# JEE (Main) 2017 Questions with Solution (9th April – online)

1. The function $f: \mathbf{N} \to \mathbf{N}$ defined by $f(x) = x - 5\left[\frac{x}{5}\right]$, where **N** is the set of natural numbers and $[x]$ denotes the greatest integer less than or equal to $x$, is:
   (a) one–one and onto.
   (b) one–one but not onto
   (c) onto but not one–one
   (d) neither one–one or onto

2. The sum of all the real values of x satisfying the equation $2^{(x-1)(x^2+5x-50)} = 1$ is:
   (a) 16 (b) 14
   (c) –4 (d) –5

3. The equation $I_m\left(\frac{iz-2}{z-i}\right)+1 = 0$, $n \in C, z \neq i$ represents a part of a circle having radius equal to:
   (a) 2 (b) 1
   (c) $\frac{3}{4}$ (d) $\frac{1}{2}$

4. For two 3 × 3 matrices A and B, let A + B = 2B′ and 3A + 2B = $I_3$, where B′ is the transpose of B and $I_3$ is 3 × 3 identity matrix. Then:
   (a) 5A + 10B = $2I_3$ (b) 10 A + 5B = $3I_3$
   (c) B + 2A = $I_3$ (d) 3A + 6B = $2I_3$

5. If $x = a$, $y = b$, $z = c$ is a solution of the system of linear equations
   $x + 8y + 7z = 0$
   $9x + 2y + 3z = 0$
   $x + y + z = 0$
   such that the point $(a, b, c)$ lies on the plane $x + 2y + z = 6$, then $2a + b + c$ equals:
   (a) –1 (b) 0
   (c) 1 (d) 2

6. The number of ways in which 5 boys and 3 girls can be seated on a round table if a particular boy $B_1$ and a particular girl $G_1$ never sit adjacent to each other, is:
   (a) 5 × 6! (b) 6 × 6!
   (c) 7! (d) 5 × 7!

7. The coefficient of $x^{-5}$ in the binomial expansion of $\left(\frac{x+1}{x^{\frac{2}{3}} - x^{\frac{1}{3}} + 1} - \frac{x-1}{x - x^{\frac{1}{2}}}\right)^{10}$, where $x \neq 0, 1$, is:
   (a) 1 (b) 4
   (c) –4 (d) –1

8. If three positive numbers $a$, $b$ and $c$ are in A.P. such that $abc = 8$, then the minimum possible value of $b$ is:
   (a) 2 (b) $4^{\frac{1}{3}}$
   (c) $4^{\frac{2}{3}}$ (d) 4

9. Let $S_n = \frac{1}{1^3} + \frac{1+2}{1^3+2^3} + \frac{1+2+3}{1^3+2^3+3^3} + \ldots\ldots\ldots + \frac{1+2+\ldots\ldots+n}{1^3+2^3+\ldots..n^3}$. If 100 $S_n = n$ then $n$ is equal to:
   (a) 199 (b) 99
   (c) 200 (d) 19

10. The value of $k$ for which the function $f(x) = \begin{cases} \left(\frac{4}{5}\right)^{\frac{\tan 4x}{\tan 5x}}, 0 < x < \frac{\pi}{2} \\ k + \frac{2}{5}, x = \frac{\pi}{2} \end{cases}$ is continuous at $x = \frac{\pi}{2}$, is
    (a) $\frac{17}{20}$ (b) $\frac{2}{5}$
    (c) $\frac{3}{5}$ (d) $-\frac{2}{5}$

