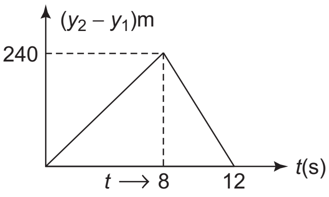
Solutions of Physics JEE Main—2015

1. Two stones are thrown up simultaneously from the edge of a cliff 240 m high with initial speed of 10 m/s and 40 m/s respectively. Which of the following graph best represents the time variation of relative position of the second stone with respect to the first?

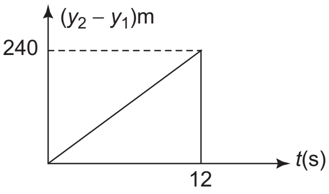
   (Assume stones do not rebound after hitting the ground and neglect air resistance, take, $g = 10$ m/s^2)

   (*The figures are schematic and not drawn to scale*)

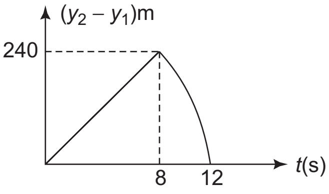
   (a)



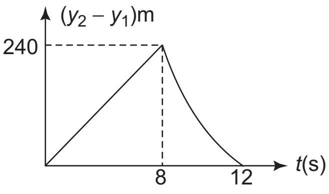
   (b)



   (c)



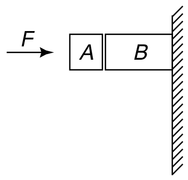
   (d)



2. The period of oscillation of a simple pendulum is $T = 2\pi\sqrt{\dfrac{L}{g}}$. Measured value of L is 20.0 cm known to 1 mm accuracy and time for 100 oscillations of the pendulum is found to be 90 s using a wrist watch of 1s resolution. The accuracy in the determination of g is:

   (a) 2% (b) 3%
   (c) 1% (d) 5%

3. Given in the figure are two blocks A and B of weight 20 N and 100 N, respectively. These are being pressed against a wall by a force F as shown. If the coefficient of friction between the blocks is 0.1 and between block B and the wall is 0.15, the frictional force applied by the wall on block B is



   (a) 100 N (b) 80 N
   (c) 120 N (d) 150 N

4. A particle of mass m moving in the x direction with speed $2v$ is hit by another particle of mass $2m$ moving in the y direction with speed v. If the collision is perfectly inelastic, the percentage loss in the energy during the collision is close to:

   (a) 44% (b) 50%
   (c) 56% (d) 62%

5. Distance of the centre of mass of a solid uniform cone from its vertex is z_0. If the radius of its base is R and its height is h then z_0 is equal to:

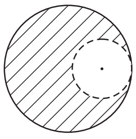
   (a) $\dfrac{h^2}{4R}$ (b) $\dfrac{3h}{4}$
   (c) $\dfrac{5h}{8}$ (d) $\dfrac{3h^2}{8R}$

6. From a solid sphere of mass M and radius R a cube of maximum possible volume is cut. Moment of inertia of cube about an axis passing through its centre and perpendicular to one of its faces is:

   (a) $\frac{MR^2}{32\sqrt{2}\pi}$ (b) $\frac{MR^2}{16\sqrt{2}\pi}$

   (c) $\frac{4MR^2}{9\sqrt{3}\pi}$ (d) $\frac{4MR^2}{3\sqrt{3}\pi}$

7. From a solid sphere of mass M and radius R, a spherical portion of radius $\frac{R}{2}$ is removed, as shown in the figure. Taking gravitational potential $V = 0$ at $r = \infty$, the potential at the centre of the cavity thus formed is:
   (G = *gravitational constant*)



   (a) $\frac{-GM}{2R}$ (b) $\frac{-GM}{R}$

   (c) $\frac{-2GM}{3R}$ (d) $\frac{-2GM}{R}$

8. A pendulum made of a uniform wire of cross sectional area A has time period T. When an additional mass M is added to its bob, the time period changes to T_M. If the Young's modulus of the material of the wire is Y then $\frac{1}{Y}$ is equal to:
   (g = *gravitational acceleration*)

   (a) $\left[\left(\frac{T_M}{T}\right)^2-1\right]\frac{A}{Mg}$ (b) $\left[\left(\frac{T_M}{T}\right)^2-1\right]\frac{Mg}{A}$

   (c) $\left[1-\left(\frac{T_M}{T}\right)^2\right]\frac{A}{Mg}$ (d) $\left[1-\left(\frac{T}{T_M}\right)^2\right]\frac{A}{Mg}$

9. Consider a spherical shell of radius R at temperature T. The black body radiation inside it can be considered as an ideal gas of photons with internal energy per unit volume $u = \frac{U}{V} \propto T^4$ and pressure $p = \frac{1}{3}\left(\frac{U}{V}\right)$. If the shell now undergoes an adiabatic expansion the relation between T and R is:

   (a) $T \propto e^{-R}$ (b) $T \propto e^{-3R}$

   (c) $T \propto \frac{1}{R}$ (d) $T \propto \frac{1}{R^3}$

10. A solid body of constant heat capacity 1 J/°C is being heated by keeping it in contact with reservoirs in two ways:

    (i) Sequentially keeping in contact with 2 reservoirs such that each reservoir supplies same amount of heat.
    (ii) Sequentially keeping in contact with 8 reservoirs such that each reservoir supplies same amount of heat.

