefore \quad \alpha^2 = 6\alpha + 2$

$\Rightarrow \quad \alpha^{10} = 6\alpha^9 + 2\alpha^8 \quad \ldots(i)$

Similarly, $\beta^{10} = 6\beta^9 + 2\beta^8 \quad \ldots(ii)$

On subtracting Eq. (ii) from Eq. (i), we get

$\alpha^{10} - \beta^{10} = 6(\alpha^9 - \beta^9) + 2(\alpha^8 - \beta^8)$

$\Rightarrow \quad a_{10} = 6a_9 + 2a_8 \quad (\because a_n = \alpha^n - \beta^n)$

$\Rightarrow \quad a_{10} - 2a_8 = 6a_9 \Rightarrow \frac{a_{10} - 2a_8}{2a_9} = 3$

4. ***(d)*** Given, $A = \begin{bmatrix} 1 & 2 & 2 \\ 2 & 1 & -2 \\ a & 2 & b \end{bmatrix}$

$$A^T = \begin{bmatrix} 1 & 2 & a \\ 2 & 1 & 2 \\ 2 & -2 & b \end{bmatrix}$$

$$AA^T = \begin{bmatrix} 1 & 2 & 2 \\ 2 & 1 & -2 \\ a & 2 & b \end{bmatrix} \begin{bmatrix} 1 & 2 & a \\ 2 & 1 & 2 \\ 2 & -2 & b \end{bmatrix}$$

$$= \begin{bmatrix} 9 & 0 & a + 4 + 2b \\ 0 & 9 & 2a + 2 - 2b \\ a + 4 + 2b & 2a + 2 - 2b & a^2 + 4 + b^2 \end{bmatrix}$$

It is given that $AA^T = 9I$

$\Rightarrow$

$$\begin{bmatrix} 9 & 0 & a + 4 + 2b \\ 0 & 9 & 2a + 2 - 2b \\ a + 4 + 2b & 2a + 2 - 2b & a^2 + 4 + b^2 \end{bmatrix} = 9\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$\Rightarrow$

$$\begin{bmatrix} 9 & 0 & a + 4 + 2b \\ 0 & 9 & 2a + 2 - 2b \\ a + 4 + 2b & 2a + 2 - 2b & a^2 + 4 + b^2 \end{bmatrix} = \begin{bmatrix} 9 & 0 & 0 \\ 0 & 9 & 0 \\ 0 & 0 & 9 \end{bmatrix}$$

On comparing, we get

$a + 4 + 2b = 0$

$\Rightarrow \quad a + 2b = -4 \quad \ldots(i)$

$2a + 2 - 2b = 0$

$\Rightarrow \quad a - b = -1 \quad \ldots(ii)$

and $\quad a^2 + 4 + b^2 = 9 \quad \ldots(iii)$

On solving Eqs. (i) and (ii), we get

$a = -2, b = -1$

This satisfies Eq. (iii).

Hence, $(a, b) \equiv (-2, -1)$

5. ***(c)*** Given system of linear equations

$2x_1 - 2x_2 + x_3 = \lambda x_1$

$\Rightarrow \quad (2 - \lambda)x_1 - 2x_2 + x_3 = 0 \quad \ldots(i)$

$2x_1 - 3x_2 + 2x_3 = \lambda x_2$

$\Rightarrow \quad 2x_1 - (3 + \lambda)x_2 + 2x_3 = 0 \quad \ldots(ii)$

$-x_1 + 2x_2 = \lambda x_3$

$\Rightarrow \quad -x_1 + 2x_2 - \lambda x_3 = 0 \quad \ldots(iii)$

Since, the system has non-trivial solution.

$$\therefore \quad \begin{vmatrix} 2 - \lambda & -2 & 1 \\ 2 & -(3 + \lambda) & 2 \\ -1 & 2 & -\lambda \end{vmatrix} = 0$$

$\Rightarrow (2 - \lambda)(3\lambda + \lambda^2 - 4) + 2(-2\lambda + 2)$

$+ 1(4 - 3 - \lambda) = 0$

$\Rightarrow (2 - \lambda)(\lambda^2 + 3\lambda - 4) + 4(1 - \lambda)$

$+ (1 - \lambda) = 0$

$\Rightarrow (2 - \lambda)(\lambda + 4)(\lambda - 1) + 5(1 - \lambda) = 0$

$\Rightarrow \quad (\lambda - 1)[(2 - \lambda)(\lambda + 4) - 5] = 0$

$\Rightarrow \quad (\lambda - 1)(\lambda^2 + 2\lambda - 3) = 0$

$\Rightarrow \quad (\lambda - 1)[(\lambda - 1)(\lambda + 3)] = 0$

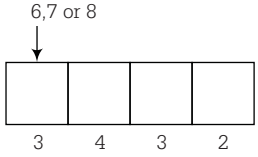
$\Rightarrow \quad (\lambda - 1)^2(\lambda + 3) = 0$

$\Rightarrow \quad \lambda = 1, 1, -3$

6. *(b)* The integer greater than 6000 may be of 4 digit or 5 digit. So, here two cases arise.

