JEE (Main) 2017 Questions with Solutions Mathematics (8th April - online)

1. Let $f(x) = 2^{10} x + 1$ and $g(x) = 3^{10} x - 1$. If $(fog)(x) = x$, then x is equal to:

   (a) $\frac{3^{10}-1}{3^{10}-2^{-10}}$ (b) $\frac{2^{10}-1}{2^{10}-3^{-10}}$

   (c) $\frac{1-3^{-10}}{2^{10}-3^{-10}}$ (d) $\frac{1-2^{-10}}{3^{10}-2^{-10}}$

2. Let $p(x)$ be a quadratic polynomial such that $p(0) = 1$. If $p(x)$ leaves remainder 4 when divided by $x - 1$ and it leaves remainder 6 when divided by $x + 1$; then:

   (a) $p(2) = 11$ (b) $p(2) = 19$

   (c) $p(-2) = 19$ (d) $p(-2) = 11$

3. Let $z \in C$, the set of complex numbers. Then the equation, $2|z + 3i| - |z - i| = 0$ Represents:

   (a) a circle with radius $\frac{8}{3}$.

   (b) a circle with diameter $\frac{10}{3}$.

   (c) an ellipse with length of major axis $\frac{16}{3}$.

   (d) an ellipse with length of minor axis $\frac{16}{9}$.

4. The number of real values of λ for which the system of linear equations

   $2x + 4y - \lambda z = 0$

   $4x + \lambda y + 2z = 0$

   $\lambda x + 2y + 2z = 0$

   has infinitely many solutions, is:

   (a) 0 (b) 1

   (c) 2 (d) 3

5. Let A be any 3 × 3 invertible matrix. Then which one of the following is not always true?

   (a) adj (A) $=|A|\cdot A^{-1}$

   (b) adj (adj(A)) $= |A|\cdot A$

   (c) adj (adj(A)) $= |A|^2\cdot(\text{adj}(A))^{-1}$

   (d) adj (adj(A)) $= |A|.(\text{adj}(A))^{-1}$

6. If all the words, with or without meaning, are written using the letters of the word QUEEN and are arranged as in English dictionary, then the position of the word QUEEN is:

   (a) 44^{th} (b) 45^{th}

   (c) 46^{th} (d) 47^{th}

7. If $(27)^{999}$ is divided by 7, then the remainder is:

   (a) 1 (b) 2

   (c) 3 (d) 6

8. If the arithmetic mean of two numbers a and b $a > b > 0$, is five times their goemetric mean, then $\frac{a+b}{a-b}$ is equal to:

   (a) $\frac{\sqrt{6}}{2}$ (b) $\frac{3\sqrt{2}}{4}$

   (c) $\frac{7\sqrt{3}}{12}$ (d) $\frac{5\sqrt{6}}{12}$

9. If the sum of the first n terms of the series $\sqrt{3}+\sqrt{75}+\sqrt{243}+\sqrt{507}+....$ is $435\sqrt{3}$ then n equals:

   (a) 18 (b) 15

   (c) 13 (d) 29

10. $\lim_{x\to 3}\frac{\sqrt{3x}-3}{\sqrt{2x-4}-\sqrt{2}}$ is equal to:

    (a) $\sqrt{3}$ (b) $\frac{1}{\sqrt{2}}$

    (c) $\frac{\sqrt{3}}{2}$ (d) $\frac{1}{2\sqrt{2}}$

11. The tangent at the point (2, –2) to the curve, $x^2y^2 - 2x = 4(1 - y)$ does not pass through the point:

(a) $\left(4, \frac{1}{3}\right)$ (b) (8, 5)

(c) (–4, –9) (d) (–2, –7)

12. If y = $\left[x+\sqrt{x^2-1}\right]^{15}+\left[x-\sqrt{x^2-1}\right]^{15}$ then $\left(x^2-1\right)\frac{d^2y}{dx^2}+x\frac{dy}{dx}$ is equal to:

(a) 125 y (b) 224 y^2

(c) 225 y^2 (d) 225 y

13. If a point P has co-ordinates (0, –2) and Q is any point on the circle, $x^2 + y^2 - 5x - y + 5 = 0$, then the maximum value of $(PQ)^2$ is:

(a) $\frac{25+\sqrt{6}}{2}$ (b) $14+5\sqrt{3}$

(c) $\frac{47+10\sqrt{6}}{2}$ (d) $8+5\sqrt{3}$

14. The integral I = $\int\sqrt{1+2\cot x\left(\operatorname{cosec} x+\cot x\right)}dx$ $\left(0<x<\frac{\pi}{2}\right)$ is equal to:

(where C is a constant of integration)

(a) $4\log\left(\sin\frac{x}{2}\right)+C$ (b) $2\log\left(\sin\frac{x}{2}\right)+C$

(c) $2\log\left(\cos\frac{x}{2}\right)+C$ (d) $4\log\left(\cos\frac{x}{2}\right)+C$

15. the integral I = $\int_{\frac{\pi}{12}}^{\frac{\pi}{4}}\frac{8\cos 2x}{\left(\tan x+\cot x\right)^3}dx$ equals:

(a) $\frac{15}{128}$ (b) $\frac{15}{64}$

(c) $\frac{13}{32}$ (d) $\frac{13}{256}$

16. The area (in sq. units) of the smaller portion enclosed between the curves, $x^2 + y^2 = 4$ and $y^2 = 3x$, is:

(a) $\frac{1}{2\sqrt{3}}+\frac{\pi}{3}$ (b) $\frac{1}{\sqrt{3}}+\frac{2\pi}{3}$

(c) $\frac{1}{2\sqrt{3}}+\frac{2\pi}{3}$ (d) $\frac{1}{\sqrt{3}}+\frac{4\pi}{3}$

17. The curve satisfying the differential equation, $y\mathrm{d}x - (x + 3y^2)\mathrm{d}y = 0$ and passing through the point (1, 1), also passes through the point:

(a) $\left(\frac{1}{4}, -\frac{1}{2}\right)$ (b) $\left(-\frac{1}{3}, \frac{1}{3}\right)$

(c) $\left(\frac{1}{3}, -\frac{1}{3}\right)$ (d) $\left(\frac{1}{4}, \frac{1}{2}\right)$

18. The locus of the point of intersection of the straight lines,

$tx - 2y - 3t = 0$

$x - 2ty + 3 = 0$ ($r \in \mathbf{R}$), is :

(a) an ellipse with eccentricity $\frac{2}{\sqrt{5}}$

(b) an ellipse with the length of major axis 6

(c) a hyperbola with eccentricity $\sqrt{5}$

(d) a hyperbola with the length of conjugate axis 3

19. If two parallel chords of a circle, having diameter 4 units, lie on the opposite sides of the centre and subtend angles $\cos^{-1}\left(\frac{1}{7}\right)$ and $\sec^{-1}(7)$ at the centre respectively, then the distance between these chords, is:

(a) $\frac{4}{\sqrt{7}}$ (b) $\frac{8}{\sqrt{7}}$

(c) $\frac{8}{7}$ (d) $\frac{16}{7}$

20. If the common tangents to the parabola, $x^2 = 4y$ and the circle, $x^2 + y^2 = 4$ intersect at the point P, then the distance of P from the origin, is:

(a) $\sqrt{2}+1$ (b) $2\left(3+2\sqrt{2}\right)$

(c) $2\left(\sqrt{2}+1\right)$ (d) $3+2\sqrt{2}$

21. Consider an ellipse, whose centre is at the origin and its major axis is along the x-axis. If its eccentricity is $\frac{3}{5}$ and the distance between its foci is 6, then the area (in sq. units) of the quadrilateral inscribed in the ellipse, whith the vertices as the vertices of the ellipse, is:

(a) 8 (b) 32

(c) 80 (d) 40

22. The coordinates of the foot of the perpendicular from the point (1, –2, 1) on the plane containing the lines, $\frac{x+1}{6}=\frac{y-1}{7}=\frac{z-3}{8}$ and $\frac{x-1}{3}=\frac{y-2}{5}=\frac{z-3}{7}$, is:

(a) (2, –4, 2) (b) (–1, 2, –1)

(c) (0, 0, 0) (d) (1, 1, 1)

23. The line of intersection of the planes $\mathbf{r}\cdot(3\mathbf{i}-\mathbf{j}+\mathbf{k})=1$ and $\mathbf{r}\cdot(\mathbf{i}+4\mathbf{j}-2\mathbf{k})=2$, is:

(a) $\dfrac{x-\frac{4}{7}}{-2}=\dfrac{y}{7}=\dfrac{z-\frac{5}{7}}{13}$

(b) $\dfrac{x-\frac{4}{7}}{2}=\dfrac{y}{-7}=\dfrac{z+\frac{5}{7}}{13}$

(c) $\dfrac{x-\frac{6}{13}}{2}=\dfrac{y-\frac{5}{13}}{-7}=\dfrac{z}{-13}$

(d) $\dfrac{x-\frac{6}{13}}{2}=\dfrac{y-\frac{5}{13}}{7}=\dfrac{z}{-13}$

24. The area (in sq. units) of the parallelogram whose diagonals are along the vectors $8\mathbf{i}-6\mathbf{j}$ and $3\mathbf{i}+4\mathbf{j}-12\mathbf{k}$, is:

(a) 26 (b) 65
(c) 20 (d) 52

25. The mean age of 25 teachers in a school is 40 years. A teacher retires at the age of 60 years and a new teacher is appointed in his place. If now the mean age of the teachers in this school is 39 years, then the age (in years) of the newly appointed teacher is:

(a) 25 (b) 30
(c) 35 (d) 40

26. Three persons P,Q and R independentely try to hit a target. If the probabilities of their hitting the target are $\frac{3}{4}$, $\frac{1}{2}$ and $\frac{5}{8}$ respectively, then the probabiity that the target is hit by P or Q but not by R is:

(a) $\frac{21}{64}$ (b) $\frac{9}{64}$
(c) $\frac{15}{64}$ (d) $\frac{39}{64}$

27. An unbiased coin is tossed eight times. The probability of obtaining at least one head and at least one tail is:

(a) $\frac{255}{256}$ (b) $\frac{127}{128}$
(c) $\frac{63}{64}$ (d) $\frac{1}{2}$

28. If S = $\left\{x\in[0,2\pi]:\begin{vmatrix}0 & \cos x & -\sin x\\ \sin x & 0 & \cos x\\ \cos x & \sin x & 0\end{vmatrix}=0\right\}$,

Then $\sum_{x\in s}\tan\left(\frac{\pi}{3}+x\right)$ is equal to:

(a) $4+2\sqrt{3}$ (b) $-2+\sqrt{3}$
(c) $-2-\sqrt{3}$ (d) $-4-2\sqrt{3}$

29. the value of $y=\tan^{-1}\left[\dfrac{\sqrt{1+x^2}+\sqrt{1-x^2}}{\sqrt{1+x^2}-\sqrt{1-x^2}}\right]$,

$|x|<\frac{1}{2}$, $x\neq 0$, is equal to:

(a) $\frac{\pi}{4}+\frac{1}{2}\cos^{-1}x^2$ (b) $\frac{\pi}{4}+\cos^{-1}x^2$
(c) $\frac{\pi}{4}-\frac{1}{2}\cos^{-1}x^2$ (d) $\frac{\pi}{4}-\cos^{-1}x^2$

30. The proposition $(\sim p)\vee(p\wedge\sim q)$ is equivalent to:

(a) $p\vee\sim q$ (b) $p\rightarrow\sim q$
(c) $p\wedge\sim q$ (d) $q\rightarrow p$

Answers

1. (d)	**2.** (c)	**3.** (a)	**4.** (b)	**5.** (d)
6. (c)	**7.** (d)	**8.** (d)	**9.** (b)	**10.** (b)
11. (d)	**12.** (d)	**13.** (b)	**14.** (b)	**15.** (a)
16. (d)	**17.** (b)	**18.** (d)	**19.** (b)	**20.** (c)
21. (d)	**22.** (c)	**23.** (c)	**24.** (b)	**25.** (c)
26. (a)	**27.** (b)	**28.** (d)	**29.** (a)	**30.** (b)

Hints and Solutions

1. $(fog)(x)=x$

$\Rightarrow f(g(x))=x$

$\Rightarrow f(3^{10}x-1)=x$

$\Rightarrow 2^{10}(3^{10}x-1)+1=x$

$\Rightarrow 3^{10}x-1+2^{-10}=2^{-10}x$

$\Rightarrow (3^{10}-2^{-10})x=1-2^{-10}$

$\Rightarrow x=\dfrac{1-2^{-10}}{3^{10}-2^{-10}}$

2. Let $p(x) = (x - 1)(ax + b) + 4$

$p(0) = (-1)(0 + b) + 4 = 1$

$\Rightarrow -b + 4 = 1 \Rightarrow b = 3$

Also, $6 = p(-1) = (-1 - 1)(-a + b) + 4$

$\Rightarrow 1 = a - b \Rightarrow a = 4$

$\therefore\ p(x) = (x - 1)(4x + 3) + 4$

$\Rightarrow p(2) = (2 - 1)(11) + 4 = 15$

$p(-2) = (-2 - 1)(-8 + 3) + 4 = 19$

3. Let $z = x + iy$

$2|x + iy + 3i| = |x + iy - i|$

$\Rightarrow 4[x^2 + (y + 3)^2] = x^2 + (y - 1)^2$

$\Rightarrow 3x^2 + 3y^2 + 26y + 35 = 0$

It's a circle with radius

$$= \sqrt{\left(\frac{13}{3}\right)^2 - \frac{35}{3}} = \frac{8}{3}$$

4. As the system has infinitely many solutions

$$\begin{vmatrix} 2 & 4 & -\lambda \\ 4 & \lambda & 2 \\ \lambda & 2 & 2 \end{vmatrix} = 0$$

$$\Rightarrow 2\begin{vmatrix} \lambda & 2 \\ 2 & 2 \end{vmatrix} - 4\begin{vmatrix} 4 & 2 \\ \lambda & 2 \end{vmatrix} - \lambda\begin{vmatrix} 4 & \lambda \\ \lambda & 2 \end{vmatrix} = 0$$