    In both the cases body is brought from initial temperature 100°C to final temperature 200°C. Entropy change of the body in the two cases respectively is:

    (a) In 2, 4 In 2 (b) In 2, In 2
    (c) In 2, 2 In 2 (d) 2 In 2, 8 In 2

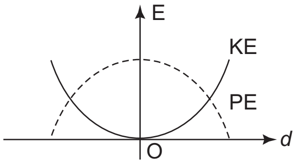
11. Consider an ideal gas confined in an isolated closed chamber. As the gas undergoes an adiabatic expansion, the average time of collision between molecules increases as V^q, where V is the volume of the gas. The value of q is:
    $\left(\gamma = \frac{C_p}{C_v}\right)$

    (a) $\frac{3\gamma+5}{6}$ (b) $\frac{3\gamma-5}{6}$

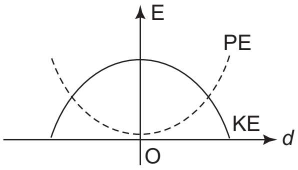
    (c) $\frac{\gamma+1}{2}$ (d) $\frac{\gamma-1}{2}$

12. For a simple pendulum, a graph is plotted between its kinetic energy (KE) and potential energy (PE) against its displacement d. Which one of the following represents these correctly?
    (graphs are schematic and not drawn to scale)

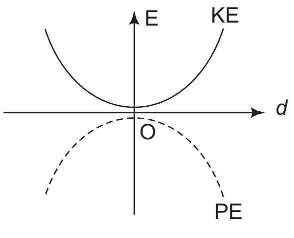
(a)



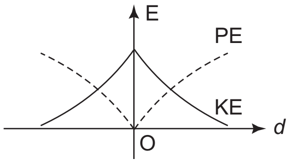
(b)



(c)



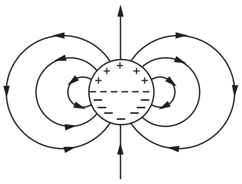
(d)



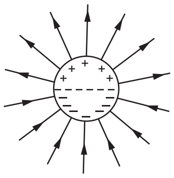
13. A train is moving on a straight track with speed 20 ms^{-1}. It is blowing its whistle at the frequency of 1000 Hz. The percentage change in the frequency heard by a person standing near the track as the train passes him is (speed of sound = 320 ms^{-1}) close to:
    (a) 6% (b) 12%
    (c) 18% (d) 24%

14. A long cylindrical shell carries positive surface charge σ in the upper half and negative surface charge $-\sigma$ in the lower half. The electric field lines around the cylinder will look like figure given in : (*figures are schematic and not drawn to scale*)

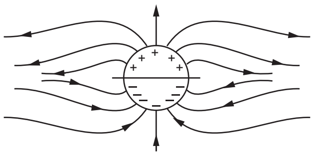
(a)



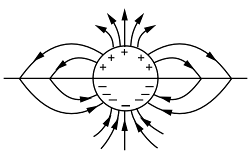
(b)



(c)

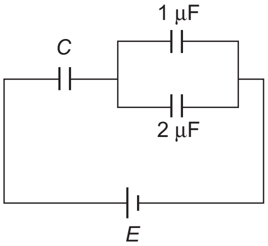


(d)

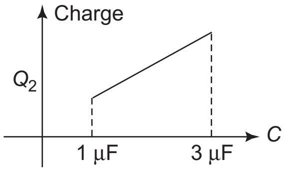


15. A uniformly charged solid sphere of radius R has potential V_0 (measured with respect to ∞) on its surface. For this sphere the equipotential surfaces with potentials $\frac{3V_0}{2}, \frac{5V_0}{4}, \frac{3V_0}{4}$ and $\frac{V_0}{4}$ have radius R_1, R_2, R_3 and R_4 respectively. Then
    (a) $R_1 = 0$ and $R_2 > (R_4 - R_3)$
    (b) $R_1 \neq 0$ and $(R_2 - R_1) > (R_4 - R_3)$
    (c) $R_1 = 0$ and $R_2 < (R_4 - R_3)$
    (d) $2R < R_4$

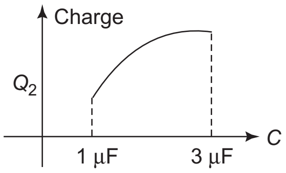
16. In the given circuit, charge Q_2 on the 2μF capacitor changes as C is varied from 1μF to 3μF. Q_2 as a function of 'C' is given properly by : (*figures are drawn schematically and are not to scale*)



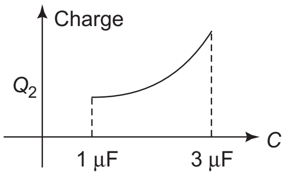
(a)



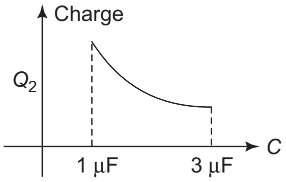
(b)



(c)

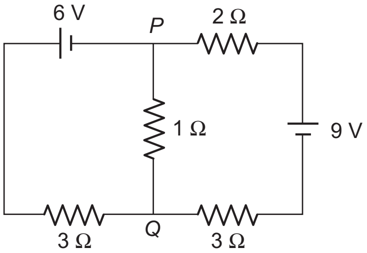


(d)



17. When 5 V potential difference is applied across a wire of length 0.1 m, the drift speed of electrons is 2.5×10^{-4} ms^{-1}. If the electron density in the wire is 8×10^{28} m^{-3}, the resistivity of the material is close to:
    (a) 1.6×10^{-8} Ωm (b) 1.6×10^{-7} Ωm
    (c) 1.6×10^{-6} Ωm (d) 1.6×10^{-5} Ωm

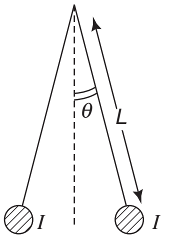
18. In the circuit shown, the current in the 1Ω resistor is:



    (a) 1.3 A, from P to Q
    (b) 0 A
    (c) 0.13 A, from Q to P
    (d) 0.13 A, from P to Q

19. Two coaxial solenoids of different radii carry current I in the same direction. Let $\vec{F}_1$ be the magnetic force on the inner solenoid due to the outer one and $\vec{F}_2$ be the magnetic force on the outer solenoid due to the inner one. Then.
    (a) $\vec{F}_1 = \vec{F}_2 = 0$
    (b) $\vec{F}_1$ is radially inwards and $\vec{F}_2$ is radially outwards
    (c) $\vec{F}_1$ is radially inwards and $\vec{F}_2 = 0$
    (d) $\vec{F}_1$ is radially outwards and $\vec{F}_2 = 0$

20. Two long current carrying thin wires, both with current I, are held by insulating threads of length L and are in equilibrium as shown in the figure, with threads making an angle 'θ' with the vertical. If wires have mass λ per unit length then the value of I is: (g = gravitational acceleration)