***Case* I** When number is of 4 digit.

Four digit number can starts from 6, 7 or 8



Thus, total number of 4 digit number, which are greater than 6000 $= 3 \times 4 \times 3 \times 2 = 72$

***Case* II** When number is of 5 digit.

Total number of five digit number which are greater than 6000 $= 5! = 120$

$\therefore$ Total number of integers $= 72 + 120$
$= 192$

7. *(a)* Let T_{r+1} be the general term in the expansion of $(1 - 2\sqrt{x})^{50}$

$$\therefore \quad T_{r+1} = {}^{50}C_r (1)^{50-r}(-2x^{1/2})^r$$
$$= {}^{50}C_r 2^r x^{r/2}(-1)^r$$

For the integral power of x, r should be even integer.

$$\therefore \text{ Sum of coefficients} = \sum_{r=0}^{25} {}^{50}C_{2r}(2)^{2r}$$
$$= \frac{1}{2}[(1+2)^{50} + (1-2)^{50}]$$
$$= \frac{1}{2}[3^{50} + 1]$$

Aliter

We have,

$$(1 - 2\sqrt{x})^{50} = C_0 - C_1 2\sqrt{x} + C_2(2\sqrt{x})^2 + \ldots + C_{50}(2\sqrt{x})^{50} \ldots(\text{i})$$

$$(1 + 2\sqrt{x})^{50} = C_0 + C_1 2\sqrt{x} + C_2(2\sqrt{x})^2 + \ldots + C_{50}(2\sqrt{x})^{50} \ldots(\text{ii})$$

On adding Eqs. (i) and (ii), we get

$$(1 - 2\sqrt{x})^{50} + (1 + 2\sqrt{x})^{50} = 2[C_0 + C_2(2\sqrt{x})^2 + \ldots + C_{50}(2\sqrt{x})^{50}]$$

$$\Rightarrow \quad \frac{(1-2\sqrt{x})^{50} + (1+2\sqrt{x})^{50}}{2} = C_0 + C_2(2\sqrt{x})^2 + \ldots + C_{50}(2\sqrt{x})^{50}$$

On putting $x = 1$, we get

$$\frac{(1-2\sqrt{1})^{50} + (1+2\sqrt{1})^{50}}{2} = C_0 + C_2(2)^2 + \ldots + C_{50}(2)^{50}$$

$$\Rightarrow \quad \frac{(-1)^{50} + (3)^{50}}{2} = C_0 + C_2(2)^2 + \ldots + C_{50}(2)^{50}$$

$$\Rightarrow \quad \frac{1 + 3^{50}}{2} = C_0 + C_2(2)^2 + \ldots + C_{50}(2)^{50}$$

8. *(b)* Given, m is the AM of l and n

$$\therefore \quad l + n = 2m \qquad \ldots(\text{i})$$

and G_1, G_2, G_3 are geometric means between l and n

$\therefore$ l, G_1, G_2, G_3, n are in GP.

Let r be the common ratio of this GP.

$$\therefore \quad G_1 = lr$$
$$G_2 = lr^2$$
$$G_3 = lr^3$$
$$n = lr^4$$

$$\Rightarrow \quad r = \left(\frac{n}{l}\right)^{\frac{1}{4}}$$

Now, $G_1^4 + 2G_2^4 + G_3^4 = (lr)^4 + 2(lr^2)^4 + (lr^3)^4$

$$= l^4 \times r^4(1 + 2r^4 + r^8)$$
$$= l^4 \times r^4(r^4 + 1)^2$$
$$= l^4 \times \frac{n}{l}\left(\frac{n+l}{l}\right)^2$$
$$= ln \times 4m^2 = 4lm^2 n$$

9. *(b)* ***Central Idea*** *Write the nth term of the given series and simplify it to get its lowest form. Then, apply,* $S_n = \Sigma T_n$.

Given series is

$$\frac{1^3}{1} + \frac{1^3 + 2^3}{1+3} + \frac{1^3 + 2^3 + 3^3}{1+3+5} + \ldots \infty$$

Let T_n be the nth term of the given series.

$$\therefore \quad T_n = \frac{1^3 + 2^3 + 3^3 + \ldots + n^3}{1 + 3 + 5 + \ldots + \text{to } n \text{ terms}}$$
$$= \frac{\left\{\frac{n(n+1)}{2}\right\}^2}{n^2}$$
$$= \frac{(n+1)^2}{4}$$

$$S_9 = \sum_{n=1}^{9} \frac{(n+1)^2}{4} = \frac{1}{4}[(2^2 + 3^2 + \ldots + 10^2) + 1^2 - 1^2]$$
$$= \frac{1}{4}\left[\frac{10\,(10+1)(20+1)}{6} - 1\right] = \frac{384}{4} = 96$$