$\Rightarrow 2(2\lambda - 4) - 4(8 - 2\lambda) - \lambda(8 - \lambda^2) = 0$

$\Rightarrow \lambda^3 - 4\lambda - 40 = 0$

Let $g(x) = x^3 - 4x - 40$

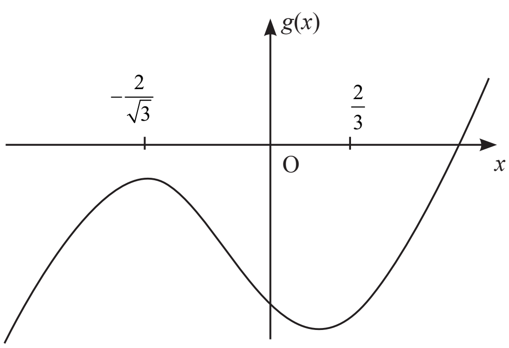
$g'(x) = 3x^2 - 4$

$g'(x) = 0 \Rightarrow x = \pm\ 2/\sqrt{3}$

$g''(x) = 6x$

As $g''\left(\frac{2}{\sqrt{3}}\right) > 0$, $g''\left(-\frac{2}{\sqrt{3}}\right) < 0$

Graph of $y(x)$ is



$\therefore\ g(x) = 0$ has exactly one root.

$\Rightarrow$ there is exactly one value of λ

5. We know $A^{-1} = \frac{1}{|A|}\,\text{adj}A$

$\Rightarrow |A|A^{-1} = \text{adj}A$

$\Rightarrow \text{adj}(\text{adj}A) = \text{adj}(|A|A^{-1})$

$= |A|^2\,\text{adj}(A^{-1})$

$= |A|^2(|A^{-1}|(A^{-1})^{-1})$

$= |A|A$

Also, $(\text{adj}A)^{-1} = \frac{1}{|\text{adj}A|}\,\text{adj}(\text{adj}A)$

$= \frac{1}{|A|^2}\,\text{adj}(\text{adj}A)$

$\Rightarrow \text{adj}(\text{adj}A) = |A|^2\,(\text{adj}(A))^{-1}$

Thus, $\text{adj}(\text{adj}\ A) = |A|(\text{adj}\ A)^{-1}$

is not necessarily true.

6. Letters of the word QUEEN are

E,E,N,Q,U

Words beginning with E (4!)	R 1 to 24
Words beginning with N $\left(\frac{4!}{2!}\right)$	25 to 36
Words beginning with QE (3!)	37 to 42
Words beginning with QN $\left(\frac{3!}{2!}\right)$	43 to 45

QUEEN is the next word and has rank 46^{th}.

7. $(27)^{994} = (28 - 1)^{999} = (7 \times 4 - 1)^{999}$

$= 7m - 1 = 7(m - 1) + 6$

$\therefore$ remainder is 6

8. $\frac{a+b}{2} = 5\sqrt{ab}$

$\Rightarrow \frac{a+b}{\sqrt{ab}} = 10 \Rightarrow \sqrt{\frac{a}{b}} + \sqrt{\frac{b}{a}} = 10$

$\Rightarrow \frac{a}{b} + \frac{b}{a} + 2 = 100$

$\Rightarrow \frac{a}{b} + \frac{b}{a} = 98$

$\Rightarrow a^2 + b^2 + 2ab = 100ab$

And $a^2 + b^2 - 2ab = 96ab$

$\Rightarrow \frac{(a+b)^2}{(a-b)^2} = \frac{100}{96} = \frac{25}{24}$

$\Rightarrow \frac{a+b}{a-b} = \frac{5}{2\sqrt{6}} = \frac{5\sqrt{6}}{12}$

9. $\sqrt{3}+\sqrt{75}+\sqrt{243}+\sqrt{507}+...$

Upto n terms $= 435\sqrt{3}$

$\Rightarrow \sqrt{3}+5\sqrt{3}+9\sqrt{3}+13\sqrt{3}+....$ upto n terms $= 435\sqrt{3}$

$\Rightarrow$ 1 + 5 + 9 + 13 + …. Upto n terms = 435

$\Rightarrow \frac{n}{2}\{2(1)+4(n-1)\} = 435$

$\Rightarrow 2n^2 - 30n + 29n - 435 = 0$

$\Rightarrow 2n(n - 15) + 29(n - 15) = 0$

$\Rightarrow (2n + 29)(n - 15) = 0$

$\Rightarrow n = 15$ as $n \in \mathbf{N}$.