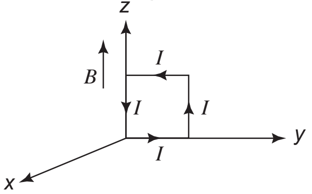


(a) $\sin\theta\sqrt{\dfrac{\pi\lambda g L}{\mu_0\cos\theta}}$ (b) $2\sin\theta\sqrt{\dfrac{\pi\lambda g L}{\mu_0\cos\theta}}$

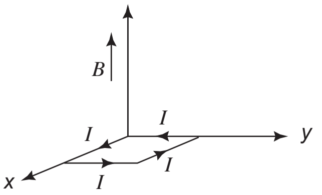
(c) $2\sqrt{\dfrac{\pi g L}{\mu_0}\tan\theta}$ (d) $2\sqrt{\dfrac{\pi\lambda g L}{\mu_0}\tan\theta}$

21. A rectangular loop of sides 10 cm and 5 cm carrying a current I of 12 A is placed in different orientations as shown in the figures below:

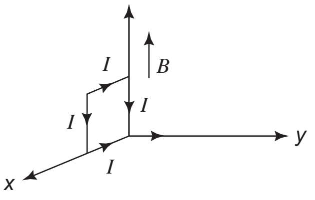
(i)



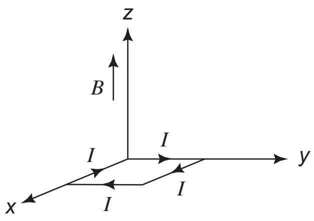
(ii)



(ii)

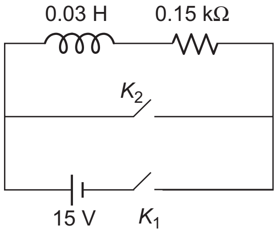


(iv)



If there is a uniform magnetic field of 0.3 T in the positive z direction, in which orientations the loop be in (i) stable equilibrium and (ii) unstable equilibrium?
    (a) (i) and (ii), respectively
    (b) (i) and (iii), respectively
    (c) (ii) and (iv), respectively
    (d) (ii) and (iii), respectively

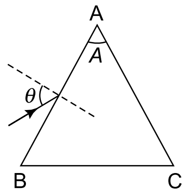
22. An inductor (L = 0.03 H) and a resistor (R = 0.15 kΩ) are connected in series to a battery of 15 V EMF in a circuit shown below. The key K_1 has been kept closed for a long time. Then at t = 0, K_1 is opened and key K_2 is closed simultaneously. At t = 1 ms, the current in the circuit will be : ($e^5 \cong 150$)



(a) 100 mA (b) 67 mA
(c) 6.7 mA (d) 0.67 mA

23. A red LED emits light at 0.1 watt uniformly around it. The amplitude of the electric field of the light at a distance of 1 m from the diode is:
    (a) 1.73 V/m (b) 2.45 V/m
    (c) 5.48 V/m (d) 7.75 V/m

24. Monochromatic light is incident on a glass prism of angle A. If the refractive index of the material of the prism is μ, a ray, incident at an angle θ, on the face AB would get transmitted through the face AC of the prism provided:



(a) $\theta > \sin^{-1}\left[\mu\sin\left(A - \sin^{-1}\left(\dfrac{1}{\mu}\right)\right)\right]$

(b) $\theta < \sin^{-1}\left[\mu\sin\left(A - \sin^{-1}\left(\dfrac{1}{\mu}\right)\right)\right]$

(c) $\theta > \cos^{-1}\left[\mu \sin\left(A + \sin^{-1}\left(\frac{1}{\mu}\right)\right)\right]$

(d) $\theta < \cos^{-1}\left[\mu \sin\left(A + \sin^{-1}\left(\frac{1}{\mu}\right)\right)\right]$

25. On a hot summer night, the refractive index of air is smallest near the ground and increases with height form the ground. When a light beam is directed horizontally, the Huygens' principle leads us to conclude that as it travels, the light beam:
    (a) becomes narrower
    (b) goes horizontally without any deflection
    (c) bends downwards
    (d) bends upwards

26. Assuming human pupil to have a radius of 0.25 cm and a comfortable viewing distance of 25 cm, the minimum separation between two objects that human eye can resolve at 500 nm wavelength is:
    (a) 1 μm (b) 20 μm
    (c) 100 μm (d) 300 μm

27. As an electron makes a transition from an excited state to the ground state of a hydrogen-like atom/ion:
    (a) its kinetic energy increases but potential energy and total energy decrease
    (b) kinetic energy, potential energy and total energy decrease
    (c) kinetic energy decreases, potential energy increases but total energy remains same.
    (d) kinetic energy and total energy decrease but potential energy increases

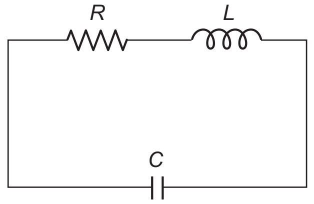
28. Match List-I (Fundamental Experiment) with List-II (its conclusion) and select the correct option from the choice given below the list

	List-I		List-II
(A)	Franck-Hertz Experiment.	(i)	Particle nature of light
(B)	Photo-electric experiment.	(ii)	Discrete energy levels of atom
(C)	Davison-Germer Experiment.	(iii)	Wave nature of electron
		(iv)	Structure of atom

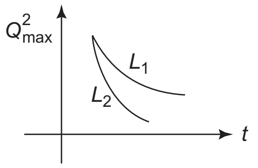
(a) (A) – (i) (B) – (iv) (C) – (iii)
(b) (A) – (ii) (B) – (iv) (C) – (iii)
(c) (A) – (ii) (B) – (i) (C) – (iii)
(d) (A) – (iv) (B) – (iii) (C) – (ii)

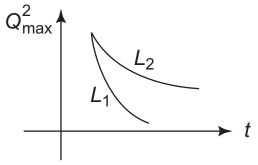
29. A signal of 5 kHz frequency is amplitude modulated on a carrier wave of frequency 2 MHz. The frequencies of the resultant signal is/are:
    (a) 2 MHz only
    (b) 2005 kHz, and 1995 kHz
    (c) 2005 kHz, 2000 kHz and 1995 kHz
    (d) 2000 kHz and 1995 kHz

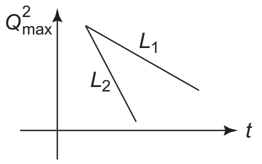
30. An *LCR* circuit is equivalent to a damped pendulum. In an *LCR* circuit the capacitor is charged to Q_0 and then connected to the *L* and *R* as shown below:

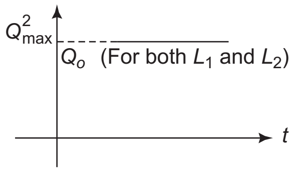


If a student plots graphs of the square of maximum charge (Q^2_{Max}) on the capacitor with time (t) for two different values L_1 and L_2 ($L_1 > L_2$) of L then which of the following represents this graph correctly? (*plots are schematic and not drawn to scale*)

(a)


(b)


(c)


(d)


Answers

1. (c) 2. (b) 3. (c) 4. (c)
5. (b) 6. (3) 7. (b) 8. (a)
9. (c) 10. None 11. (c) 12. (b)
13. (b) 14. (a) 15. (c), (d) 16. (b)
17. (d) 18. (c) 19. (a) 20. (a)
21. (c) 22. (d) 23. (b) 24. (a)
25. (d) 26. (b) 27. (a) 28. (c)
29. (c) 30. (a)

Solutions

1. Let y_1 and y_2 be the positions of the stones at time t then.

$$y_1 = 10t - \frac{1}{2}gt^2 \qquad (1)$$

and
$$y_2 = 40t - \frac{1}{2}gt^2 \qquad (2)$$

where $g = 10\ \text{ms}^{-2}$. The relative position of the second stone with respect to the first is

$$y_2 - y_1 = 30t \qquad (3)$$

If t_1 and t_2 are times taken by the first stone and by the second stone to hit the ground, then $y_1 = y_2 = -240$ m and Eqs. (1) and (2) become

$$-240 = 10t_1 - 5t_1^2 \qquad (4)$$

and
$$-240 = 40t_2 - 5t_2^2 \qquad (5)$$

The positive roots of Eqs. (4) and (5) are $t_1 = 8$ s and $t_2 = 12$ s.

Thus the first stone reaehes the ground earlier. Since the stones do not rebound, at times $t \geq 8$ s, $y_1 = 0$ and y_2 is given by Eq. (2) which is a parabola. But up to $t = 8$s, it follows from Eq. (3) that $(y_2 - y_1)$ is proportional to t. Hence up to $t = 8$ s, graph is $(y_2 - y_1)$ is a straight line, and in the interval $t = 8$ s and $t = 12$ s, the graph is parabolic. So the correct graph is (c).

2. $T = 2\pi\sqrt{\dfrac{L}{g}} \Rightarrow g = \dfrac{4\pi^2 L}{T^2}$

If n is the number of oscillations completed in t seconds, then $T = \dfrac{t}{n}$. Thus, in terms of measured quantities,

$$g = \frac{4\pi^2 L n^2}{t^2}$$

$$\frac{\Delta g}{g} = \frac{\Delta L}{L} + 2\frac{\Delta t}{t} \quad (\because n = 100 \text{ is exact})$$

$$= \frac{0.1\ \text{cm}}{20\ \text{cm}} + 2 \times \frac{1\ \text{s}}{90\ \text{s}}$$

$$= \frac{1}{200} + \frac{1}{45}$$

$$\therefore \quad 100 \times \frac{\Delta g}{g} = 100 \times \left(\frac{1}{200} + \frac{1}{45}\right)$$

$$= 0.5 + 2.2 = 2.7$$

The closest choice is (b)

3. Since the blocks are held stationary, they are in translational as well as rotational equilibrium. Hence no net force and no net torque acts on the blocks. Hence normal reaction on block A due to block $B = F$. Also normal reaction on B by the wall will also be equal to F. Also frictional force $f = mg$. Hence frictional force on A due to $B = m_A g = 20$ N. Therefore, frictional force exerted by the wall on block B is

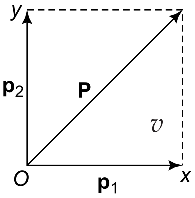
$$f = m_A g + m_B g$$
$$= 20\ \text{N} + 100\ \text{N} = 120\ \text{N}$$

So the correct choice is (c).

4. Linear momentum of the first particle is

$\mathbf{p}_1 = m(2v)$ along the x-axis.

$\mathbf{p}_2 = 2m(v)$ along the y-axis.



In a perfectly inelastic collisions, the two particle stick together. The mass of the composite particle is $M = m + 2m = 3m$. If $\mathbf{V}$ is the velocity of the composite particle, its momentum after the collision is

$$\mathbf{P} = M\mathbf{V} = (3m)\mathbf{V}$$

From the figure, it follows that

$$P = \sqrt{p_1^2 + p_2^2}$$

$$\Rightarrow \quad (3m)V = \sqrt{(2mv)^2 + (2mv)^2}$$

$$\Rightarrow \quad 3mV = \sqrt{8}\ mv$$

$$\Rightarrow \quad V = \frac{\sqrt{8}\,v}{3}$$

Total initial kinetic energy is

$$K_i = \frac{1}{2} \times m(2v)^2 + \frac{1}{2} \times (2m)v^2 = 3mv^2$$

Total final kinetic energy is

$$K_f = \frac{1}{2}MV^2 = \frac{1}{2} \times (3m) \times \left(\frac{\sqrt{8}}{3}v\right)^2$$

$$= \frac{4mv^2}{3}$$

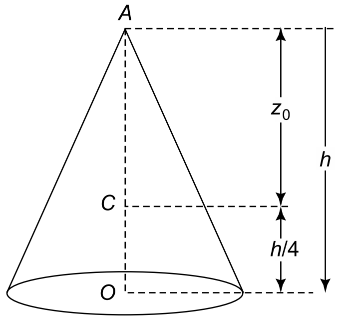
Percentage loss is

$$\frac{K_i - K_f}{K_i} \times 100 = \frac{3mv^2 - \dfrac{4mv^2}{3}}{3mv^2} \times 100$$

$$= \frac{5}{9} \times 100 = 55.56\% \simeq 56\%$$

So the correct choice is (c).

5. The centre of mass C of cone of height is at a distance



$$AC = AO - CO$$

or $$z_0 = h - \frac{h}{4} = \frac{3h}{4}$$

So the correct choice is (b).