10. *(c)* We have,

$$\lim_{x\to 0}\frac{(1-\cos 2x)(3+\cos x)}{x\tan 4x}$$

$$=\lim_{x\to 0}\frac{2\sin^2 x(3+\cos x)}{x\times\frac{\tan 4x}{4x}\times 4x}$$

$$=\lim_{x\to 0}\frac{2\sin^2 x}{x^2}\times\lim_{x\to 0}\frac{(3+\cos x)}{4}\times\frac{1}{\lim\limits_{x\to 0}\frac{\tan 4x}{4x}}$$

$$=2\times\frac{4}{4}\times 1 \qquad \left(\because \lim_{\theta\to 0}\frac{\sin\theta}{\theta}=1 \text{ and } \lim_{\theta\to 0}\frac{\tan\theta}{\theta}=1\right)$$

$$=2$$

11. *(a)* Since, $g(x)$ is differentiable $\Rightarrow g(x)$ must be continuous.

$$\therefore\quad g(x)=\begin{cases}k\sqrt{x+1} & ,\ 0\le x\le 3\\ mx+2 & ,\ 3<x\le 5\end{cases}$$

At $x=3$, RHL $=3m+2$

and at $x=3$, LHL $=2k$

$$\therefore\quad 2k=3m+2 \qquad \ldots(i)$$

$$\text{Also, } g'(x)=\begin{cases}\dfrac{k}{2\sqrt{x+1}} & ,\ 0\le x<3\\ m & ,\ 3<x\le 5\end{cases}$$

$$\therefore L\{g'(3)\}=\frac{k}{4} \text{ and } R\{g'(3)\}=m$$

$$\Rightarrow\quad \frac{k}{4}=m \text{ i.e. } k=4m \qquad \ldots(ii)$$

On solving Eqs. (i) and (ii), we get

$$k=\frac{8}{5},\ m=\frac{2}{5}$$

$$\Rightarrow\quad k+m=2$$

12. *(d)* Given equation of curve is

$$x^2+2xy-3y^2=0 \qquad \ldots(i)$$

On differentiating w.r.t. x, we get

$$2x+2xy'+2y-6yy'=0$$

$$\Rightarrow\quad y'=\frac{x+y}{3y-x}$$

At $x=1,\ y=1,\ y'=1$

i.e. $\left(\dfrac{dy}{dx}\right)_{(1,\,1)}=1$

Equation of normal at $(1, 1)$ is

$$y-1=-\frac{1}{1}(x-1)$$

$$\Rightarrow\quad y-1=-(x-1)$$

$$\Rightarrow\quad x+y=2 \qquad \ldots(ii)$$

On solving Eqs. (i) and (ii) simultaneously, we get

$$x^2+2x(2-x)-3(2-x)^2=0$$

$$\Rightarrow\quad x^2+4x-2x^2-3(4+x^2-4x)=0$$

$$\Rightarrow\quad -x^2+4x-12-3x^2+12x=0$$

$$\Rightarrow\quad -4x^2+16x-12=0$$

$$\Rightarrow\quad 4x^2-16x+12=0$$

$$\Rightarrow\quad x^2-4x+3=0$$

$$\Rightarrow\quad (x-1)(x-3)=0$$

$\Rightarrow x=1, 3$

Now, when $x=1$, then $y=1$

and when $x=3$, then $y=-1$

$\therefore\ P=(1, 1)$ and $Q=(3, -1)$

Hence, normal meets the curve again at $(3, -1)$ in fourth quadrant.

Aliter

Given,

$$x^2+2xy-3y^2=0$$

$$\Rightarrow\quad (x-y)(x+3y)=0$$

$$\Rightarrow\quad x-y=0 \text{ or } x+3y=0$$

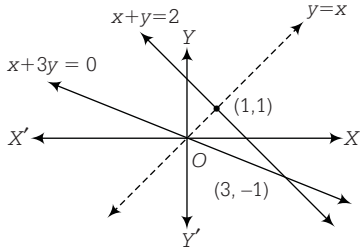
Equation of normal at $(1, 1)$ is

$$y-1=-1(x-1)$$

$$\Rightarrow\quad x+y-2=0$$

It intersects $x+3y=0$ at $(3, -1)$

and hence normal meet the curve in fourth quadrant.



13. *(c)* ***Central Idea*** *Any function have extreme values (maximum or minimum) at its critical points, where* $f'(x)=0$.