11. If $2x = y^{\frac{1}{5}} + y^{-\frac{1}{5}}$ and $(x^2-1)\frac{d^2y}{dx^2} + \lambda x\frac{dy}{dx} + ky = 0$ then $\lambda + k$ is equal to
    (a) –23 (b) –24
    (c) 26 (d) –26
12. The function $f$ defined by $f(x) = x^3 - 3x^2 + 5x + 7$, is
    (a) increasing in **R**
    (b) decreasing in **R**
    (c) decreasing in $(0, \infty)$ and increasing in $(-\infty, 0)$
    (d) increasing in $(0, \infty)$ and decreasing in $(-\infty, 0)$
13. Let $f$ be polynomial function such that $f(3x) = f'(x) \cdot f''(x)$, for all $x \in \mathbf{R}$, then
    (a) $f(2) + f'(2) = 28$
    (b) $f''(2) - f'(2) = 0$
    (c) $f''(2) - f(2) = 4$
    (d) $f(2) - f'(2) + f''(2) = 10$
14. If $f\left(\frac{3x-4}{3x+4}\right) = x+2, x \neq -\frac{4}{3}$, and $\int f(x)dx$ = A log |1 – $x$| + B$x$ + C, then the ordered pair (A, B) is equal to (where C is a constant of integration,
    (a) $\left(\frac{8}{3}, \frac{2}{3}\right)$ (b) $\left(-\frac{8}{3}, \frac{2}{3}\right)$
    (c) $\left(-\frac{8}{3}, -\frac{2}{3}\right)$ (d) $\left(\frac{8}{3}, -\frac{2}{3}\right)$
15. If $\int_1^2 \frac{dx}{(x^2-2x+4)^{\frac{3}{2}}} = \frac{k}{k+5'}$, then $k$ is equal to:
    (a) 1 (b) 2
    (c) 3 (d) 4
16. If $\lim_{n\to\infty} \frac{1^a + 2^a + \ldots\ldots + n^a}{(n+1)^{a-1}[(na+1)+(na+2)+\ldots..+(na+n)]} = \frac{1}{60}$ for some positive real number $a$, then $a$ is equal to
    (a) 7 (b) 8
    (c) $\frac{15}{2}$ (d0 $\frac{17}{2}$
17. A tangent to the curve, $y = f(x)$ at P($x$, $y$) meets $x$–axis at A and $y$–axis at B. If AP : BP = 1 : 3 and $f(1) = 1$, then the curve also passes through the point:
    (a) $\left(\frac{1}{3}, 24\right)$ (b) $\left(\frac{1}{2}, 4\right)$
    (c) $\left(2, \frac{1}{8}\right)$ (d) $\left(3, \frac{1}{28}\right)$
18. A square, of each side 2, lies above the $x$–axis and has one vertex at the origin. If one of the sides passing through the origin makes an angle 30° with the positive direction of the $x$–axis, then the sum of the $x$–coordinates of the vertices of the square is:
    (a) $2\sqrt{3}-1$ (b) $2\sqrt{3}-2$
    (c) $\sqrt{3}-2$ (d) $\sqrt{3}-1$
19. A line drawn through the point P(4, 7) cuts the circle $x^2 + y^2 = 9$ at the points A and B. Then PA · PB is equal to:
    (a) 53 (b) 56
    (c) 74 (d) 65
20. The eccentricity of an ellipse having centre at the origin, axes along the co–ordinate axes and passing through the points (4, –1) and (–2, 2) is:
    (a) $\frac{1}{2}$ (b) $\frac{2}{\sqrt{5}}$
    (c) $\frac{\sqrt{3}}{2}$ (d) $\frac{\sqrt{3}}{4}$
21. If $y = mx + c$ is the normal at a point on the parabola $y^2 = 8x$ whose focal distance is 8 units, then $|c|$ is equal to:
    (a) $2\sqrt{3}$ (b) $8\sqrt{3}$
    (c) $10\sqrt{3}$ (d) $16\sqrt{3}$
22. If a variable plane, at a distance of 3 units from the origin, intersects the coordinate axes at A, B and C, then the locus of the centroid of ΔABC is
    (a) $\frac{1}{x^2}+\frac{1}{y^2}+\frac{1}{z^2}=1$ (b) $\frac{1}{x^2}+\frac{1}{y^2}+\frac{1}{z^2}=3$
    (c) $\frac{1}{x^2}+\frac{1}{y^2}+\frac{1}{z^2}=\frac{1}{9}$ (d) $\frac{1}{x^2}+\frac{1}{y^2}+\frac{1}{z^2}=9$
23. If the line, $\frac{x-3}{1} = \frac{y+2}{-1} = \frac{z+\lambda}{-2}$ lies in the plane, $2x - 4y + 3z = 2$, then the shortest distance between this line and the line $\frac{x-1}{12} = \frac{y}{9} = \frac{z}{4}$ is:
    (a) 2 (b) 1
    (c) 0 (d) 3
24. If the vector $\mathbf{b} = 3\mathbf{j} + 4\mathbf{k}$ is written as the sum of a vector $\mathbf{b}_1$ parallel to $\mathbf{a} = \mathbf{i} + \mathbf{j}$ and a vector $\mathbf{b}_2$, perpendicular to **a**, then $\mathbf{b}_1 \times \mathbf{b}_2$ is equal to:
    (a) $-3\mathbf{i}+3\mathbf{j}-9\mathbf{k}$ (b) $6\mathbf{i}-6\mathbf{j}+\frac{9}{2}\mathbf{k}$
    (c) $-6\mathbf{i}+6\mathbf{j}-\frac{9}{2}\mathbf{k}$ (d) $3\mathbf{i}-3\mathbf{j}+9\mathbf{k}$

