JEE Main 2017 (Offline)

1. Let k be an integer such that the triangle with vertices $(k, -3k)$, $(5, k)$ and $(-k, 2)$ has area 28 sq. units. Then the orthocentre of this triangle is at the point:
   (a) $\left(2, \frac{1}{2}\right)$ (b) $\left(2, -\frac{1}{2}\right)$
   (c) $\left(1, \frac{3}{4}\right)$ (d) $\left(1, -\frac{3}{4}\right)$

2. If, for a positive integer n, the quadratic equation,
   $x(x+1)+(x+1)(x+2)+....$
   $+(x+\overline{n-1})(x+n)=10n$
   has two consecutive intergal solutions, then n is equal to:
   (a) 11 (b) 12
   (c) 9 (d) 10

3. The function $f:\mathbf{R}\rightarrow\left[-\frac{1}{2},\frac{1}{2}\right]$ defined as $f(x)=\frac{x}{1+x^2}$, is
   (a) neither injective nor surjective.
   (b) invertible.
   (c) injective but not surjective.
   (d) surjective but not injective.

4. The following statement $(p\rightarrow q)\rightarrow[(\sim p\rightarrow q)]$ is:
   (a) a fallacy
   (b) a tautology
   (c) equivalent to $\sim p\rightarrow q$
   (d) equivalent to $p\rightarrow\sim q$

5. If S is the set of distinct values of 'b' for which the following, system of linear equations
   $x+y+z=1$
   $x+ay+z=1$
   $ax+by+z=0$
   has no solution, then S is:
   (a) A singleton
   (b) An empty set
   (c) An infinite set
   (d) A finite set contaning two or more elements

6. The area (in sq. units) of the region $\{(x,y):\ x\geq 0, x+y\leq 3, x^2\leq 4y$ and $y\leq 1+\sqrt{x}\}$ is:
   (a) $\frac{5}{2}$ (b) $\frac{59}{12}$
   (c) $\frac{3}{2}$ (d) $\frac{7}{3}$

7. For any three positive real numbers a, b and c, $9(25a^2+b^2)+25(c^2-3ac)=15b(3a+c)$.
   Then:
   (a) a, b and c are in G.P.
   (b) b, c and a are in G.P.
   (c) b, c and a are in A.P.
   (d) a, b and c are in A.P.

8. A man X has 7 friends, 4 of them are ladies and 3 are men. His wife Y also has 7 friends, 3 of them are ladies and 4 are men. Assume X and Y have no common friends. Then the total number of ways in which X and Y together can throw a party inviting 3 ladies and 3 men, so that 3 friends of each of X and Y are in this party, is:
   (a) 484 (b) 485
   (c) 468 (d) 469

9. The normal to the curve $y(x-2)(x-3)=x+6$ at the point where the curve intersects the y-axis passes through the point:

(a) $\left(\frac{1}{2},\frac{1}{3}\right)$ (b) $\left(-\frac{1}{2},-\frac{1}{2}\right)$

(c) $\left(\frac{1}{2},\frac{1}{2}\right)$ (d) $\left(\frac{1}{2},-\frac{1}{3}\right)$

10. A hyperbola passes through the point $P(\sqrt{2},\sqrt{3})$ and has foci at $(\pm 2, 0)$. Then the tangent to this hyperbola at P also passes through the point:

(a) $\left(-\sqrt{2},-\sqrt{3}\right)$ (b) $\left(3\sqrt{2},2\sqrt{3}\right)$

(c) $\left(2\sqrt{2},3\sqrt{3}\right)$ (d) $\left(\sqrt{3},\sqrt{2}\right)$

11. Let $a, b, c \in \mathbf{R}$. If $f(x)=ax^2+bx+c$ is such that $a+b+c=3$ and $f(x+y)=f(x)+f(y)+xy, \forall\, x, y \in \mathbf{R}$, then $\sum_{n=1}^{10} f(n)$ is equal to:

(a) 255 (b) 330

(c) 165 (d) 190

12. Let $\mathbf{a}=2\mathbf{i}+\mathbf{j}-2\mathbf{k}$ and $\mathbf{b}=\mathbf{i}+\mathbf{j}$ Let $\mathbf{c}$ be a vector such that $|\mathbf{c}-\mathbf{a}|=3, |(\mathbf{a}\times\mathbf{b})\times\mathbf{c}|=3$ and the angle between $\mathbf{c}$ and $\mathbf{a}\times\mathbf{b}$ be 30^o. Then $\mathbf{a}\cdot\mathbf{c}$ is equal to:

(a) $\frac{1}{8}$ (b) $\frac{25}{8}$

(c) 2 (d) 5

13. Let a vertical tower AB have its end A on the level ground. Let C be the mid-point of AB and P be a point on the ground such that AP = 2AB. If $\angle BPC=\beta$ then $\tan\beta$ is equal to:

(a) $\frac{4}{9}$ (b) $\frac{6}{7}$

(c) $\frac{1}{4}$ (d) $\frac{2}{9}$

14. Twenty meters of wire is available for fencing off a flower-bed in the form of a circular sector. Then the maximum area (in sq. m) of the flower-bed, is:

(a) 30 (b) 12.5

(c) 10 (d) 25

15. The integral $\int_{\frac{\pi}{4}}^{\frac{3\pi}{4}} \frac{dx}{1+\cos x}$ is equal to:

(a) –1 (b) –2

(c) 2 (d) 4

16. If $(2+\sin x)\frac{dy}{dx}+(y+1)\cos x=0$ and $y(0)=1$, then $y\left(\frac{\pi}{2}\right)$ is equal to:

(a) $\frac{4}{3}$ (b) $\frac{1}{3}$

(c) $-\frac{2}{3}$ (d) $-\frac{1}{3}$

17. Let $I_n=\int \tan^n x\, dx$, $(n > 1)$. If $I_4 + I_6 = a\tan^5 x + bx^5 + c$, where c is a constant of integration, then the ordered pair (a, b) is equal to:

(a) $\left(-\frac{1}{5},0\right)$ (b) $\left(-\frac{1}{5},1\right)$

(c) $\left(\frac{1}{5},0\right)$ (d) $\left(\frac{1}{5},-1\right)$

18. Let ω be a complex number such that $2\omega + 1 = z$ where $z=\sqrt{-3}$. If

$$\begin{vmatrix} 1 & 1 & 1 \\ 1 & -\omega^2-1 & \omega^2 \\ 1 & \omega^2 & \omega^7 \end{vmatrix} = 3k$$

then k is equal to :

(a) 1 (b) $-z$

(c) z (d) –1

19. The value of

$\left({}^{21}C_1-{}^{10}C_1\right)+\left({}^{21}C_2-{}^{10}C_2\right)+\left({}^{21}C_3-{}^{10}C_3\right)+\left({}^{21}C_4-{}^{10}C_4\right)+\ldots\ldots\left({}^{21}C_{10}-{}^{10}C_{10}\right)$ is :

(a) $2^{20}-2^{10}$ (b) $2^{21}-2^{11}$

(c) $2^{21}-2^{10}$ (d) $2^{20}-2^{9}$

20. $\lim_{x\to\frac{\pi}{2}} \frac{\cot x-\cos x}{(\pi-2x)^3}$ equals:

(a) $\frac{1}{4}$ (b) $\frac{1}{24}$

(c) $\frac{1}{16}$ (d) $\frac{1}{8}$

21. If $5(\tan^2 x - \cos^2 x) = 2\cos 2x + 9$, then the value of $\cos 4x$ is:

(a) $-\frac{7}{9}$ (b) $-\frac{3}{5}$

(c) $\frac{1}{3}$ (d) $\frac{2}{9}$

22. If the image of the point P(1, –2, 3) in the plane, $2x + 3y - 4z + 22 = 0$ measured parallel to the line, $\frac{x}{1} = \frac{y}{4} = \frac{7}{5}$ is Q, then PQ is equal to:

(a) $6\sqrt{5}$ (b) $3\sqrt{5}$

(c) $2\sqrt{42}$ (d) $\sqrt{42}$

23. The distance of the point (1,3, –7) from the plane passing through the point (1, –1, –1), having normal perpendicular to both the lines $\frac{x-1}{1} = \frac{y+2}{-2} = \frac{z-4}{-2}$ and $\frac{x-2}{2} = \frac{y+1}{-1} = \frac{z+7}{-1}$, is :

(a) $\frac{10}{\sqrt{74}}$ (b) $\frac{20}{\sqrt{74}}$

(c) $\frac{10}{\sqrt{83}}$ (d) $\frac{5}{\sqrt{83}}$

24. If for $x \in \left(0, \frac{1}{4}\right)$, the derivative of $\tan^{-1}\left(\frac{6x\sqrt{x}}{1-9x^3}\right)$ is $\sqrt{x}$. $g(x)$, then $g(x)$ equal:

(a) $\frac{3}{1+9x^3}$ (b) $\frac{9}{1+9x^3}$

(c) $\frac{3x\sqrt{x}}{1-9x^3}$ (d) $\frac{3x}{1-9x^3}$

25. The radius of a circle, having minimum area, which touches the curve $y = 4 - x^2$ and the lines, $y = |x|$ is:

(a) $4\left(\sqrt{2}+1\right)$ (b) $2\left(\sqrt{2}+1\right)$

(c) $2\left(\sqrt{2}-1\right)$ (d) $4\left(\sqrt{2}-1\right)$

26. A box contains 15 green and 10 yellow balls. If 10 balls are randomly drawn, one-by-one, with replacement, then the variance of the number of green balls drawn is:

(a) $\frac{6}{25}$ (b) $\frac{12}{5}$

(c) 6 (d) 4

27. The eccentricity of an ellipse whose centre is at the origin is $\frac{1}{2}$. If one of its directrices is $x = -4$, then the equation of the normal to it at $\left(1, \frac{3}{2}\right)$ is:

(a) $x + 2y = 4$ (b) $2y - x = 2$

(c) $4x - 2y = 1$ (d) $4x + 2y = 7$

28. If two different numbers are taken from the set {0, 1, 3,….., 10}; then the probability that their sum as well as absolute difference are both multiple of 4, is :

(a) $\frac{7}{55}$ (b) $\frac{6}{55}$

(c) $\frac{12}{55}$ (d) $\frac{14}{45}$

29. For three events A, B and C, P (Exactly one of A or B occurs) = P(Exactly one of B or C occurs) =P(Exactly one of C or A occurs)=$\frac{1}{4}$ and P(All the three events occur simultaneously) =$\frac{1}{16}$.

Then the probability that at least one of the events occurs, is:

(a) $\frac{3}{16}$ (b) $\frac{7}{32}$

(c) $\frac{7}{16}$ (d) $\frac{7}{46}$

30. If A = $\begin{bmatrix} 2 & -3 \\ -4 & 1 \end{bmatrix}$, then adj $(3A^2+12A)$ is equal to:

(a) $\begin{bmatrix} 72 & -63 \\ -84 & 51 \end{bmatrix}$ (b) $\begin{bmatrix} 72 & -84 \\ -63 & 51 \end{bmatrix}$

(c) $\begin{bmatrix} 51 & 63 \\ 84 & 72 \end{bmatrix}$ (d) $\begin{bmatrix} 51 & 84 \\ 63 & 72 \end{bmatrix}$

Answers

1. (a)	**2.** (a)	**3.** (d)	**4.** (b)	**5.** (a)
6. (a)	**7.** (c)	**8.** (b)	**9.** (c)	**10.** (c)
11. (b)	**12.** (c)	**13.** (d)	**14.** (d)	**15.** (c)
16. (b)	**17.** (c)	**18.** (b)	**19.** (a)	**20.** (c)
21. (a)	**22.** (d)	**23.** (c)	**24.** (b)	**25.** (d)
26. (b)	**27.** (c)	**28.** (b)	**29.** (c)	**30.** (c)

Hints and solutions

1. $\frac{1}{2}\left|\begin{vmatrix} k & -3k & 1 \\ 5 & k & 1 \\ -k & 2 & 1 \end{vmatrix}\right| = 28$

$R_2 \rightarrow R_2 - R_1$, $R_3 \rightarrow R_3 - R_1$

To get

$$\begin{vmatrix} k & -3k & 1 \\ 5-k & 4k & 0 \\ -2k & 2+3k & 0 \end{vmatrix} = \pm 56$$

$\Rightarrow (5 - k)(2 + 3k) + 8k^2 = \pm 56$

$\Rightarrow 10 + 13k - 3k^2 + 8k^2 = \pm 56$

$\Rightarrow 5k^2 + 13k^2 + 66 = 0$ or $5k^2 + 13k - 46 = 0$

But $5k^2 + 13k + 66 = 0$

does not have real roots.

Next, $5k^2 + 13k - 46 = 0$ has two roots,

viz. 2 and $-\frac{23}{5}$

As k is an integer, $k = 2$.

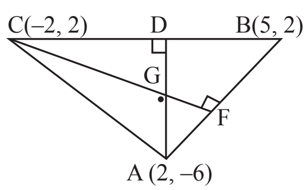
Thus, vertices of triangle are

(2, –6), (5, 2) and (–2, 2)

Equation of altitude AD is $x = 2$

Slope of AB is $\frac{-6-2}{2-5} = \frac{8}{3}$

$\therefore$ Slope os CF is $-\frac{3}{8}$



Equation of CF is

$y - 2 = -\frac{3}{8}(x + 2)$

Intersection of $x = 2$, is $y = \frac{1}{2}$.

Thus, orthocenter H of ΔABC is $(2, \frac{1}{2})$

2. Equation

$x(x + 1) + (x + 1)(x + 2) + \ldots\ldots + (x + n - 1)(x + n) = 10n$

Can be written as

$nx^2 + (1 + 3 + \ldots.. + 2n - 1)x + \sum_{k=1}^{n-1} k(k+1) = 10n$

$\Rightarrow nx^2 + n^2x + \frac{1}{6}(n - 1)n(2n - 1) + \frac{1}{2}n(n - 1)$

$= 10n$

$\Rightarrow x^2 + nx + \frac{1}{3}(n^2 - 31) = 0$ (i)

If α, β are roots of (i), then

$|\alpha - \beta| = 1$

$\Rightarrow (\alpha + \beta)^2 - 4\alpha\beta = 1$

$\Rightarrow n^2 - \frac{4}{3}(n^2 - 31) = 1$

$\Rightarrow -n^2 + 121 = 0$

$\Rightarrow n = 11.$

3. Note that

$\frac{x}{1+x^2} = \frac{1}{3}$

$\Rightarrow x^2 - 3x + 1 = 0$

$\Rightarrow x = \frac{3 \pm \sqrt{5}}{2}$

$\therefore f\left(\frac{3-\sqrt{5}}{2}\right) = \frac{1}{3} = f\left(\frac{3+\sqrt{5}}{2}\right)$

Thus, f is not injective.