10. $\lim\limits_{x\to 3}\frac{\sqrt{3x}-3}{\sqrt{2x-4}-\sqrt{2}}$

$= \lim\limits_{x\to 3}\frac{\left(\sqrt{3x}-3\right)\left(\sqrt{3x}+3\right)\left(\sqrt{2x-4}+\sqrt{2}\right)}{\left(\sqrt{2x-4}-\sqrt{2}\right)\left(\sqrt{2x-4}+\sqrt{2}\right)\left(\sqrt{3x}+3\right)}$

$= \lim\limits_{x\to 3}\frac{(3x-9)\left(\sqrt{2x-4}+\sqrt{2}\right)}{(2x-4-2)\left(\sqrt{3x}+3\right)}$

$= \frac{3}{2}\lim\limits_{x\to 3}\frac{\sqrt{2x-4}+\sqrt{2}}{\sqrt{3x}+3}$

$= \frac{3}{2}\;\frac{\sqrt{2}+\sqrt{2}}{\sqrt{3^2}+3} = \frac{1}{\sqrt{2}}$.

11. $x^2y^2 - 2x = 4(1 - y)$

$\Rightarrow \quad x^2y^2 - 2x + 4y - 4 = 0$

Differentiating w.r.t. x, we get

$2xy^2 + 2x^2y\,\frac{dy}{dx} - 2 + 4\,\frac{dy}{dx} = 0$

When $x = 2$, $y = -2$

$16 + 2(2^2)(-2)\left.\frac{dy}{dx}\right]_{(2,-2)} - 2 + 4\left.\frac{dy}{dx}\right]_{(2,-2)} = 0$

$\Rightarrow \left.\frac{dy}{dx}\right]_{(2,-2)} = \frac{14}{12} = \frac{7}{6}$

Equation of tangent at (2, –2) is

$y + 2 = \frac{7}{6}(x - 2)$

It does not pass through (–2, –7).

12. $y = \left[x+\sqrt{x^2-1}\right]^{15}+\left[x-\sqrt{x^2-1}\right]^{15}$

$\frac{dy}{dx} = 15\left[x+\sqrt{x^2-1}\right]^{14}\left\{1+\frac{x}{\sqrt{x^2-1}}\right\}$

$\qquad + 15\left[x-\sqrt{x^2-1}\right]^{14}\left\{1-\frac{x}{\sqrt{x^2-1}}\right\}$

$= \frac{15}{\sqrt{x^2-1}}\left[\left(x+\sqrt{x^2-1}\right)^{15}-\left(x-\sqrt{x^2-1}\right)^{15}\right]$

$\Rightarrow \sqrt{x^2-1}\,\frac{dy}{dx} = 15\left[\left(x+\sqrt{x^2-1}\right)^{15}-\left(x-\sqrt{x^2-1}\right)^{15}\right]$

Differentiating w.r.t. x, we get

$\sqrt{x^2-1}\frac{d^2y}{dx^2}+\frac{x}{\sqrt{x^2-1}}\frac{dy}{dx}$

$= \frac{225}{\sqrt{x^2-1}}\left(1+\sqrt{x^2-1}\right)^{15} + \frac{225}{\sqrt{x^2-1}}\left(1+\sqrt{x^2-1}\right)^{15}$

$\Rightarrow (x^2 - 1)\;\frac{d^2y}{dx^2}+x\frac{dy}{dx}=225y$

13. Rewrite the circle as

$$\left(x-\frac{5}{2}\right)^2+\left(y-\frac{1}{2}\right)^2 = \frac{3}{2} \qquad (1)$$

A point on (1) is

$x = \frac{5}{2}+\sqrt{\frac{3}{2}}\cos\theta$, $y = \frac{1}{2}+\sqrt{\frac{3}{2}}\sin\theta$, $0 \le \theta < 2\pi$.