25. From a group of 10 men and 5 women, four member committees are to be formed each of which must contain at least one woman. Then the probability for these committees to have more women than men, is:

(a) $\frac{21}{220}$ (b) $\frac{3}{11}$

(c) $\frac{1}{11}$ (d) $\frac{2}{23}$

26. Let E and F be two independent events. The probability that both E and F happen is $\frac{1}{12}$ and the probability that neither E nor F happens is $\frac{1}{2}$, then a value of $\frac{P(E)}{P(F)}$ is

(a) $\frac{4}{3}$ (b) $\frac{3}{2}$

(c) $\frac{1}{3}$ (d) $\frac{5}{12}$

27. The sum of 100 observations and the sum of their squares are 400 and 2475, respectively. Later on, three observations, 3, 4 and 5, were found to be incorrect. If the incorrect observations are omitted, then the variance of the remaining observations is:

(a) 8.25 (b) 8.50

(c) 8.00 (d) 9.00

28. A value of $x$ satisfying the equation $\sin[\cot^{-1}(1 + x)] = \cos[\tan^{-1}x]$ is

(a) $-\frac{1}{2}$ (b) $-1$

(c) 0 (d) $\frac{1}{2}$

29. The two adjacent sides of a cyclic quadrilateral are 2 and 5 and the angle between them is 60°. If the area of the quadrilateral is $4\sqrt{3}$, then the perimeter of the quadrilateral is:

(a) 12.5 (b) 13.2

(c) 12 (d) 13

30. Contra positive of the statement

'If two numbers are not equal, then their squares are not equal', is:

(a) If the squares of two numbers are equal, then the numbers are equal.

(b) If the squares of two numbers are equal, then the numbers are not equal.

(c) If the squares of two numbers are not equal, then the numbers are not equal.

(d) If the squares of two numbers are not equal, then the numbers are equal.

## Answers

| | | | | |
|---|---|---|---|---|
| **1.** (d) | **2.** (c) | **3.** (c) | **4.** (b) | **5.** (c) |
| **6.** (a) | **7.** (a) | **8.** (a) | **9.** (a) | **10.** (c) |
| **11.** (b) | **12.** (a) | **13.** (b) | **14.** (b) | **15.** (a) |
| **16.** (a) | **17.** (c) | **18.** (b) | **19.** (b) | **20.** (c) |
| **21.** (c) | **22.** (a) | **23.** (c) | **24.** (b) | **25.** (c) |
| **26.** (a) | **27.** (d) | **28.** (a) | **29.** (d) | **30.** (a) |

## Hints and Solutions

1. Note that $f(5) = 5-5\left[\frac{5}{5}\right] = 0 = f(10)$.

Thus, function $f$ is not defined as $0 \notin \mathbf{N}$, the codomain.

As, for $0 \le r \le 4$

$f(5m + r)$

$= 5m + r - 5m$

$= r$

Range $f = (0, 1, 2, 3, 4)$

In view of one may answer it as (d).

2. $2^{(x-1)(x^2+5x-50)} = 1$ (i)

$\Rightarrow (x-1)(x^2 + 5x - 50) = 0$

$\Rightarrow (x-1)(x+10)(x-5) = 0$

$\therefore x = 1, 5, -10$

$\therefore$ sum of the real values satisfying (i) is $1 + 5 - 10 = -4$.

3. Let $z = x + iy$

Now, $\text{Im}\left(\frac{iz-2}{z-1}\right)+1 = 0$

$\Rightarrow \text{Im}\frac{ix-y-2}{x+i(y-1)}+1 = 0$

$\Rightarrow \text{Im}\frac{(ix-y-2)(x-i(y-1))}{x^2+(y-1)^2}+1 = 0$

$\Rightarrow \frac{x^2+(y+2)(y-1)}{x^2+(y-1)^2}+1 = 0$

$\Rightarrow 2x^2+(y-1)(2y+1) = 0$

$\Rightarrow 2x^2 + 2y^2 - y - 1 = 0$

$x^2 + y^2 - \frac{1}{2}y - \frac{1}{2} = 0$

$\Rightarrow\ r^2 = \left(\frac{1}{4}\right)^2 + \frac{1}{2} = \frac{9}{16}$

$\Rightarrow r = \frac{3}{4}$

4. A + B = 2B′ (1)

3A + 2B = $I_3$ (2)

From (1), (2), we get

A' + B' = 2B (3)

and 3A' + 2B' = $I_3$ (4)

From (1) and (3)

A – A′ + B – B′ = 2(B′ – B)

$\Rightarrow$ A – A′ = 3(B′ – B) (5)

From (2) and (4)

3(A – A′) + 2(B – B′) = 0 (6)

From (5) and (6)

9(B′ – B) – 2(B′ – B) = 0

$\Rightarrow$ 7(B' – B) = 0 $\Rightarrow$ B′ = B

$\Rightarrow$ From (1) we get A = B

and from (2) A = B = $\frac{1}{5}I_3$

Now, 5A + 10B = 15A = $3I_3 \neq 2I_2$

10A + 5B = 15A = $3I_3$

It is easy to check (c), (d) are not true.