6. If a is the side of the cube of maximum possible volume, then

$$2R = \sqrt{a^2 + a^2 + a^2} = \sqrt{3}\, a$$

$$\Rightarrow \quad a = \frac{2R}{\sqrt{3}}$$

Density of sphere is $\rho = \dfrac{M}{4\dfrac{\pi}{3}R^3}$

Volume of cube $= a^3 = \left(\dfrac{2R}{\sqrt{3}}\right)^3 = \dfrac{8R^3}{3\sqrt{3}}$

$\therefore$ Mass of cube is $m = \rho a^3 = \dfrac{3M}{4\pi R^3} \times \dfrac{8R^3}{3\sqrt{3}} = \dfrac{2M}{\pi\sqrt{3}}$

The moment of inertia of the cube about the axis passing through its centre and perpendicular to one of its faces is

$$I = \frac{ma^2}{6} = \frac{2M}{\pi\sqrt{3}} \times \frac{a^2}{6} = \frac{2M}{\pi\sqrt{3}} \times \frac{1}{6} \times \left(\frac{2R}{\sqrt{3}}\right)^2$$

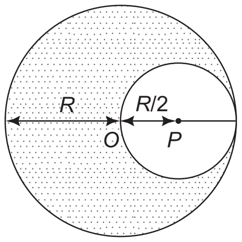
$$= \frac{4MR^2}{9\pi\sqrt{3}}$$

So the correct choice is (c).

7. Volume of solid sphere is $\rho = \dfrac{M}{\dfrac{4\pi}{3}R^3}$

Volume of spherical portion removed $= \dfrac{4\pi}{3}\left(\dfrac{R}{2}\right)^3$
$= \dfrac{4\pi}{3}\dfrac{R^3}{8}$

$\therefore$ Mass of portion removed $(m) = \dfrac{3M}{4\pi R^3} \times \dfrac{4\pi}{3}\dfrac{R^3}{8}$
$= \dfrac{M}{8}$



Gravitational potential due to a solid sphere of mass M and radius R at a point inside it at a distance r from its centre is given by

$$V = -\frac{3\,GM}{R^3}\left(\frac{R^2}{2} - \frac{r^2}{6}\right)$$

For point P, $r = \dfrac{R}{2}$, we have

$$V = -\frac{3\,GM}{R^3}\left(\frac{R^2}{2} - \frac{(R/2)^2}{6}\right) = -\frac{11\,GM}{8R}$$

Gravitational potential at P due to spherical cavity of mass $m = \dfrac{M}{8}$ at point P is

$$V' = -\frac{3GM/8}{2R/2} = -\frac{3GM}{8R}$$

Therefore gravitational potential at P due to the sphere with cavity (i.e. due to shaded portion is)

$$V'' = V - V'$$

$$= -\frac{11\,GM}{8R} - \left(-\frac{3\,GM}{8R}\right) = -\frac{GM}{R}$$

So the correct choice is (b).

8. $T = 2\pi\sqrt{\dfrac{L}{g}}$; L = length of pendulum string. Let l be the increase in the length of the string when an additional mass M is added to its bob. Then

$$T_M = 2\pi\sqrt{\frac{L+l}{g}}$$

Now $$Y = \frac{Mg\,L}{Al}$$

$$\Rightarrow \quad l = \frac{Mg\,L}{AY}$$

$$\therefore \quad T_M = 2\pi\sqrt{\frac{L + \dfrac{Mg\,L}{AY}}{g}}$$

$$\Rightarrow \quad T_M{}^2 = 4\pi^2\left(\frac{L}{g} + \frac{ML}{AY}\right) \qquad (1)$$

Now $$T^2 = 4\pi^2\,\frac{L}{g} \qquad (2)$$

Dividing (1) by (2)

$$\frac{T_M^2}{T^2} = 1 + \frac{Mg}{AY}$$

$$\Rightarrow \frac{1}{Y} = \frac{A}{Mg}\left[\left(\frac{T_M}{T}\right)^2 - 1\right]\text{, which is choice (a).}$$

9. From the first law of thermodynamics,

$$dQ = dU + dW$$

For an adiabatic process, $dQ = 0$. Hence

$$0 = dU + dW$$

$$\Rightarrow \quad dU = -\,dW = -\,P\,dV \qquad (\because\ dW = \rho\,dV)$$

Given $P = -\,\dfrac{1}{3}\dfrac{U}{V}$. Therefore,

$$dU = -\,\frac{1}{3}\frac{U}{V}\,dV \qquad \left(\because\ P = \frac{1}{3}\frac{U}{V}\right)$$

$$\Rightarrow \quad \frac{dU}{U} = -\frac{1}{3}\frac{dV}{V}$$

Integrating

$$\ln U = -\,\frac{1}{3}\ln V + C; \qquad (C = \text{integration constant})$$

$$\Rightarrow \ln\,(UV^{1/3}) = C \qquad (1)$$

Now it is given that $\dfrac{U}{V} \propto T^4 \Rightarrow U = kVT^4$, where k is a proportionality constant. Using this in (1), we get

$$\ln\,(kT^4\,V^{4/3}) = C$$

$$\Rightarrow \quad \ln\left[kT^4 \times \left(\frac{4\pi}{3}R^3\right)^{4/3}\right] = C$$

$$\Rightarrow \quad \ln\,[kT^4\,R^4] = C$$

$\Rightarrow T^4R^4 = \text{constant} \Rightarrow T \propto \dfrac{1}{R}$. So the correct choice is (c).

10. In cases (i) and (ii), the temperature of the body is increased from 100°C to 200°C. In case (i) there are 2 reservoirs. Since each reservoir supplies the same amount of heat, each reservoir raises the temperature of the body by 100°C/2 = 50°C = 50 K. Therefore, the change in entropy in case (i) is

$$(dS)_1 = \left[\int_{373}^{423}\frac{dT}{T} + \int_{423}^{473}\frac{dT}{T}\right]$$

$$= \ln\,(423) - \ln\,(373) + \ln\,(473) - \ln\,(423)$$

$$= \ln\,(473) - \ln\,(373)$$

$$= \ln\left(\frac{473}{373}\right)$$

In case (ii) there are 8 reservoirs. Therefore, each reservoir raises the temperature of the body by 100°C/8 = 12.5 °C = 12.5 K. Therefore, the change in entropy in case (ii) is

$$(dS)_2 = \left[\int_{373}^{385.5}\frac{dT}{T} + \int_{385.5}^{398}\frac{dT}{T} + \int_{398}^{410.5}\frac{dT}{T} + \int_{410.5}^{423}\frac{dT}{T} + \int_{423}^{435.5}\frac{dT}{T} + \int_{435.5}^{448}\frac{dT}{T} + \int_{448}^{460.5}\frac{dT}{T} + \int_{460.5}^{473}\frac{dT}{T}\right]$$

$$= \ln\left(\frac{473}{373}\right)$$

So none of the choices given in the question is correct. However, if temperatures of the body were given in kelvin, the correct choice would be (b).