$\therefore\ (PQ)^2 = \left(\frac{5}{2}+\sqrt{\frac{3}{2}}\cos\theta\right)^2 + \left(\frac{5}{2}+\sqrt{\frac{3}{2}}\sin\theta\right)^2$

$= \frac{25}{4}+5\sqrt{\frac{3}{2}}\cos\theta+\frac{3}{2}\cos^2\theta$

$\qquad + \frac{25}{4}+5\sqrt{\frac{3}{2}}\sin\theta+\frac{3}{2}\sin^2\theta$

$= 14 + 5\sqrt{3}\cos\left(\theta-\frac{\pi}{4}\right)$

$\therefore$ max $(PQ)^2 = 14 + 5\sqrt{3}$

14. $1 + 2\cot x\,(\operatorname{cosec}x + \cot x)$

$= 1 + 2\cot x\operatorname{cosec}x + 2\cot^2x$

$= (1 + \cot^2x) + 2\cot x\operatorname{cosec}x + \cot^2x$

$= \operatorname{cosec}^2x + 2\cot x\operatorname{cosec}x + \cot^2x$

$= (\operatorname{cosec}x + \cot x)^2$

$\therefore\quad I = \int\left(\operatorname{cosec}x+\cot x\right)dx = \int\frac{1+\cos x}{\sin x}dx$

$$= \int \frac{2\cos^2(x/2)}{2\cos(x/2)\sin(x/2)}dx = \int \cot\left(\frac{x}{2}\right)dx$$

$$= 2\log\left(\sin\frac{x}{2}\right) + c.$$

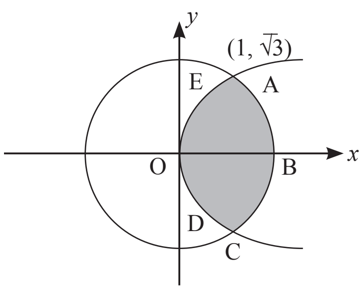
15. $\tan x + \cot x = \frac{1+\tan^2 x}{\tan x} = \frac{2}{\sin 2x}$

$$\therefore \ \mathrm{I} = \int_{\pi/12}^{\pi/4} 8\cos 2x \frac{\sin^3 2x}{2^3}dx$$

$$= \frac{1}{4}\times\frac{1}{2}\left(\sin^4 2x\right)\Big]_{\pi/12}^{\pi/4}$$

$$= \frac{1}{8}\left(1-\left(\frac{1}{2}\right)^4\right) = \frac{15}{128}.$$

16.



Required area is shaded in the figure.

Area = area (ODCBAEO)

= 2 Area (O B A E O)

$$= 2\int_0^{\sqrt{3}} \sqrt{4-y^2}\,dy - 2\int_0^{\sqrt{3}} \frac{y^2}{3}dy$$

$$= \left[y\sqrt{4-y^2} + 4\sin^{-1}\left(\frac{y}{2}\right)\right]_0^{\sqrt{3}} - \frac{2}{9}\left[y^3\right]^{\sqrt{3}}$$

$$= \sqrt{3}(1) + 4\sin^{-1}\left(\frac{\sqrt{3}}{2}\right) - \frac{2}{9}\left(3\sqrt{3}\right)$$

$$= \sqrt{3} + 4\left(\frac{\pi}{3}\right) - \frac{2}{\sqrt{3}}$$

$$= \frac{1}{\sqrt{3}} + \frac{4}{3}\pi.$$

17. Write the given differential equation as

$$\frac{dx}{dy} + \left(\frac{-1}{y}\right)x = 3y \qquad (1)$$

I.F. $= e^{\int\left(-\frac{1}{y}\right)dy} = e^{-\ln(y)} = \frac{1}{y}$

Multiply (1) by $\frac{1}{y}$ to obtain

$$\frac{d}{dy}\left(\frac{1}{y}x\right) = 3$$

$$\Rightarrow \frac{x}{y} = 3y + C$$

As it passes through (1, 1), we get

$1 = 3 + C \Rightarrow C = -2$

$\therefore \ x = (3y - 2)y$

It passes through $\left(-\frac{1}{3}, \frac{1}{3}\right)$.

18. $tx - 2y - 3t = 0$

$x - 2ty + 3 = 0$

$$\frac{x}{-6-6t^2} = \frac{-y}{3t+3t} = \frac{1}{-2t^2+2}$$

$$\Rightarrow x = \frac{-6\left(t^2+1\right)}{-2\left(t^2-1\right)} = \frac{3\left(t^2+1\right)}{t^2-1}$$

$$y = \frac{6t}{2\left(t^2-1\right)} = \frac{3t}{t^2-1}$$

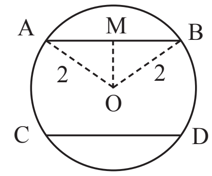
$$\therefore \ \left(\frac{x}{3}\right)^2 - 4\left(\frac{y}{3}\right)^2 = \frac{\left(t^2+1\right)^2 - 4t^2}{\left(t^2-1\right)^2} = \frac{\left(t^2-1\right)^2}{\left(t^2-1\right)^2} = 1$$

$$\Rightarrow \frac{x^2}{9} - \frac{y^2}{(3/2)^2} = 1$$

It's a hyperbola with length of conjugate axis as 3

19. As $\sec^{-1}(7) = \cos^{-1}(1/7)$, both the chards are equidistant from the centre.

Let AB = $2a$.