5. $x + 8y + 7z = 0$ (1)

$9x + 2y + 3z = 0$ (2)

$x + y + z = 0$ (3)

From (2), (3)

$\frac{x}{2-3} = \frac{-y}{9-3} = \frac{z}{9-2}$

$\Rightarrow\ \frac{x}{-1} = \frac{y}{-6} = \frac{z}{7} = k$ (say)

$\Rightarrow\ x = -k,\ y = -6k,\ z = 7k$

Note $(-k, -6k, 7k)$ satisfies (1)

Let $a = -k,\ b = -6k,\ c = 7k$

As $(a, b, c)$ lies on $x + 2y + z = 6$

we get,

$-k - 12k + 7k = 6 \Rightarrow k = -1$

$\therefore\ a = 1,\ b = 6,\ c = -7$

Now, $2a + b + c = 2 + 6 - 7 = 1$

6. Required number of ways

= Total number of ways – Number of ways in which $B_1$ and $G_1$ are together

= 7! – (2!)6! = (5)(6!)

7. $\frac{x+1}{x^{\frac{2}{3}} - x^{\frac{1}{3}} + 1} = \frac{\left(x^{\frac{1}{3}}\right)^3 + 1}{x^{\frac{2}{3}} - x^{\frac{1}{3}} + 1}$

$= \frac{\left(x^{\frac{1}{3}} + 1\right)\left(x^{\frac{2}{3}} - x^{\frac{1}{3}} + 1\right)}{\left(x^{\frac{2}{3}} - x^{\frac{1}{3}} + 1\right)}$

Also, $\frac{x-1}{x - x^{\frac{1}{2}}} = \frac{\left(\sqrt{x}-1\right)\left(\sqrt{x}+1\right)}{\sqrt{x}\left(\sqrt{x}-1\right)} = 1 + x^{-\frac{1}{2}}$

$\therefore\ \left(\frac{x+1}{x^{\frac{2}{3}} - x^{\frac{1}{3}} + 1} - \frac{x-1}{x - x^{\frac{1}{2}}}\right)^{10}$

$= \left(x^{\frac{1}{3}} - x^{-\frac{1}{2}}\right)^{10}$

$T_{r+1} = {}^{10}C_r \left(x^{\frac{1}{3}}\right)^{10-r} \left(-x^{\frac{-1}{2}}\right)^r$

$= {}^{10}C_r x^{\frac{10-r}{3} - \frac{r}{2}} (-1)^r$

For coefficient of $x^{-5}$, set

$\frac{10-r}{3} - \frac{r}{2} = -5$

$\Rightarrow\ 20 - 2r - 3r = -30$

$\Rightarrow\ r = 10$

$\therefore$ coefficient of $x^{-5}$ is ${}^{10}C_{10}(-1)^{10} = 1$

8. $3b = a + b + c \geq 3(abc)^{\frac{1}{3}} = 3(8)^{\frac{1}{3}}$

$\Rightarrow\ b \geq 2$

$\therefore$ minimum value of b is 2.

9. $t_r = \frac{1 + 2 + ..... + r}{1^3 + 2^3 + ..... + r^3}$

$= \frac{\frac{r(r+1)}{2}}{\frac{r^2(r+1)^2}{4}} = \frac{2}{r(r+1)}$

$= 2\left(\frac{1}{r} - \frac{1}{r+1}\right)$

$\Rightarrow \quad S_n = 2\sum_{r=1}^{n}\left(\frac{1}{r}-\frac{1}{r+1}\right) = 2\left(1-\frac{1}{n+1}\right)$

$\Rightarrow \quad S_n = \frac{2n}{n+1}$

Now, $S_n = \frac{n}{100} \quad \therefore \quad \frac{2n}{n+1} = \frac{n}{100}$

$\Rightarrow \quad n = 199$

10. As f is continuous at $x = \frac{\pi}{2}$,

$$f\left(\frac{\pi}{2}\right) = \lim_{x\to\frac{\pi}{2}}\left(\frac{4}{5}\right)^{\frac{\tan 4x}{\tan 5x}} = \left(\frac{4}{5}\right)^{\lim_{x\to\frac{\pi}{2}}\frac{\tan 4x}{\tan 5x}} = \left(\frac{4}{5}\right)^{\circ} = 1$$

$\therefore \quad k + \frac{2}{5} = 1 \qquad \Rightarrow k = \frac{3}{5}$.