For $\alpha \neq 0$,

$\frac{x}{1+x^2} = \alpha \Leftrightarrow \alpha x^2 - x + \alpha = 0$

$x = \frac{1 \pm \sqrt{1-4\alpha^2}}{2\alpha}$

$\therefore f\left(\frac{1 \pm \sqrt{1-4\alpha^2}}{2\alpha}\right) = \alpha$

Also, $f(0) = 0$.

Thus, f is surjective.

4. Let P be the statement $p \to q$;

Q be the statement $\sim p \to q$ and

R be the statement $Q \to q$.

p	q	~p	P	Q	Q→P	P→(Q→q)
T	T	F	T	T	T	T
T	F	F	F	T	F	T
F	T	T	T	T	T	T
F	F	T	T	F	T	T

Thus, $(p \to q) \to [(\sim p \to q) \to q]$

is a tautology.

5. Let

$$D = \begin{vmatrix} 1 & 1 & 1 \\ 1 & a & 1 \\ a & b & 1 \end{vmatrix}$$

Applying, $C_1 \to C_1 - C_3$, $C_2 \to C_2 - C_3$, to obtain

$$D = \begin{vmatrix} 0 & 0 & 1 \\ 0 & a-1 & 1 \\ a-1 & b-1 & 1 \end{vmatrix} = -(a-1)^2$$

If $D \neq 0$, the system has unique solution, therefore,

$D = 0 \Rightarrow a = 1$

System of Equations become

$x + y + z = 1$

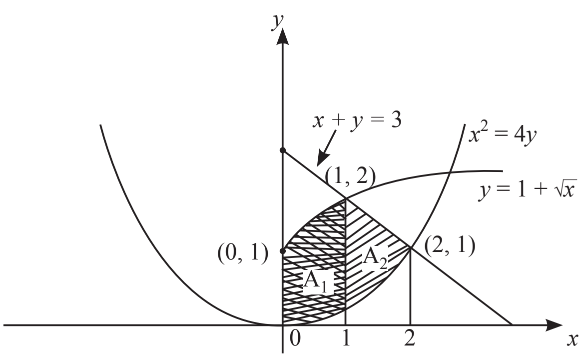
$x + by + z = 0$

$\Rightarrow (1 - b)y = 1$

For system to have no solution, $b = 1$

6. $y = 1 + \sqrt{x}$ and

$y = 3 - x$ meet at (1, 2)



Also, $x^2 = 4y$ and $x + y = 3$ meet at (2, 1)

Required area

$= A_1 + A_2$

$$= \int_0^1 \left(1 + \sqrt{x} - \frac{1}{4}x^2\right)dx + \int_1^2 \left(3 - x - \frac{1}{4}x^2\right)dx$$

$$= \left(x + \frac{2}{3}x^{\frac{3}{2}} - \frac{1}{12}x^3\right)\Bigg]_0^1 + \left(3x - \frac{1}{2}x^2 - \frac{1}{12}x^3\right)\Bigg]_1^2$$

$$= \left(1 + \frac{2}{3} - \frac{1}{12}\right) + \left(6 - 2 - \frac{8}{12} - 3 + \frac{1}{2} + \frac{1}{12}\right)$$

$$= \frac{5}{2}\text{ sq.units}$$

7. $9(25a^2 + b^2) + 25(c^2 - 3ac) = 15b(3a + c)$

$\Rightarrow (15a)^2 + (3b)^2 + (5c)^2 - 45ab - 15bc - 75ac = 0$

$\Rightarrow ((15a)^2 + (3b)^2 - 90ab) + ((3b)^2 + (5c)^2 - 30bc) + ((15a)^2 + (5c)^2 - 150ac) = 0$

$\Rightarrow (15a - 3b)^2 + (3b - 5c)^2 + (15a - 5c)^2 = 0$

$\Rightarrow 15a - 3b = 0,\ 3b - 5c = 0$ and $15a - 5c = 0$

$\Rightarrow \frac{a}{1} = \frac{b}{5} = \frac{c}{3} = k$(say)

$\Rightarrow a = k,\ b = 5k,\ c = 3k$

$\therefore b, c, a$ are in A.P.