Since, the function have extreme values at $x=1$ and $x=2$.

$$\therefore\quad f'(x)=0 \text{ at } x=1 \text{ and } x=2$$

$$\Rightarrow\quad f'(1)=0 \text{ and } f'(2)=0$$

Also it is given that

$$\lim_{x\to 0}\left[1+\frac{f(x)}{x^2}\right]=3$$

$$\Rightarrow\quad 1+\lim_{x\to 0}\frac{f(x)}{x^2}=3$$

$$\Rightarrow\quad \lim_{x\to 0}\frac{f(x)}{x^2}=2$$

$\Rightarrow f(x)$ will be of the form

$$ax^4 + bx^3 + 2x^2$$

[$\because f(x)$ is of four degree polynomial]

Let $f(x) = ax^4 + bx^3 + 2x^2$

$\Rightarrow f'(x) = 4ax^3 + 3bx^2 + 4x$

$\Rightarrow f'(1) = 4a + 3b + 4 = 0$ ...(i)

and $f'(2) = 32a + 12b + 8 = 0$

$\Rightarrow 8a + 3b + 2 = 0$ ...(ii)

On solving Eqs. (i) and (ii), we get

$$a = \frac{1}{2}, b = -2$$

$\therefore f(x) = \frac{x^4}{2} - 2x^3 + 2x^2$

$\Rightarrow f(2) = 8 - 16 + 8 = 0$

14. *(d)* $\int \frac{dx}{x^2(x^4+1)^{\frac{3}{4}}} = \int \frac{dx}{x^5\left(1+\frac{1}{x^4}\right)^{\frac{3}{4}}}$

Put $1 + \frac{1}{x^4} = t^4$

$\Rightarrow -\frac{4}{x^5}dx = 4t^3 dt$

$\Rightarrow \frac{dx}{x^5} = -t^3 dt = \int \frac{-t^3 dt}{t^3}$

$= -\int dt = -t + c = -\left(1 + \frac{1}{x^4}\right)^{1/4} + c$

15. *(c)* ***Central Idea*** *Apply the property* $\int_a^b f(x)dx = \int_a^b f(a+b-x)dx$ *and then add.*

Let $I = \int_2^4 \frac{\log x^2}{\log x^2 + \log(36 - 12x + x^2)}dx$

$= \int_2^4 \frac{2\log x}{2\log x + \log(6-x)^2}dx$

$= \int_2^4 \frac{2\log x\, dx}{2[\log x + \log(6-x)]}$

$\Rightarrow I = \int_2^4 \frac{\log x\, dx}{[\log x + \log(6-x)]}$ ...(i)

$\Rightarrow I = \int_2^4 \frac{\log(6-x)}{\log(6-x) + \log x}dx$ ...(ii)

$\left[\because \int_a^b f(x)dx = \int_a^b f(a+b-x)dx\right]$

On adding Eqs. (i) and (ii), we get

$$2I = \int_2^4 \frac{\log x + \log(6-x)}{\log x + \log(6-x)}dx$$

$\Rightarrow 2I = \int_2^4 dx = [x]_2^4$

$\Rightarrow 2I = 2 \Rightarrow I = 1$

16. *(d)* Given region is $\{(x, y): y^2 \le 2x$ and $y \ge 4x - 1\}$

$y^2 \le 2x$ represents a region inside the parabola

$$y^2 = 2x \quad \text{...(i)}$$

and $y \ge 4x - 1$ represents a region to the left of the line

$$y = 4x - 1 \quad \text{...(ii)}$$

The point of intersection of the curve (i) and (ii) is

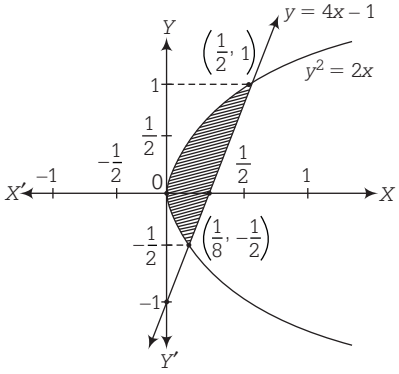
$(4x-1)^2 = 2x$

$\Rightarrow 16x^2 + 1 - 8x = 2x$

$\Rightarrow 16x^2 - 10x + 1 = 0$

$\Rightarrow x = \frac{1}{2}, \frac{1}{8}$

$\therefore$ The points where these curves intersect, are $\left(\frac{1}{2}, 1\right)$ and $\left(\frac{1}{8}, -\frac{1}{2}\right)$.



Hence, required area

$= \int_{-1/2}^{1}\left(\frac{y+1}{4} - \frac{y^2}{2}\right)dy$

$= \frac{1}{4}\left(\frac{y^2}{2} + y\right)_{-1/2}^{1} - \frac{1}{6}(y^3)_{-1/2}^{1}$

$= \frac{1}{4}\left\{\left(\frac{1}{2}+1\right) - \left(\frac{1}{8} - \frac{1}{2}\right)\right\} - \frac{1}{6}\left\{1 + \frac{1}{8}\right\}$

$= \frac{1}{4}\left\{\frac{3}{2} + \frac{3}{8}\right\} - \frac{1}{6}\left\{\frac{9}{8}\right\}$

$= \frac{1}{4} \times \frac{15}{8} - \frac{3}{16} = \frac{9}{32}$

17. *(c)* Given differential equation is

$$(x\log x)\frac{dy}{dx} + y = 2x\log x, \quad (x \ge 1)$$

$\Rightarrow \frac{dy}{dx} + \frac{y}{x\log x} = 2$

This is a linear differential equation.