From ΔOAB

Cos(∠AOB)

$$= \frac{2^2 + 2^2 - (2a)^2}{(2)(2)(2)}$$

$$\Rightarrow \frac{1}{7} = \frac{8-4a^2}{8} = 1 - \frac{a^2}{2}$$

$$\Rightarrow \frac{a^2}{2} = 1 - \frac{1}{7} = \frac{6}{7} \Rightarrow a = 2\sqrt{\frac{3}{7}}$$

Now, $OM^2 = OA^2 - AM^2$

$$= 2^2 - a^2 = 4 - \frac{12}{7} = \frac{16}{7}$$

$$\Rightarrow \text{OM} = \frac{4}{\sqrt{7}}$$

$\therefore$ distance between porallel chards

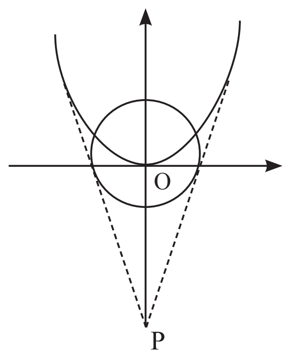
$$= 2(\text{OM}) = \frac{8}{\sqrt{7}}.$$

20. Equation of any tangent to $x^2 = 4y$ is

$$x = my + \frac{1}{m}$$

It will be tangent to $x^2 + y^2 = 4$ if

$$\frac{|m(0) - 0 + \frac{1}{m}|}{\sqrt{m^2+1}} = 2$$



$\Rightarrow \left(\frac{1}{m}\right)^2 = 4(m^2+1)$

$\Rightarrow 4m^4 + 4m^2 + 1 = 2$

$\Rightarrow (2m^2 + 1)^2 = 2$

$\Rightarrow 2m^2 + 1 = \sqrt{2}$

$\Rightarrow m^2 = \frac{\sqrt{2}-1}{2}$

Let $m_1 = \sqrt{\frac{\sqrt{2}-1}{2}}$, $m_2 = -\sqrt{\frac{\sqrt{2}-1}{2}}$

$\therefore$ Two common tangents are

$$x = m_1y + \frac{1}{m_1},\ x = m_2y + \frac{1}{m_2}$$

These two intersect at

$$p\left(0, \frac{1}{m_1m_2}\right)$$

$\therefore \quad OP = \frac{1}{m_1m_2} = \frac{2}{\sqrt{2}-1} = 2(\sqrt{2}+1).$

21. $2ae = 6 \Rightarrow 2a\left(\frac{3}{5}\right) = 6$

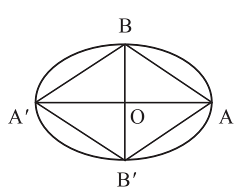
$\Rightarrow 2a = 10 \Rightarrow a = 5$

Now, $b^2 = a^2 - a^2e^2$
$= 25 - 9 = 16$

$\Rightarrow b = 4$

Area of quadrilateral

$= 4$[area ΔOAB]



$= (4)\left(\frac{1}{2}\right)(5)(4) = 40.$

22. Equation of plane containing

Both the lines is

$$\begin{vmatrix} x+1 & y-1 & z-3 \\ 6 & 7 & 8 \\ 3 & 5 & 7 \end{vmatrix} = 0$$

$\Rightarrow (49 - 40)(x + 1) - (42 - 24)(y - 1) + (30 - 21)(z - 3) = 0$

$\Rightarrow (x + 1) - 2(y - 1) + (z - 3) = 0$

Note that it passes through (1, 2, 3)

As points (2, –4, 2), (–1, –2, –1) do not lie on (1), ensures (a), (b) are not possible.