8. Different possible arrangements are given in the following table

	H W	H W	H W	H W
M	3 0	2 1	1 2	0 3
F	0 3	1 2	2 1	3 0

$\therefore$ The number of ways

$= ({}^4C_3)\ ({}^4C_3) + ({}^4C_2)({}^3C_1)({}^3C_1)({}^4C_2) + ({}^4C_1)({}^3C_2)({}^3C_2)({}^4C_1) + ({}^3C_3)({}^3C_3)$

$= 485$

9. The curve meets the y–axis at (0, 1). We have

$$y = \frac{x+6}{(x-2)(x-3)}$$

$$= \frac{9}{x-3} - \frac{8}{x-2}$$

$$\frac{dy}{dx} = -\frac{9}{(x-3)^2} + \frac{8}{(x-2)^2}$$

$$\therefore \frac{dy}{dx}\Bigg]_{(0,1)} = -\frac{9}{9} + \frac{8}{4} = 1$$

Thus, slope of normal at (0, 1) is –1.

Equation of normal at (0, 1) is

$y - 1 = -1(x - 0)$ or $x + y = 1$.

It passes through $\left(\frac{1}{2}, \frac{1}{2}\right)$

10. Let the equation of the hyperbola be

$$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$$

We know $b^2 = a^2(e^2 - 1)$

$\Rightarrow a^2 + b^2 = (ae)^2 = 2^2 = 4$

Also,

$$\frac{2}{a^2} - \frac{3}{b^2} = 1$$

$$\Rightarrow \frac{2}{a^2} - \frac{3}{4 - a^2} = 1$$

$\Rightarrow 8 - 2a^2 - 3a^2 = a^2(4 - a^2)$

$\Rightarrow a^4 - 9a^2 + 8 = 0 \Rightarrow a^2 = 1, 8$

As $a^2 + b^2 = 4$, $a^2 \neq 8$

Thus, $a^2 = 1$ and $b^2 = 3$

$\therefore$ equation of hyperbola is

$x^2 - \frac{1}{3}y^2 = 1$

An equation of tangent at $\left(\sqrt{2}, \sqrt{3}\right)$ is

$\sqrt{2}x - \frac{\sqrt{3}}{3}y = 1$

It passes through $\left(2\sqrt{2}, 3\sqrt{3}\right)$.

11. $f(x + y) = f(x) + f(y) + xy$

Also, $f(1) = 3$

$f(k + 1) = f(k) + f(1) + (k)(1)$

$= f(k) + (k + 3)$

Let

$S = f(1) + f(2) + \ldots\ldots + f(k-1) + f(k)$

$S = \quad f(1) + \ldots.+ f(k-2) + f(k-1) + f(k)$

Subtract to obtain

$0 = f(1) + (f(2) - f(1)) + (f(3) - f(2)) + \ldots. + (f(k) - f(k-1)) - f(k)$

$\Rightarrow f(k) = 3 + 4 + 5 + \ldots\ldots + (k + 2)$

$= \frac{1}{2}k(3 + k + 2) = \frac{1}{2}k(k+5)$

Now, $\sum_{n=1}^{10} f(n) = \frac{1}{2}\sum_{n=1}^{10}\left(n^2 + 5n\right)$

$= \frac{1}{2}\left[\frac{(10)(11)(21)}{6} + \frac{5}{2}(10)(11)\right]$

$= \frac{1}{4}(10)(11)(7+5) = 330$

12. $\mathbf{a} \times \mathbf{b} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 2 & 1 & -2 \\ 1 & 1 & 0 \end{vmatrix}$

$= 2\mathbf{i} - 2\mathbf{j} + \mathbf{k}$

$\Rightarrow |\mathbf{a} \times \mathbf{b}| = 3$

As $|(\mathbf{a} \times \mathbf{b}) \times \mathbf{c}| = 3$

$\Rightarrow |\mathbf{a} \times \mathbf{b}||\mathbf{c}| \sin 30° = 3$

$\Rightarrow 3|\mathbf{c}|\left(\frac{1}{2}\right) = 3 \Rightarrow |\mathbf{c}| = 2$

Also,

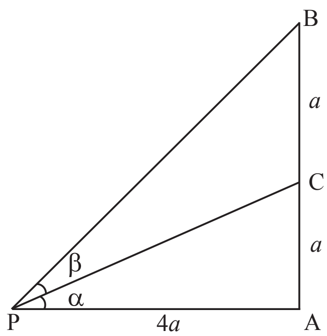
$3 = |\mathbf{c} - \mathbf{a}|$

$\Rightarrow 9 = |\mathbf{c}|^2 + |\mathbf{a}|^2 - 2\mathbf{a}\cdot\mathbf{c}$

$\Rightarrow \mathbf{a}\cdot\mathbf{c} = \frac{1}{2}(9 + 4 - 9) = 2$

13. Let AC = a, then AB = $2a$

Also, AP = 2(AB) = $4a$



We have

$PB^2 = 16a^2 + 4a^2 = 20a^2$

$PC^2 = 16a^2 + a^2 = 17a^2$

$\therefore \cos\beta = \frac{20a^2 + 17a^2 - a^2}{2\left(\sqrt{20}a\right)\left(\sqrt{17}a\right)} = \frac{36}{4\sqrt{85}} = \frac{9}{\sqrt{85}}$