$\therefore$ IF $= e^{\int \frac{1}{x \log x} dx} = e^{\log(\log x)} = \log x$

Now, the solution of given differential equation is given by

$$y \cdot \log x = \int \log x \cdot 2dx$$

$$\Rightarrow \quad y \cdot \log x = 2\int \log x \, dx$$

$$\Rightarrow \quad y \cdot \log x = 2[x \log x - x] + c$$

At $x = 1, c = 2$

$$\Rightarrow \quad y \cdot \log x = 2[x \log x - x] + 2$$

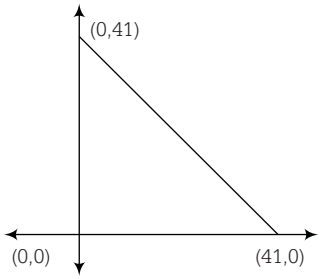
At $x = e$,

$y = 2(e - e) + 2 \Rightarrow y = 2$

18. *(d)* Required points (x, y) are such that it satisfy $x + y < 41$

and $x > 0, y > 0$

Number of positive integral solution of the equation $x + y + k = 41$ will be number of integral coordinates in the bounded region.

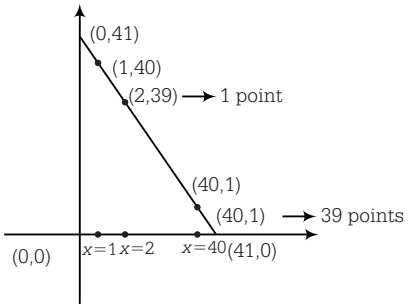


$\therefore$ Total number of integral coordinates

$$= {}^{41-1}C_{3-1} = {}^{40}C_2 = \frac{40!}{2!\,38!} = 780$$

Aliter

Consider the following figure :



Clearly, the number of required points

$$= 1 + 2 + 3 + \ldots + 39$$

$$= \frac{39}{2}(39 + 1) = 780$$

19. *(c)* ***Central Idea*** *First of all find the point of intersection of the lines* $2x - 3y + 4 = 0$ *and* $x - 2y + 3 = 0$ *(say A). Now, the line* $(2x - 3y + 4) + k(x - 2y + 3) = 0$ *is the perpendicular bisector of the line joining points* $P(2, 3)$ *and image* $P'(h, k)$. *Now,* $AP = AP'$ *and simplify*.

Given line is

$(2x - 3y + 4) + k(x - 2y + 3) = 0, k \in R$ ...(i)

This line will pass through the point of intersection of the lines

$$2x - 3y + 4 = 0 \quad \ldots(ii)$$

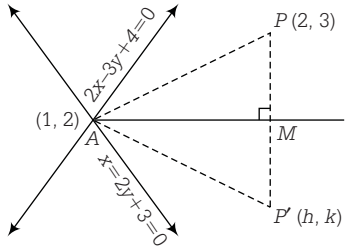
and $x - 2y + 3 = 0 \quad \ldots(iii)$

On solving Eqs. (ii) and (iii), we get

$x = 1, y = 2$

$\therefore$ Point of intersection of lines (ii) and (iii) is $(1, 2)$.

Let M be the mid-point of PP', then AM is perpendicular bisector of PP' (where, A is the point of intersection of given lines).



$\therefore \quad AP = AP'$

$\Rightarrow$

$$\sqrt{(2-1)^2 + (3-2)^2} = \sqrt{(h-1)^2 + (k-2)^2}$$

$$\Rightarrow \quad \sqrt{2} = \sqrt{h^2 + k^2 - 2h - 4k + 1 + 4}$$

$$\Rightarrow \quad \sqrt{2} = \sqrt{h^2 + k^2 - 2h - 4k + 5}$$

$$\Rightarrow \quad h^2 + k^2 - 2h - 4k + 5 = 2$$

$$\Rightarrow \quad h^2 + k^2 - 2h - 4k + 3 = 0$$

Thus, the required locus is

$$x^2 + y^2 - 2x - 4y + 3 = 0$$

which is a equation of circle with

$$\text{radius} = \sqrt{1 + 4 - 3} = \sqrt{2}$$

Aliter

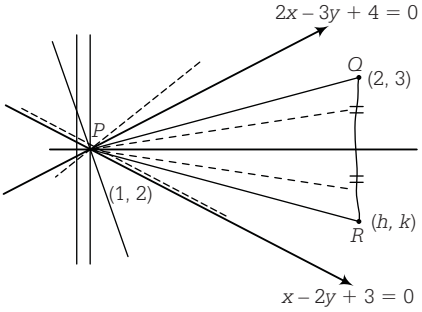
$(2x - 3y + 4) + k(x - 2y + 3) = 0$ is family of lines passing through (1, 2). By congruency of triangles, we can prove that mirror image (h, k) and the point (2, 3) will be equidistant from (1, 2).