Also, (0, 0, 0) lies on (1) and line joining (1, –2, 1) and (0, 0, 0) is perpendicular to (1)

23. Let a, b, c, be direction ratios of the line of intersection of the planes

$$3x - y + z = 1 \qquad (1)$$

and

$$x + 4y - 2z = 2 \qquad (2)$$

Then

$3a - b + c = 0$

$a + 4b - 2c = 0$

Solving we get

$$\frac{a}{2-4} = \frac{-b}{-6-1} = \frac{c}{12+1}$$

$\Rightarrow \frac{a}{-2} = \frac{b}{7} = \frac{c}{13}$

Note that (4/7, 0, 5/7) doesn't lie on (1)

Also (6/13, 5/13, 0) lies on both (1) and (2)

$\therefore$ the correct answer is (c)

24. Area of parallelogram is

$= \frac{1}{2}|\mathbf{d}_1 \times \mathbf{d}_2|$

$= \frac{1}{2}\left|\begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 8 & -6 & 0 \\ 3 & 4 & -12 \end{vmatrix}\right|$

$= \frac{1}{2}|72\mathbf{i} + 96\mathbf{j} + 50\mathbf{k}|$

$= \sqrt{36^2 + 48^2 + 25^2} = \sqrt{4225} = 65.$

25. Let age of new teacher be x years, then

$$\frac{40 \times 25 - 60 + x}{25} = 39$$

$\Rightarrow 40 - \frac{60 - x}{25} = 39$

$\Rightarrow \frac{60 - x}{25} = 1 \Rightarrow x = 35.$

26. P(P or Q but not R hits the target)

$= P((P \cup Q) \cap R')$

$= P(P \cup Q)\ P(R') = [1 - P\ (P' \cap Q')]\ P\ (R')$

$= [1 - P(P')\ P\ (Q')]P\ (R')$

$= \left[1 - \left(\frac{1}{4}\right)\left(\frac{1}{2}\right)\right]\left(\frac{3}{8}\right) = \frac{21}{64}.$

27. X = number of heads obtained,

Then X ~ B (n, p) where $n = 8$ $p = ½$

P(obtaining at leart one head and at leant one tail)

= 1 – P(no head) – P(no tail)

$= 1 - \left(\frac{1}{2}\right)^8 - \left(\frac{1}{2}\right)^8 = 1 - \frac{1}{2^7}$

$= \frac{127}{128}$

28. Let $\Delta(x) = \begin{vmatrix} 0 & \cos x & -\sin x \\ \sin x & 0 & \cos x \\ \cos x & \sin x & 0 \end{vmatrix}$

$= \cos^3 x - \sin^3 x$

$= (\cos x - \sin x)(\cos^2 x + \sin^2 x + \cos x \sin x)$

$= (\cos x - \sin x)\ \left(1 + \frac{1}{2}\sin 2x\right)$

Now, $\Delta(x) = 0 \Rightarrow \tan x = 1$

$\Rightarrow\ x = \frac{\pi}{4}, \frac{5\pi}{4}$

$\therefore\ S = \left\{\frac{\pi}{4}, \frac{5\pi}{4}\right\}$

Now, $\tan\left(\frac{\pi}{4} + \frac{\pi}{3}\right) + \tan\left(\frac{5\pi}{4} + \frac{\pi}{3}\right)$

$= \tan\left(\frac{\pi}{4} + \frac{\pi}{3}\right) + \tan\left(\frac{\pi}{4} + \frac{\pi}{3}\right)$

$= 2\frac{\tan\left(\frac{\pi}{4}\right) + \tan\left(\frac{\pi}{3}\right)}{1 - \tan\left(\frac{\pi}{4}\right)\tan\left(\frac{\pi}{3}\right)}$

$= 2\frac{1+\sqrt{3}}{1-\sqrt{3}} = -2\ \frac{\left(\sqrt{3}+1\right)^2}{3-1}$

$= -(3 + 1 + 2\sqrt{3}) = -4 - 2\sqrt{3}$

29. Put $x^2 = \cos 2\theta$, then

$1 + x^2 = 1 + \cos 2\theta = 2\cos^2\theta$

And $1 - x^2 = 1 - \cos 2\theta = 2\sin^2\theta$

$\therefore\ y = \tan^{-1}\left(\frac{\sqrt{2}\cos\theta + \sqrt{2}\sin\theta}{\sqrt{2}\cos\theta - \sqrt{2}\sin\theta}\right)$

$= \tan^{-1}\left(\frac{1+\tan\theta}{1-\tan\theta}\right) = \tan^{-1}\left(\tan\left(\frac{\pi}{4} + \theta\right)\right)$

$= \frac{\pi}{4} + \theta = \frac{\pi}{4} + \frac{1}{2}\cos^{-1} x^2$

30. $(\sim p) \vee (p \wedge \sim q)$

$= (\sim p \vee p) \wedge (\sim p \vee \sim q)$

$= T \wedge (\sim p \vee \sim q)$

$= \sim p \vee \sim q$

$= p \rightarrow \sim q$