11. The mean free path l is defined as the average distance covered by a molecule between two successive collisions and is given by

$$l = \frac{1}{\sqrt{2}\,n\,\pi\,d^2}$$

where n = number of molecules per unit volume = $\dfrac{N}{V}$ and d is the diameter of a molecule. The average time between successive collisions is

$$t = \frac{\text{mean free path}}{\text{thermal speed}} = \frac{l}{v}$$

where $\quad v = \sqrt{\dfrac{3RT}{M}}$; M = molecule mass.

$$\therefore \quad t = \frac{1}{\sqrt{2}\,\dfrac{N}{V}\,\pi d^2} \times \sqrt{\frac{M}{3\,RT}}$$

or $\quad t = \dfrac{KV}{\sqrt{T}}$, where K is a constant given by

$$K = \frac{\sqrt{M}}{\sqrt{2}\,N\,\pi\,d^2\,\sqrt{3R}}$$

$$\therefore \quad T = \frac{K^2\,V^2}{t^2} \text{ or } T \propto \frac{V^2}{t^2}$$

For adiabatic process $TV^{\gamma-1}$ = constant

$$\therefore \quad \frac{V^2}{t^2}\,V^{\gamma-1} = \text{constant}$$

$$\Rightarrow \quad \frac{V^{\gamma+1}}{t^2} = \text{constant}$$

$$\Rightarrow \quad t \propto V^{\frac{\gamma+1}{2}} \qquad (1)$$

Given $\quad t \propto V^q \quad (2)$

Comparing (1) and (2), we get $q = \frac{\gamma + 1}{2}$. So the correct choice is (c).

12. At the mean position O i.e. at $d = 0$, KE is maximum and PE is zero. Also PE $\propto d^2$. Hence the correct choice is (b).

13. When the train is approaching, the apparent frequency is

$$\nu_1 = \nu\left(\frac{v}{v - u}\right)$$

When the train is receding,

$$\nu_2 = \nu\left(\frac{v}{v + u}\right)$$

Change in frequency is

$$\Delta\nu = \nu_1 - \nu_2$$

$$= \nu\, v\left(\frac{1}{v - u} - \frac{1}{v + u}\right)$$

$$= \frac{\nu\, v \times 2u}{v^2 - u^2}$$

$$= \frac{2\nu\, u\, v}{v^2\left(1 - \frac{u^2}{v^2}\right)} = \frac{2\nu u}{v}$$

$(\because u << v)$

$$\Rightarrow \quad \frac{\Delta\nu}{\nu} = \frac{2u}{v}$$

$\therefore$ Percentage change is $\frac{\Delta\nu}{\nu} \times 100 = \frac{2u}{v} \times 100$

$$= \frac{2 \times 20}{320} \times 100$$

$$= 12.5\%$$

So the closest choice is (b).

14. Electric field lines originate from a positive charge and terminate on a negative charge. They do not form closed loops. So the correct choice is (a).

15. Electric potential due to a solid sphere of radius R and carrying a charge Q at a point P inside it at a distance r from its centre C is given by

$$V = \frac{3kQ}{2R^3}\left(R^2 - \frac{r^2}{3}\right) \quad (1)$$

where $\quad k = \frac{1}{4\pi\,\varepsilon_0}$

(i) At the centre C, $r = 0$ and Eq. (1) gives.

$$V_c = \frac{3kQ}{2R}$$

At the susface, $r = R$ and Eq. (1) givens

$$V_0 = \frac{3kQ}{2R^3}\left(R^2 - \frac{R^2}{3}\right) = \frac{kQ}{R}$$

Therefore, $V_C = \frac{3}{2}V_0$ at the centre. The equipotential surface on which the potential is $\frac{3V_0}{2}$ has $r = 0$ which gives $R_1 = 0$

(ii) For points outside the sphere (i.e. for $r > R$), the entire charge of the sphere may be assumed to be concentrated at its centre. Hence for $r > R$,

$$V = \frac{kQ}{r} \quad (2)$$

It is given that at $r = R_2$, potential $= \frac{5V_0}{4}$.

Putting $V = \frac{5V_0}{4} = \frac{5kQ}{4R}$ and $r = R_2$ is Eq. (1) we get

$$R_2 = \frac{R}{\sqrt{2}}$$

It is given at $r = R_3$, potential $= \frac{3V_0}{4}$. Hence from Eq. (2), we have

$$\frac{kQ}{R_3} = \frac{3V_0}{4} = \frac{3}{4}\frac{kQ}{R}$$

$$\Rightarrow \quad R_3 = \frac{4R}{3}$$

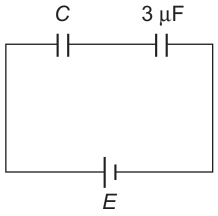
Similarly at $r = R_4$,

$$\frac{kQ}{R_4} = \frac{V_0}{4} = \frac{kQ}{4R}$$

$$\Rightarrow \quad R_4 = 4R$$

We have seen that $R_1 = 0$. $R_2 = \frac{R}{\sqrt{2}}$, $R_3 = \frac{4R}{3}$ and $R_4 = 4R$. Thus choices (c) and (d) are both correct.