$\tan\beta = \frac{\sqrt{1-\cos^2\beta}}{\cos\beta} = \frac{\sqrt{85-81}}{9} = \frac{2}{9}$

14. Let radius of circle be r, and $\angle AOB = \theta$,

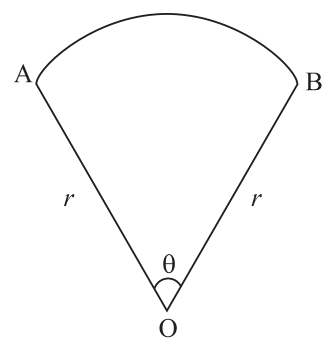
Then arc(AB) = θr

We are given

$r + r + \theta r = 20$

$\Rightarrow \theta = \frac{20 - 2r}{r}$

Also, area A of sector is given by



$A = \frac{\theta}{2\pi}\left(\pi r^2\right)$

$= \frac{1}{2}r^2\theta$

$= \frac{1}{2}r^2\left(\frac{20-2r}{r}\right) = r(10 - r)$

$= 25 - (5 - r)^2$

Thus, A is maximum when $r = 5$ and maximum A is $25m^2$

15. Let $I = \int_{\frac{\pi}{4}}^{\frac{3\pi}{4}} \frac{dx}{1+\cos x}$ (1)

$= \int_{\frac{\pi}{4}}^{\frac{3\pi}{4}} \frac{dx}{1+\cos\left(\frac{3\pi}{4}+\frac{\pi}{4}-x\right)}$

$I = \int_{\frac{\pi}{4}}^{\frac{3\pi}{4}} \frac{dx}{1-\cos x}$ (2)

Adding (1) and (2), we get

$2I = \int_{\frac{\pi}{4}}^{\frac{3\pi}{4}} \left(\frac{1}{1+\cos x}+\frac{1}{1-\cos x}\right)dx$

$= \int_{\frac{\pi}{4}}^{\frac{3\pi}{4}} \frac{2}{1-\cos^2 x}dx = 2\int_{\frac{\pi}{4}}^{\frac{3\pi}{4}} \cos ec^2 x dx$

$\Rightarrow I = \left(-\cot x\right)\Big]_{\frac{\pi}{4}}^{\frac{3\pi}{4}} = -\cot\left(\frac{3\pi}{4}\right)+\cot\left(\frac{\pi}{4}\right)$

$\Rightarrow I = 2.$

16. Rewrite the equation as

$\frac{dy}{y+1} = -\frac{\cos x}{2+\sin x}dx$

Integrating, we get

$\ln|y + 1| = -\ln|2 + \sin x| + \ln c$

$\Rightarrow \ln|(y + 1)(2 + \sin x) = \ln c$

$\Rightarrow (y + 1)(2 + \sin x) = \pm c = A$ (say)

We are given $y(0) = 1$

$\therefore (1 + 1)(2 + \sin 0) = A \Rightarrow A = 4$

Thus, $y = -1 + \frac{4}{2+\sin x}$

$\Rightarrow y\left(\frac{\pi}{2}\right) = -1+\frac{4}{2+1} = \frac{1}{3}$

17. $I_4 + I_6 = \int\left(\tan^4 x+\tan^6 x\right)dx$

$= \int \tan^4 x \sec^2 x dx$

$= \frac{1}{5}\tan^5 x + c$

$\therefore a = \frac{1}{5}, b = 0$

18. $w = \frac{1}{2}\left(-1+\sqrt{3}i\right) \Rightarrow 2w + 1 = \sqrt{3}i$

Also, $w^7 = w$ and $w^2 + 1 = -w$

Using $R_1 \to R_1 + R_2 + R_3$ in the given determinant, We get

$$\Delta = \begin{vmatrix} 3 & 0 & 0 \\ 1 & w & w^2 \\ 1 & w^2 & w \end{vmatrix}$$

$= 3(w^2 - w^4) = 3(w^2 - w)$

$\therefore k = w^2 - w$

$= w^2 + w - 2w$

$= -1 - 2w = -z$

19. Let

$S = {}^{21}C_1 + {}^{21}C_2 + \ldots\ldots + {}^{21}C_{10}$ (1)

Using ${}^nC_r = {}^nC_{n-r}$, We get

$S = {}^{21}C_{20} + {}^{21}C_{19} + \ldots + {}^{21}C_{11}$ (2)

Adding (1), (2) we get

$2s = \sum_{k=1}^{20}\left({}^{21}C_k\right) = 2^{21} - 1 - 1 = 2^{21} - 2$

$\Rightarrow S = 2^{20}-1$

Next, let

$T = {}^{10}C_1 + {}^{10}C_2 + \ldots + {}^{10}C_{10}$

$= 2^{10} - 1$

$\therefore ({}^{21}C_1 - {}^{10}C_1) + ({}^{21}C_2 - {}^{10}C_2) + \ldots + ({}^{21}C_{10} - {}^{10}C_{10})$