11. $2x = y^{\frac{1}{5}} + y^{-\frac{1}{5}}$

$\Rightarrow \quad \left(y^{\frac{1}{5}}\right)^2 - 2x\left(y^{\frac{1}{5}}\right) + 1 = 0$

$\Rightarrow \quad y^{\frac{1}{5}} = x \pm \sqrt{x^2-1}$

$\Rightarrow \quad y = \left(x \pm \sqrt{x^2-1}\right)^5$

$\Rightarrow \quad \frac{dy}{dx} = 5\left(x \pm \sqrt{x^2-1}\right)^4\left(1 \pm \frac{x}{\sqrt{x^2-1}}\right)$

$= \pm\frac{5y}{\sqrt{x^2-1}}$

$\Rightarrow \quad \sqrt{x^2-1}\frac{dy}{dx} = \pm 5y$

$\Rightarrow \quad \sqrt{x^2-1}\frac{d^2y}{dx^2} + \frac{x}{\sqrt{x^2-1}}\frac{dy}{dx} = \pm 5\frac{dy}{dx}$

$\Rightarrow \quad \left(x^2-1\right)\frac{d^2y}{dx^2} + x\frac{dy}{dx} = \pm 5\left(\sqrt{x^2-1}\frac{dy}{dx}\right)$

$= \pm 5(\pm 5y) = 25y$

$\therefore \quad \lambda = 1, k = -25$

$\Rightarrow \quad \lambda + k = -24$

12. $f(x) = x^3 - 3x^2 + 5x + 7$

$f'(x) = 3x^2 - 6x + 5 = 3(x-1)^2 + 2 > 0 \quad \vee x \in R$

Thus, $f$ is increasing in **R**.

13. Let $\deg(f(x)) = n$, then

$f(3x) = f'(x)f''(x)$

$\Rightarrow \quad \deg(f(3x)) = \deg(f'(x)) + \deg(f''(x))$

$\Rightarrow \quad n = (n-1) + (n-2)$

$\Rightarrow \quad n = 3$

Let $f(x) = ax^3 + bx^2 + cx + d$, then

$f(3x) = f'(x)f''(x)$

$\Rightarrow \quad 27ax^3 + 9bx^2 + 3cx + d$

$= (3ax^2 + 2bx + c)(6ax + 2b)$

$\Rightarrow \quad 27a = 18a^2$

$\Rightarrow \quad a = \frac{3}{2} \qquad (\because a \neq 0)$

Next, $9b = 6ab + 12ab = 18ab = 27b$

$\Rightarrow \quad b = 0$

and $3c = 6ac + 4b^2 = 9c + 0$

$\Rightarrow \quad c = 0$

and $d = 2bc = 0$

Thus, $f(x) = \frac{3}{2}x^3$

$\Rightarrow f'(x) = \frac{9}{2}x^2$ , $f''(x) = 9x$

$f''(2) - f'(2) = 0$.

14. $f\left(\frac{3x-4}{3x+4}\right) = x + 2$

Put $\frac{3x-4}{3x+4} = t \quad \Rightarrow 1 - \frac{8}{3x+4} = t$

$1 - t = \frac{8}{3x+4}$

$\Rightarrow \quad 3x + 4 = \frac{8}{1-t}$

$\Rightarrow \quad x = \frac{1}{3}\left(\frac{8}{1-t} - 4\right) = \frac{1}{3}\left(\frac{8-4+4t}{1-t}\right)$

$= \frac{4}{3}\left(\frac{1+t}{1-t}\right)$

$\therefore \; f(x) = \frac{4}{3}\left(\frac{1+x}{1-x}\right) + 2 = \frac{4}{3}\left(\frac{2}{1-x} - 1\right) + 2$

$= \frac{8}{3}\frac{1}{1-x} + \frac{2}{3}$

$\Rightarrow \quad \int f(x)dx = -\frac{8}{3}\log|1-x| + \frac{2}{3}x + C$

$\therefore \quad A = -\frac{8}{3}$

$B = \frac{2}{3}$.