$\therefore$ Locus of (h, k) is $PR = PQ$

$\Rightarrow (h-1)^2 + (k-2)^2 = (2-1)^2 + (3-2)^2$

or $(x-1)^2 + (y-2)^2 = 2$

$\therefore$ Locus is a circle of radius $= \sqrt{2}$



20. *(c)* ***Central Idea*** *Number of common tangents depend on the position of the circle with respect to each other.*

(i) If circles touch externally $\Rightarrow C_1C_2 = r_1 + r_2$, *3 common tangents*

(ii) If circles touch internally $\Rightarrow C_1C_2 = r_2 - r_1$, *1 common tangent*

(iii) If circles do not touch each other, 4 *common tangents*

Given equations of circles are

$x^2 + y^2 - 4x - 6y - 12 = 0$ ...(i)

$x^2 + y^2 + 6x + 18y + 26 = 0$ ...(ii)

Centre of circle (i) is $C_1(2, 3)$ and radius

$= \sqrt{4 + 9 + 12} = 5(r_1)$ (say)

Centre of circle (ii) is $C_2(-3, -9)$ and radius

$= \sqrt{9 + 81 - 26} = 8(r_2)$ (say)

Now, $C_1C_2 = \sqrt{(2+3)^2 + (3+9)^2}$

$\Rightarrow C_1C_2 = \sqrt{5^2 + 12^2}$

$\Rightarrow C_1C_2 = \sqrt{25 + 144} = 13$

$\therefore r_1 + r_2 = 5 + 8 = 13$

Also, $C_1C_2 = r_1 + r_2$

Thus, both circles touch each other externally. Hence, there are three common tangents.

21. *(d)* Given equation of ellipse is

$$\frac{x^2}{9} + \frac{y^2}{5} = 1 \quad \text{...(i)}$$

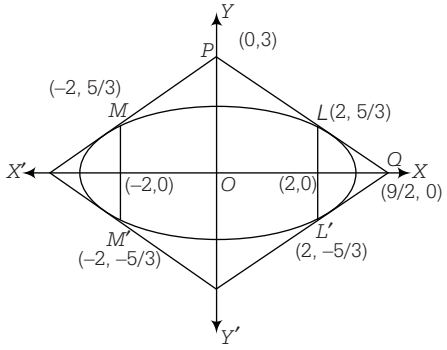
$\therefore a^2 = 9, b^2 = 5$

$\Rightarrow a = 3, b = \sqrt{5}$

Now, $e = \sqrt{1 - \frac{b^2}{a^2}} = \sqrt{1 - \frac{5}{9}} = \frac{2}{3}$

foci $= (\pm ae, 0) = (\pm 2, 0)$

and $\frac{b^2}{a} = \frac{5}{3}$



$\therefore$ Extremities of one of latusrectum are

$$\left(2, \frac{5}{3}\right)$$

and $\left(2, \frac{-5}{3}\right)$.

$\therefore$ Equation of tangent at $\left(2, \frac{5}{3}\right)$ is,

$$\frac{x(2)}{9} + \frac{y(5/3)}{5} = 1$$

or $2x + 3y = 9$ ...(ii)

Eq.(ii) intersects X and Y-axes at $\left(\frac{9}{2}, 0\right)$ and $(0, 3)$, respectively.

$\therefore$ Area of quadrilateral

$= 4 \times$ Area of ΔPOQ

$= 4 \times \left(\frac{1}{2} \times \frac{9}{2} \times 3\right)$

$= 27$ sq units

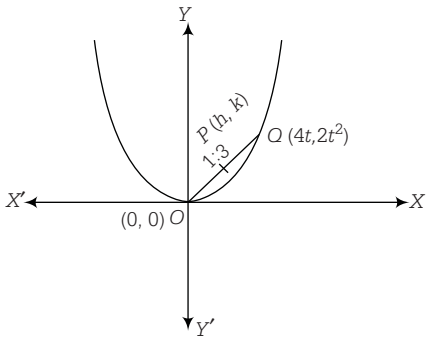
22. *(d)* ***Central Idea*** *Any point on the parabola* $x^2 = 8y$ *is* $(4t, 2t^2)$. *Point* P *divides the line segment joining of* O(0, 0) *and* $Q(4t, 2t^2)$ *in the ratio* 1 : 3. *Apply the section formula for internal division.*

Equation of parabola is

$x^2 = 8y$ ...(i)

Let any point Q on the parabola (i) is $(4t, 2t^2)$.