$= (2^{20} - 1) - (2^{10} - 1) = 2^{20} - 2^{10}$

20. $\lim_{x\to\frac{\pi}{2}} \frac{\cot x - \cos x}{(\pi - 2x)^3}$

$= \lim_{x\to\frac{\pi}{2}} \frac{(\cos x)(1-\sin x)}{8\sin x\left(\frac{\pi}{2}-x\right)^3}$

$= \lim_{\theta\to 0} \frac{\cos\left(\frac{\pi}{2}+\theta\right)\left[1-\sin\left(\frac{\pi}{2}+\theta\right)\right]}{8\left(\sin\left(\frac{\pi}{2}+\theta\right)\right)(-\theta)^3}$ $\quad [x = \frac{\pi}{2}+\theta]$

$= \lim_{\theta\to 0}\frac{(\sin\theta)(1-\cos\theta)}{8(\cos\theta)\theta^3}$

$= \frac{1}{8}\lim_{\theta\to 0}\frac{(\sin\theta)(1-\cos\theta)(1+\cos\theta)}{(\cos\theta)(1+\cos\theta)\theta^3}$

$= \frac{1}{8}\lim_{\theta\to 0}\left(\frac{\sin\theta}{\theta}\right)^3\frac{1}{(\cos\theta)(1+\cos\theta)}$

$= \frac{1}{8}(1)^3\,\frac{1}{(1)(1+1)} = \frac{1}{16}.$

21. $5(\tan^2 x - \cos^2 x) = 2\cos(2x) + 9$

$\Rightarrow 5\tan^2 x = 5\cos^2 x + 2(2\cos^2 x - 1) + 9$

$\Rightarrow 5\tan^2 x = 9\cos^2 x + 7$

$\Rightarrow 5\sec^2 x = 9\cos^2 x + 12$

$\Rightarrow 9\cos^4 x + 12\cos^2 x - 5 = 0$

$\Rightarrow (3\cos^2 x - 1)(3\cos^2 x + 5) = 0$

$\Rightarrow \cos^2 x = \frac{1}{3}$ $\quad [\because 3\cos^2 x + 5 > 0]$

$\Rightarrow \cos 2x = 2\cos^2 x - 1 = -\frac{1}{3}$

$\therefore \cos(4x) = 2\cos^2(2x) - 1 = -\frac{7}{9}.$

22. Equation of line through P(1, –2, 3)

And parallel to $\frac{x}{1} = \frac{y}{4} = \frac{z}{5}$ is

$\frac{x-1}{1} = \frac{y+2}{4} = \frac{z-3}{5} = r(\text{say})$

$\Rightarrow x = r + 1,\ y = 4r - 2,\ z = 5r + 3.$

Let coordinates of point Q be $(r + 1,\ 4r - 2,\ 5r + 3)$

As Q lies on $2x + 3y - 4z + 22 = 0$,

$2(r + 1) + 3(4r - 2) - 4(5r + 3) + 22 = 0$

$\Rightarrow -6r + 6 = 0$

$\Rightarrow r = 1$

Now,

$PQ^2 = (r + 1 - 1)^2 + (4r - 2 + 2)^2 + (5r + 3 - 3)^2$

$= r^2(1 + 16 + 25) = 42r^2 = 42$

$\Rightarrow PQ = \sqrt{42}$

23. Note that normal to plane is

$\mathbf{N} = (\mathbf{i} - 2\mathbf{j} + 3\mathbf{k}) \times (2\mathbf{i} - \mathbf{j} - \mathbf{k})$

$= 5\mathbf{i} + 7\mathbf{j} + 3\mathbf{k}$

Therefore equation of plane is

$5(x - 1) + 7(y + 1) + 3(z + 1) = 0$ (1)

Distance of (1, 3, -7) from (1) is

$\frac{|5(1-1)+7(3+1)+3(-7+1)|}{\sqrt{25+49+9}} = \frac{10}{\sqrt{83}}$

24. $y = \tan^{-1}\left(\frac{6x\sqrt{x}}{1-9x^3}\right)$

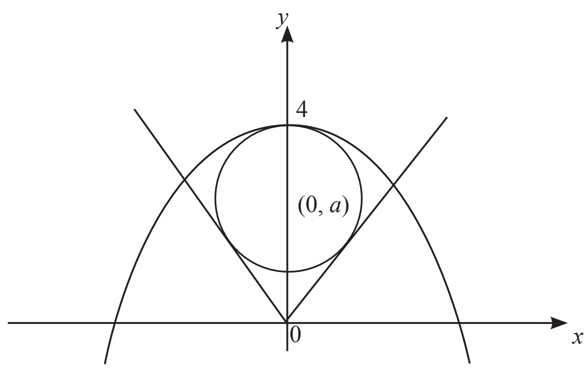
$= \tan^{-1}\left(\frac{2\left(3x^{\frac{3}{2}}\right)}{1-\left(3x^{\frac{3}{2}}\right)^2}\right)$