15. $I = \int_1^2 \frac{dx}{\left(x^2-2x+4\right)^{\frac{3}{2}}}$

$= \int_1^2 \frac{dx}{\left[(x-1)^2+3\right]^{\frac{3}{2}}}$

Put $x - 1 = \sqrt{3}\tan\theta$

$$I = \int_0^{\frac{\pi}{6}} \frac{\sqrt{3}\sec^2\theta}{3\sqrt{3}\sec^3\theta} d\theta = \frac{1}{3}\int_0^{\frac{\pi}{6}} \cos\theta d\theta$$

$$= \frac{1}{3}[\sin\theta]_0^{\frac{\pi}{6}} = \frac{1}{6}$$

Now, $\frac{k}{k+5} = \frac{1}{6}$

$\Rightarrow$ $k = 1$

16. $$L = \lim_{n\to\infty} \frac{1^a + 2^a + .......+ n^a}{(n+1)^{a-1}\left[(na+1)+(na+2)+....(na+n)\right]}$$

$$= \lim_{n\to\infty} \frac{n^a \sum_{k=1}^{n}\left(\frac{k}{n}\right)^a}{n^a\left(1+\frac{1}{n}\right)^{a-1}\sum_{k=1}^{n}\left(a+\frac{k}{n}\right)}$$

$$= \lim_{n\to\infty} \frac{\frac{1}{n}\sum_{k=1}^{n}\left(\frac{k}{n}\right)^a}{\left(1+\frac{1}{n}\right)^{a-1}\frac{1}{n}\sum_{k=1}^{n}\left(a+\frac{k}{n}\right)}$$

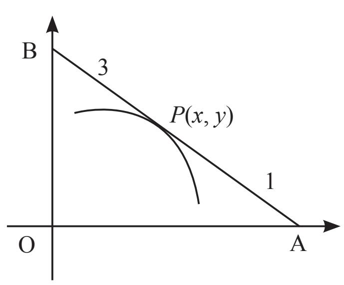
$$= \frac{\int_0^1 x^a dx}{\int_0^1 (a+x)dx} = \frac{\frac{1}{a+1}}{\frac{1}{2}\left((a+1)^2 - a^2\right)}$$

$$= \frac{2}{(a+1)(2a+1)} = \frac{1}{60}$$

$\Rightarrow$ $(a+1)(2a+1) = 120$

$\Rightarrow$ $a = 7$ $\quad (\because a > 0)$

17. An equation of tangent at $P(x, y)$ is




$Y - y = \frac{dy}{dx}(X - x)$

It meets $x$ – axis in

$A\left(x - \frac{dx}{dy}y, 0\right)$

and $y$–axis in $B\left(0, y - x\frac{dy}{dx}\right)$

Coordinates of the point which divide it in the ratio 1 : 3 is

$$\left(\frac{3}{4}\left(x - \frac{dx}{dy}y\right), \frac{1}{4}\left(y - x\frac{dy}{dx}\right)\right) = (x, y)$$

$\Rightarrow$ $\frac{3}{4}\left(x - y\frac{dx}{dy}\right) = x,\ \frac{1}{4}\left(y - x\frac{dy}{dx}\right) = y$

$\Rightarrow$ $3\frac{dx}{x} + \frac{dy}{y} = 0$

$\Rightarrow$ $\log(x^3y) =$ const.

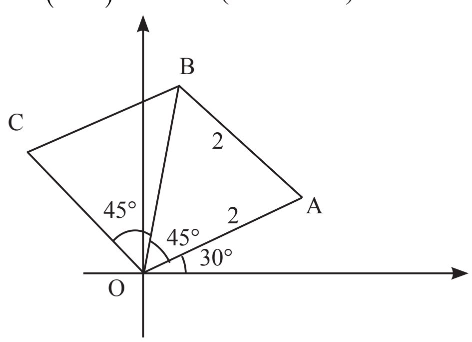
$\Rightarrow$ $x^3y = c$

When $x = 1$, $y = 1$, therefore $c = 1$

Thus, required curve $x^3y = 1$ (i)

Note that $\left(2, \frac{1}{8}\right)$ satisfies (i)

18. $x$–coordinate of A is $2\cos 30° = \sqrt{3}$, $x$–coordinate of B is $2\sqrt{2}\cos 75°$ and $x$–coordinate of $C$ is $2\cos(120°) = 2\cos(90° + 30°) = -2\sin 30° = -1$




Thus, sum of $x$–coordinates

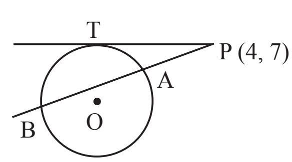
$$= 0 + \sqrt{3} + 2\sqrt{2}\cos 75° - 1$$

$$= \sqrt{3} + 2\sqrt{2}\cos(45° + 30°) - 1$$

$$= \sqrt{3} + 2\sqrt{2}\left(\frac{1}{\sqrt{2}}\cdot\frac{\sqrt{3}}{2} - \frac{1}{\sqrt{2}}\cdot\frac{1}{2}\right) - 1$$

$$= \sqrt{3} + \sqrt{3} - 1 - 1 = 2\sqrt{3} - 2$$

19.