16. The circuit can be redrawn as follows.



Here C is in μF. Let E_1 and E_2 be the voltages across C and 3μ F capacitor respectively. Then

$$CE_1 = 3E_2$$

$$\Rightarrow \quad \frac{E_2}{E_1} = \frac{C}{3} \Rightarrow E_1 = \frac{3E_2}{C}$$

Also $\quad E_1 + E_2 = E$

$$\Rightarrow \quad \frac{3E_2}{C} + E_2 = E \Rightarrow E_2 = \frac{EC}{3+C}$$

$\therefore$ Charge on 2μ F capacitor is (in μ F)

$$Q_2 = 2E_2 = \frac{2EC}{3+C} \quad (1)$$

It follows from Eq. (1) that Q_2 increases non-linearly as C increases and the slope of ($Q_2 - C$) graph decreases as C increases. Hence the correct choice is (b).

17. $I = ne\,A\,v_d$

$$\Rightarrow \quad \frac{V}{R} = ne\,A\,v_d$$

$$\Rightarrow \quad \frac{V}{\frac{\rho l}{A}} = ne\,A\,v_d$$

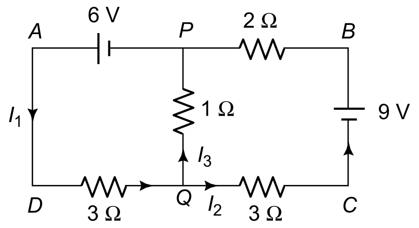
$$\Rightarrow \quad \rho = \frac{V}{l\,e\,n\,v_d}$$

$$= \frac{5}{0.1 \times (1.6 \times 10^{-19}) \times (8 \times 10^{28}) \times (2.5 \times 10^{-4})}$$

$$= 1.56 \times 10^{-5} \simeq 1.6 \times 10^{-5}\ \Omega\text{m}.$$

So the correct choice is (d).

18. Let I_1, I_2 and I_3 be the direction of currents is shown in the circuit.



From kirchhoff's junction rule

$$I_1 = I_2 + I_3 \quad (1)$$

Applying kirchhoff's loop rule to loops *APQDA* and *PBCQP*, we have

$$6 - I_3 - 3I_1 = 0 \quad (2)$$

$$\text{and } -2I_2 + 9 - 3I_2 + I_3 = 0 \quad (3)$$

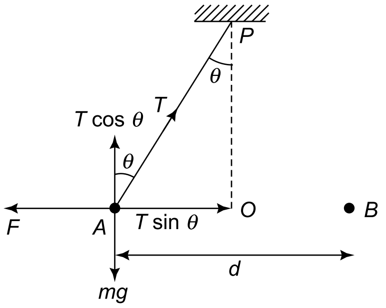
Simultaneous solution of Eqs. (1), (2) and (3) gives $I_1 = \frac{45}{23}$ A, $I_2 = \frac{42}{23}$ A and $I_3 = \frac{3}{23}$ A. The current flowing the 1Ω sesistor is $I_3 = \frac{3}{23} = 0.13$A Since I_3 is positive, it flows from Q to P. So the correct choice is (c).

19. Since the solenoids are in equilibrium, no net force is exerted by one solenoid on the other. Hence $\vec{F}_1 = 0$ and $\vec{F}_2 = 0$. So the correct choice is (a).

20. If the two wires have currents in the opposite directions, they will repel each other with a force.

$$F = \frac{\mu_0 I^2 l}{2\pi d} \quad (1)$$

where l = length each wire and d = distance between them. They will then be in equilibrium.



For equilibruim,

$$F = T\sin\theta$$

$$\text{and} \quad mg = T\cos\theta$$

$$\therefore \quad \tan\theta = \frac{F}{mg} \quad (2)$$

$$\text{Also} \quad m = \lambda l \quad (3)$$

Using (1) and (3) in (2) we get

$$\tan\theta = \frac{\frac{\mu_0 I^2 l}{2\pi d}}{\lambda\, l\, g} \quad (4)$$

In Δ *OPA*, $\sin\theta = \frac{d/2}{L} \Rightarrow d = 2L\sin\theta$. Using this in (4) and simplifying, we get

$$I = 2\sin\theta\sqrt{\frac{\pi\lambda Lg}{\mu_0\cos\theta}}$$

So the correct choice is (a).

21. In a uniform magnetic field, a loop is in stable equilibrium if (i) the torque experienced by it is zero and (ii) its potential energy is minimum. For unstable equilibrium, the torque must be zero and potential energy must be maximum

Torque is $\vec{\tau} = I\,(\vec{A} \times \vec{B})$

Potential energy is $U = -I\,(\vec{A} \cdot \vec{B})$

where $\vec{A}$ is the area vector. $\vec{B} = B\,\hat{\mathbf{k}}$. For orientation (a), $\vec{A} = A\,\vec{\mathbf{i}}$. Hence $\vec{\tau} = IAB\,(\vec{\mathbf{k}} \times \vec{\mathbf{i}}) = IAB\,\vec{\mathbf{j}}$

For orientation (c), $\vec{A} = -A\,\vec{\mathbf{i}}$ and $\vec{\tau} = -IAB\,\vec{\mathbf{j}}$, Hence choices (a) and (c) are not correct.

For orientation (b), $\vec{A} = A\hat{\mathbf{k}}$ and $\vec{\tau} = IAB\,(\hat{\mathbf{k}} \times \hat{\mathbf{k}})$ $= 0$ and $U = -\,IAB\,(\hat{\mathbf{k}} \cdot \hat{\mathbf{k}}) = -\,IAB$, which sis minimum. For orientation (d), $\vec{A} = -\,A\hat{\mathbf{k}}$ and $\vec{\tau} = -IAB\,(\hat{\mathbf{k}} \times \hat{\mathbf{k}}) = 0$ and $U = -\,IAB\,(-\,\hat{\mathbf{k}} \cdot \hat{\mathbf{k}}) =$ $+\,IAB$ which is masimum. Hence prientateon (b) corresponds to stable equilibrium and orientation (d) corresponds to unstable equilibrium. So the correct choice is (c).