Let $P(h, k)$ be the point which divides the line segment joining (0,0) and $(4t, 2t^2)$ in the ratio 1 : 3.



$$\therefore \quad h = \frac{1 \times 4t + 3 \times 0}{4}$$

$$\Rightarrow \quad h = t$$

and $$k = \frac{1 \times 2t^2 + 3 \times 0}{4}$$

$$\Rightarrow \quad k = \frac{t^2}{2}$$

$$\Rightarrow \quad k = \frac{1}{2}h^2 \qquad (\because t = h)$$

$\Rightarrow 2k = h^2 \Rightarrow 2y = x^2$,

which is required locus.

23. *(d)* Given equation of line is

$$\frac{x-2}{3} = \frac{y+1}{4} = \frac{z-2}{12} = \lambda \quad \text{(say)} \ldots(i)$$

and equation of plane is

$$x - y + z = 16 \qquad \ldots(ii)$$

Any point on the line (i) is

$$(3\lambda + 2, 4\lambda - 1, 12\lambda + 2)$$

Let this point be point of intersection of the line and plane.

$\therefore (3\lambda + 2) - (4\lambda - 1) + (12\lambda + 2) = 16$

$\Rightarrow \quad 11\lambda + 5 = 16$

$\Rightarrow \quad 11\lambda = 11$

$\Rightarrow \quad \lambda = 1$

$\therefore$ Point of intersection is (5, 3, 14).

Now, distance between the points (1, 0, 2) and (5, 3, 14)

$$= \sqrt{(5-1)^2 + (3-0)^2 + (14-2)^2}$$

$$= \sqrt{16 + 9 + 144}$$

$$= \sqrt{169} = 13$$

24. *(c)* Let equation of plane containing the lines $2x - 5y + z = 3$ and $x + y + 4z = 5$ be

$(2x - 5y + z - 3) + \lambda(x + y + 4z - 5) = 0$

$\Rightarrow (2 + \lambda)x + (\lambda - 5)y + (4\lambda + 1)z - 3 - 5\lambda = 0 \ldots(i)$

This plane is parallel to the plane $x + 3y + 6z = 1$.

$$\therefore \quad \frac{2+\lambda}{1} = \frac{\lambda - 5}{3} = \frac{4\lambda + 1}{6}$$

On taking first two equalities, we get

$$6 + 3\lambda = \lambda - 5$$

$$\Rightarrow \quad 2\lambda = -11$$

$$\Rightarrow \quad \lambda = -\frac{11}{2}$$

On taking last two equalities, we get

$$6\lambda - 30 = 3 + 12\lambda$$

$$\Rightarrow \quad -6\lambda = 33$$

$$\Rightarrow \quad \lambda = -\frac{11}{2}$$

So, the equation of required plane is

$$\left(2 - \frac{11}{2}\right)x + \left(\frac{-11}{2} - 5\right)y + \left(-\frac{44}{2} + 1\right)z - 3 + 5 \times \frac{11}{2} = 0$$

$$\Rightarrow \quad -\frac{7}{2}x - \frac{21}{2}y - \frac{42}{2}z + \frac{49}{2} = 0$$

$$\Rightarrow \quad x + 3y + 6z - 7 = 0$$

25. *(a)* Given,

$$(\mathbf{a} \times \mathbf{b}) \times \mathbf{c} = \frac{1}{3}|\mathbf{b}||\mathbf{c}|\mathbf{a}$$

$$\Rightarrow \quad -\mathbf{c} \times (\mathbf{a} \times \mathbf{b}) = \frac{1}{3}|\mathbf{b}||\mathbf{c}|\mathbf{a}$$

$$\Rightarrow \quad -(\mathbf{c} \cdot \mathbf{b}) \cdot \mathbf{a} + (\mathbf{c} \cdot \mathbf{a})\mathbf{b} = \frac{1}{3}|\mathbf{b}||\mathbf{c}|\mathbf{a}$$

$$\left[\frac{1}{3}|\mathbf{b}||\mathbf{c}| + (\mathbf{c} \cdot \mathbf{b})\right]\mathbf{a} = (\mathbf{c} \cdot \mathbf{a})\mathbf{b}$$

Since, $\mathbf{a}$ and $\mathbf{b}$ are not collinear.

$$\mathbf{c} \cdot \mathbf{b} + \frac{1}{3}|\mathbf{b}||\mathbf{c}| = 0 \text{ and } \mathbf{c} \cdot \mathbf{a} = 0$$

$$\Rightarrow \quad |\mathbf{c}||\mathbf{b}|\cos\theta + \frac{1}{3}|\mathbf{b}||\mathbf{c}| = 0$$

$$\Rightarrow \quad |\mathbf{b}||\mathbf{c}|\left(\cos\theta + \frac{1}{3}\right) = 0$$

$$\Rightarrow \quad \cos\theta + \frac{1}{3} = 0 \quad (\because |\mathbf{b}| \neq 0, |\mathbf{c}| \neq 0)$$

$$\Rightarrow \quad \cos\theta = -\frac{1}{3}$$

$$\Rightarrow \quad \sin\theta = \frac{\sqrt{8}}{3} = \frac{2\sqrt{2}}{3}$$

26. *(a)* There seems to be ambiguity in this question. It should be mentioned that boxes are different and one particular box has 3 balls.