$(PA)(PB) = PT^2$ and $PT^2 = 4^2 + 7^2 - 9 = 56$

20. Let equation of ellipse be

$$\frac{x^2}{a^2}+\frac{y^2}{b^2} = 1$$

As it passes through (4, –1) and (–2, 2) we get

$$\frac{16}{a^2}+\frac{1}{b^2}=1 \text{ and } \frac{4}{a^2}+\frac{4}{b^2}=1$$

Solving we get $b^2 = 5$, $a^2 = 20$

Now $a^2(1 - e^2) = b^2$

$\Rightarrow 20(1 - e^2) = 5$

$\Rightarrow e = \frac{\sqrt{3}}{2}$

21. $c = -2am - am^3 = -4m - 2m^3$

$\therefore\ y = mx - 4m - 2m^3$

is a normal at $P(2m^2, 4m)$

Distance of P from focus

= distance of P from directrix ($x = -2$)

$= 2(m^2 + 1)$

As $2(m^2 + 1) = 8$

$\Rightarrow m = \pm\sqrt{3}$

For $m = -\sqrt{3}$, $c = 4\sqrt{3}+2\left(3\sqrt{3}\right) = 10\sqrt{3}$

22. Let the centroid of ΔABC be $G(\alpha,\beta,\gamma)$ then coordinates of A, B, C are $(3\alpha,0,0)$, $(0,3\beta,0)$, $(0,0,3\gamma)$ respectively

Equation of the plane through A, B and C is

$$\frac{x}{3\alpha}+\frac{y}{\beta}+\frac{z}{3\gamma}=1$$

Its distance from (0, 0, 0) is

$$\frac{|0-3|}{\sqrt{\frac{1}{\alpha^2}+\frac{1}{\beta^2}+\frac{1}{\gamma^2}}}=3$$

$$\Rightarrow \frac{1}{\alpha^2}+\frac{1}{\beta^2}+\frac{1}{\gamma^2}=1$$

Thus, locus of centroid of ΔABC is

$$\frac{1}{x^2}+\frac{1}{y^2}+\frac{1}{z^2}=1$$

23. As $\frac{x-3}{1}=\frac{y+2}{-1}=\frac{z+\lambda}{-2}$ lie in the plane $2x-4y+3z=2$

$(3,-2,-\lambda)$ lies on the plane.

$\therefore\ 2(3) - 4(-2) + 3(-\lambda) = 2$

$\Rightarrow \lambda = 4$

Now $\mathbf{d}_1 = \mathbf{i}-\mathbf{j}-2\mathbf{k}$ and $\mathbf{d}_2 + 12\mathbf{i}+9\mathbf{j}-4\mathbf{k}$

$$\mathbf{d}_1\times\mathbf{d}_2 = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 1 & -1 & -2 \\ 12 & 9 & 4 \end{vmatrix}$$

$= 14\mathbf{i}-28\mathbf{j}+21\mathbf{k}$

$= 7\left(2\mathbf{i}-4\mathbf{j}+3\mathbf{k}\right)$

Also, $\begin{vmatrix} 3-1 & -2-0 & -4-0 \\ 1 & -1 & -2 \\ 12 & 9 & 4 \end{vmatrix}$

$= 14(2) - 28(-2) + 21(-4)$

$= 7(4 + 8 - 12) = 0$

$\therefore$ Shortest distance between two lines is 0.

24. We have $\mathbf{b} = \mathbf{b}_1 + \mathbf{b}_2$ where

$\mathbf{b}_1 = \alpha\mathbf{a}_1, \mathbf{b}_2\cdot\mathbf{a} = 0$

$\therefore\ \mathbf{a}\cdot\mathbf{b} = \alpha\mathbf{a}\cdot\mathbf{a}$

$\Rightarrow\ 3 = 2\alpha \Rightarrow \alpha = \frac{3}{2}$

Thus, $\mathbf{b}_1 = \frac{3}{2}\mathbf{a} \Rightarrow \vec{\mathbf{b}}_2 = \vec{\mathbf{b}} = \frac{3}{2}\vec{\mathbf{a}}$

$\Rightarrow\ \mathbf{b}_1\times\mathbf{b}_2 = \frac{3}{2}\mathbf{a}\times\left(\mathbf{b}-\frac{3}{2}\mathbf{a}\right) = \frac{3}{2}\mathbf{a}\times\mathbf{b}$

$= \frac{3}{2}(\mathbf{i}+\mathbf{j})\times(3\mathbf{j}+4\mathbf{k})$

$$= \frac{3}{2}\begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 1 & 1 & 0 \\ 0 & 3 & 4 \end{vmatrix}$$

$= \frac{3}{2}\left(4\mathbf{i}-4\mathbf{j}+3\mathbf{k}\right) = 6\mathbf{i}-6\mathbf{j}+\frac{9}{2}\mathbf{k}$.