$= 2\tan^{-1}\left(3x^{\frac{3}{2}}\right)$

$\frac{dy}{dx} = \frac{2.\left(3x^{\frac{1}{2}}\right)\left(\frac{3}{2}\right)}{1+9x^3} = \frac{9}{1+9x^3}\sqrt{x}$

$\therefore g(x) = \frac{9}{1+9x^3}$

25. As the circle touches $y = |x|$, that is, $y = x$, $y = -x$, $y > 0$, the centre of circle lies on the positive of y–axis. Let centre of the circle be



$(0, a)$ where $a > 0$.

Then the radius $r = 4 - a$

Also, length of perpendicular from $(0, a)$ to $y = x$ is r

$\therefore r = \frac{a}{\sqrt{2}}$

Thus, $4 - a = \frac{a}{\sqrt{2}} \Rightarrow 4 = \left(1+\frac{1}{\sqrt{2}}\right)a$

$\Rightarrow a = \dfrac{4\sqrt{2}}{\sqrt{2}+1}$

$\Rightarrow r = \dfrac{4}{\sqrt{2}+1} = 4\left(\sqrt{2}-1\right).$

26. Let p = probability of drawing a green ball in a single draw.

$= \dfrac{15}{25} = \dfrac{3}{5}$

n = number of balls drawn = 10, and

x = number of green balls drawn,

Then $x \sim B(n, p)$

$\text{Var}(x) = np(1 - p)$

$= 10\left(\dfrac{3}{5}\right)\left(1-\dfrac{3}{5}\right) = \dfrac{12}{5}$

27. We are given $e = \dfrac{1}{2}$, and

$-\dfrac{a}{e} = -4 \Rightarrow 2a = 4$ or $a = 2$

$\therefore\ b^2 = a^2(1 - e^2) = 4\left(1-\dfrac{1}{4}\right) = 3$

$\therefore$ Equation of ellipse is

$\dfrac{x^2}{4}+\dfrac{y^2}{3} = 1$

And equation of tangent at $\left(1,\dfrac{3}{2}\right)$ is

$\dfrac{1}{4}x+\dfrac{3}{2}\dfrac{y}{3}=1$

$x + 2y = 4$

$\therefore$ An equation of normal at $\left(1,\dfrac{3}{2}\right)$ is

$2(x - 1) - \left(y-\dfrac{3}{2}\right) = 0$

Or $2x - y - \dfrac{1}{2} = 0$ or $4x - 2y = 1$

28. Two numbers can be chosen in $^{11}C_2 = 55$ ways.

Suppose selected number are a, b with $a > b$, then

$a + b = 4r,\ a - b = 4s$

$\Rightarrow 2a = 4(r + s),\ 2b = 4(r - s)$

$\Rightarrow a = 2(r + s),\ b = 2(r - s)$

$\therefore$ both a, b are even, and possible pairs of (a, b) are

(4, 0), (8, 0), (6, 2), (10, 2), (8, 4), (10, 6),

Thus, probability of required event is $\dfrac{6}{55}$.

29. P(Exactly one of A, B) $= \dfrac{1}{4}$

$\Rightarrow P(A \cap B') + P(A' \cap B) = \dfrac{1}{4}$

$\Rightarrow P(A) + P(B) - 2P(A \cap B) = \dfrac{1}{4}$ (1)

Similarly,

$P(B) + P(C) - 2P(B \cap C) = \dfrac{1}{4}$ (2)

And $P(C) + P(A) - 2P(C \cap A) = \dfrac{1}{4}$ (3)

Adding (1), (2), (3), we get

$2[P(A) + P(B) + P(C) - P(A \cap B) - P(B \cap C) - P(C \cap A)] = \dfrac{3}{4}$

$\Rightarrow P(A) + P(B) + P(C) - P(A \cap B) - P(B \cap C) - P(C \cap A) = \dfrac{3}{8}$ (4)

Also, $P(A \cap B \cap C) = \dfrac{1}{16}$ (5)

From (4), (5), we get

$P(A \cup B \cup C) = \dfrac{3}{8}+\dfrac{1}{16} = \dfrac{7}{16}.$

30. $A^2 = \begin{bmatrix} 2 & -3 \\ -4 & 1 \end{bmatrix}\begin{bmatrix} 2 & -3 \\ -4 & 1 \end{bmatrix}$

$= \begin{bmatrix} 16 & -9 \\ -12 & 13 \end{bmatrix}$

$\therefore\ 3A^2 + 12A$

$= \begin{bmatrix} 72 & -63 \\ -84 & 51 \end{bmatrix}$

$\Rightarrow \text{adj}\ (3A^2 + 12A) = \begin{bmatrix} 51 & 63 \\ 84 & 72 \end{bmatrix}.$