Then, number of ways $= \frac{{}^{12}C_3 \times 2^9}{3^{12}}$

$= \frac{55}{3}\left(\frac{2}{3}\right)^{11}$

According to the question,

$$\frac{{}^3C_1 \times {}^{12}C_3 2^9 - {}^3C_2\,{}^{12}C_3\,{}^9C_3 + \frac{12! \times 3!}{3!3!6!3!}}{3^{12}}$$

27. *(d)* Given, $\frac{x_1 + x_2 + x_3 + \ldots + x_{16}}{16} = 16$

$\Rightarrow \quad \sum_{i=1}^{16} x_i = 16 \times 16$

Sum of new observations

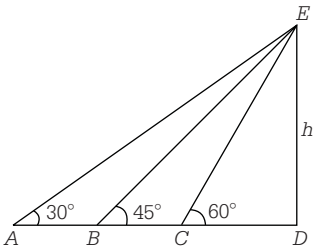
$= \sum_{i=1}^{18} y_i = (16 \times 16 - 16) + (3 + 4 + 5) = 252$

Number of observations = 18

$\therefore$ New mean $= \frac{\sum_{i=1}^{18} y_i}{18} = \frac{252}{18} = 14$

28. *(a)* According to the given information, the figure should be as follows.

Let the height of tower = h



In ΔEDA,

$$\tan 30° = \frac{ED}{AD}$$

$$\frac{1}{\sqrt{3}} = \frac{ED}{AD} = \frac{h}{AD}$$

$\Rightarrow \quad AD = h\sqrt{3}$

In ΔEDB,

$\tan 45° = \frac{h}{BD} \quad \Rightarrow \quad BD = h$

In ΔEDC,

$\tan 60° = \frac{h}{CD} \Rightarrow CD = \frac{h}{\sqrt{3}}$

Now, $\frac{AB}{BC} = \frac{AD - BD}{BD - CD}$

$\Rightarrow \quad \frac{AB}{BC} = \frac{h\sqrt{3} - h}{h - \frac{h}{\sqrt{3}}}$

$\Rightarrow \quad \frac{AB}{BC} = \frac{h(\sqrt{3} - 1)}{\frac{h(\sqrt{3} - 1)}{\sqrt{3}}}$

$\Rightarrow \quad \frac{AB}{BC} = \frac{\sqrt{3} - 1}{(\sqrt{3} - 1)} \times \sqrt{3}$

$\Rightarrow \quad \frac{AB}{BC} = \frac{\sqrt{3}}{1}$

$\therefore \quad AB : BC = \sqrt{3} : 1$

29. *(a)* Given,

$$\tan^{-1} y = \tan^{-1} x + \tan^{-1}\left(\frac{2x}{1 - x^2}\right),$$

where $|x| < \frac{1}{\sqrt{3}}$

$$\Rightarrow \quad \tan^{-1} y = \tan^{-1}\left\{\frac{x + \frac{2x}{1 - x^2}}{1 - x\left(\frac{2x}{1 - x^2}\right)}\right\}$$

$$\left[\because \tan^{-1} x + \tan^{-1} y = \tan^{-1}\left(\frac{x + y}{1 - xy}\right),\right.$$

$$\left. x > 0,\ y > 0,\ xy < 1\right]$$

$$= \tan^{-1}\left(\frac{x - x^3 + 2x}{1 - x^2 - 2x^2}\right)$$

$$\tan^{-1} y = \tan^{-1}\left(\frac{3x - x^3}{1 - 3x^2}\right)$$

$\Rightarrow \quad y = \frac{3x - x^3}{1 - 3x^2}$

Aliter

$$|x| < \frac{1}{\sqrt{3}} \Rightarrow -\frac{1}{\sqrt{3}} < x < \frac{1}{\sqrt{3}}$$

Let $x = \tan\theta$

$\Rightarrow \quad -\frac{\pi}{6} < \theta < \frac{\pi}{6}$

$\therefore \quad \tan^{-1} y = \theta + \tan^{-1}(\tan 2\theta)$

$= \theta + 2\theta = 3\theta$

$\Rightarrow \quad y = \tan 3\theta$

$\Rightarrow \quad y = \frac{3\tan\theta - \tan^3\theta}{1 - 3\tan^2\theta}$

$\Rightarrow \quad y = \frac{3x - x^3}{1 - 3x^2}$

30. *(d)* $\sim(\sim s \vee (\sim r \wedge s))$

$\equiv s \wedge (\sim(\sim r \wedge s))$

$\equiv s \wedge (r \vee \sim s)$

$\equiv (s \wedge r) \vee (s \wedge \sim s)$

$\equiv (s \wedge r) \vee \mathrm{F} \quad (\because s \wedge \sim s$ is false)

$\equiv s \wedge r$