25. Number of ways of choosing committee so that there is at least one woman is ${}^{15}C_4 - {}^{10}C_4 = 1365 - 210 = 1155$

Number of ways in which there are more women than men in the committee

$= \left({}^{10}C_0\right)\left({}^{5}C_4\right)+\left({}^{10}C_1\right)\left({}^{5}C_3\right)$

$= (1)(5) + (10)(10)$

$= 105$

$\therefore$ Probability of the required event

$= \frac{105}{1155}=\frac{1}{11}$.

26. $P(E \cap F) = \frac{1}{12} \Rightarrow P(E)P(F) = \frac{1}{12}$

and $P(E' \cap F') = \frac{1}{2} \Rightarrow P(E')P(F') = \frac{1}{2}$

$\Rightarrow (1 - P(E))(1 - P(F)) = \frac{1}{2}$

$\Rightarrow 1 - (P(E) + P(F)) + \frac{1}{12} = \frac{1}{2}$

$\Rightarrow P(E) + P(F) = \frac{7}{12}$

$\therefore$ Equation whose roots are P(E), P(F) is

$x^2 - \left(\frac{7}{12}\right)x + \frac{1}{12} = 0$

or $12x^2 - 7x + 1 = 0$

$\Rightarrow (3x - 1)(4x - 1) = 0$

$\Rightarrow x = \frac{1}{3}, \frac{1}{4}$

$\therefore \frac{P(E)}{P(F)} = \frac{\frac{1}{3}}{\frac{1}{4}}$ or $\frac{\frac{1}{4}}{\frac{1}{3}}$

$\Rightarrow \frac{P(E)}{P(F)} = \frac{4}{3}$ or $\frac{3}{4}$.

27. Correct $\sum x = 400 - 3 - 4 - 5 = 388$

Correct $\sum x^2 = 2475 - 3^2 - 4^2 - 5^2 = 2425$

$\therefore$ Correct $\sigma^2 = \frac{1}{97}(2425) - \left(\frac{1}{97}(388)\right)^2$

$= 25 - 4^2 = 9$.

28. Let $\alpha = \cot^{-1}(1 + x)$

$\Rightarrow \cot\alpha = 1 + x$

$\Rightarrow \sin\alpha = \frac{1}{\sqrt{1 + \cot^2\alpha}}$

$= \frac{1}{\sqrt{2 + 2x + x^2}}$

Next, let $\beta = \tan^{-1} x$

$\Rightarrow \tan\beta = x$

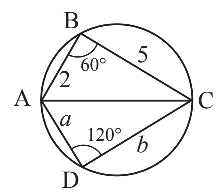
$\Rightarrow \cos\beta = \frac{1}{\sqrt{1 + \tan^2\beta}} = \frac{1}{\sqrt{1 + x^2}}$

$\therefore \sin\alpha = \cos\beta$

$\Rightarrow 2 + 2x + x^2 = 1 + x^2$

$\Rightarrow x = -\frac{1}{2}$.

29. Area of $\Delta ABC = \frac{1}{2}(2)(5)\sin 60° = \frac{5\sqrt{3}}{2}$




$\therefore$ area of $\Delta ACD = 4\sqrt{3} - \frac{5\sqrt{3}}{2} = \frac{3\sqrt{3}}{2}$

Thus, $\frac{3\sqrt{3}}{2} = \frac{1}{2}(ab)\sin 120° = \frac{1}{2}(ab)\left(\frac{\sqrt{3}}{2}\right)$

$\Rightarrow ab = 6$

Also, $AC^2 = 2^2 + 5^2 - (2)(2)(5)\cos 60°$

$= a^2 + b^2 - 2ab\cos 120°$

$\Rightarrow 19 = a^2 + b^2 - 2ab\left(-\frac{1}{2}\right) = a^2 + b^2 + 6$

$\Rightarrow a^2 + b^2 = 13$

$\Rightarrow a^2 + b^2 = 13, ab = 6$

$\Rightarrow a = 2, b = 3$

Thus, perimeter = 2 + 5 + 2 + 3 = 12.

30. Contrapositive of $p \to q$ is $\sim q \to \sim p$

$\therefore$ required contrapositive statement is

If the squares of two numbers are equal, then the numbers are equal.