22. Given $E = 15\text{V}$, $R = 0.15 \times 10^3\ \Omega$ and $L = 0.03$ H. If key K_1 is closed for a long time and K_2 is open, the current acquires a steady value I_0 given by

$$I_0 = \frac{E}{R} = \frac{15}{0.15 \times 10^3} = 0.1\ \text{A}$$

When K_2 is closed, the current decays with time t as

$$I = I_0\, e^{-t/\tau}$$

where $$\tau = \frac{L}{R} = \frac{0.03}{0.15 \times 10^3} = \frac{1}{5 \times 10^3}$$

At $$t = 1\ \text{ms} = 10^{-3}\ \text{s},$$

$$I = 0.1 \times e^{-5} \qquad \left(\because \tau = \frac{10^{-3}}{5}\right)$$

$$= \frac{0.1}{e^5} = \frac{0.1}{150} = 0.67 \times 10^{-3}\ s$$

$\Rightarrow$ $I = 0.67$ mA, which is choice (d)

23. The intensity of light emitted by LED is given by

$$I = \frac{1}{2}\ \varepsilon_0\, E_0^{\,2}\, c$$

where E_0 = amplitude of electric field and c = speed of light

If P is power, the intensity at a distance r is given by

$$I = \frac{P}{4\pi\, r^2}$$

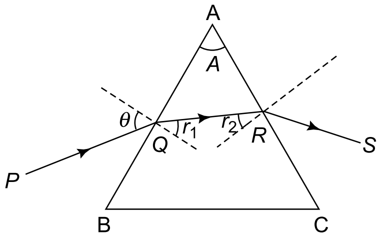
$$\frac{P}{4\pi\, r^2} = \frac{1}{2}\ \varepsilon_0\, E_0^{\,2}\, c$$

$$\Rightarrow \qquad E_0 = \sqrt{\frac{2P}{4\pi\, r^2\, \varepsilon_0\, c}} \qquad (1)$$

Given $P = 0.1$ W and $r = 1$ m.

Also $\dfrac{1}{4\pi\,\varepsilon_0} = 9 \times 10^9$ SI units and $c = 3 \times 10^8\ \text{ms}^{-1}$.

Substituting thes values in Eq. (1) we get $E_0 = 2.45$ Vm^{-1} which is choice (b)

24. 

Applying Snell's law at Q,

$$\sin\theta = \mu \sin r_1 \qquad (1)$$

Ray QR will emerge from face AC if $r_2 < i_c$ where i_c is the critical angle which is given by

$$\sin i_c = \frac{1}{\mu} \Rightarrow i_c = \sin^{-1}\left(\frac{1}{\mu}\right)$$

$$\therefore \qquad r_2 < \sin^{-1}\left(\frac{1}{\mu}\right)$$

But $r_1 + r_2 = A \Rightarrow r_2 = A - r_1$

$$\therefore\ A - r_1 < \sin^{-1}\left(\frac{1}{\mu}\right)$$

$$\Rightarrow r_1 > A - \sin^{-1}\left(\frac{1}{\mu}\right)$$

$$\therefore\ \sin r_1 > \sin\left[A - \sin^{-1}\left(\frac{1}{\mu}\right)\right]$$

Using (1) we have

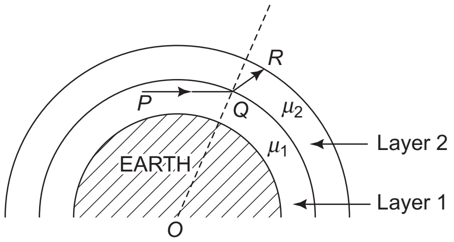
$$\frac{\sin\theta}{\mu} > \sin\left[A - \sin^{-1}\left(\frac{1}{\mu}\right)\right]$$

$$\Rightarrow \sin\theta > \mu \sin\left[A - \sin^{-1}\left(\frac{1}{\mu}\right)\right]$$

$$\Rightarrow \theta > \sin^{-1}\left[\mu \sin\,\{A - \sin^{-1}\left(\frac{1}{\mu}\right)\right]$$

So the correct choice is (a).

25. Huygens' principle can be used to prove that, if a ray of light travels from a rarer to a denser medium, it bends towards the normal at the point of incidence.



The refractive index μ_1 of the layer 1 closer to the earth is less than μ_2 of the layer 2. A horizontal ray PQ travelling in layer 1 bends along QR (towards the normal at Q). The ray bends upwards. Hence the correct choice is (d).

26. Limiting angular resolving power of human eye is given by

$$\theta_{\min} = \frac{1.22\lambda}{d}$$

where d is the diameter of the pupil of the eye. The minimum linear separation between two objects a distance D away which can be just resolved by the eye is

$$x_{\min} = D\theta_{\min}$$

$$= \frac{1.22\lambda D}{d}$$

$$= \frac{1.22 \times (500 \times 10^{-9}) \times (25 \times 10^{-2})}{2 \times 0.25 \times 10^{-2}}$$

$$= 30.5 \times 10^{-6} \text{ m} = 30.5\ \mu\text{m}$$

The closest choice is (b).

27. According to Bohr's theory, the total energy of the electron in the n state of a hydrogen like atom is

$$E = (-\ 13.6 \text{ eV})\ \frac{Z^2}{n^2}$$

Also $E = -$ K.E. and P.E. $= 2E$

As the electron falls from the higher energy state to the ground state, n decreases. Hence the total energy E decrease (because it becomes more negative). Since $E = -$ K.E., kinetic energy increases. Since P.E. $= 2E$, potential energy decreases. Hence the correct choice is (a).

28. Franck-Hertz experiment confirms that an atom has discrete energy levels. Photoelectric experiment confirms the particle nature of light. Davison-Germer experiment confirms wave nature of electron. Hence the correct choice is (c).

29. When an audio signal of frequency ν_m is superposed on a carrier wave of frequency ν_c, the resultant amplitude modulated wave contains components of frequency ν_c and upper and lower side band frequencies

$$\nu_{\text{USB}} = \nu_c + \nu_m$$

and $$\nu_{\text{LSB}} = \nu_c - \nu_m$$

Given $\nu_m = 5$ k Hz and $\nu_c = 2$MHz $= 2000$ kHz. Thus

$$\nu_{\text{USB}} = 2000 + 5 = 2005 \text{ kHz}$$

and $$\nu_{\text{LSB}} = 2000 - 5 = 1995 \text{ kHz}$$

Hence the frequencies of the modulated wave components are 2000 kHz, 1995 kHz 2005 kHz. So the correct choice is (c).

30. If the circuit did not contain resistor R, we have an $L - C$ circuit in which the charge on capacitor plates oscillates simple harmonically with angular frequency

$$\omega = \frac{1}{\sqrt{LC}}$$

At an instant of time t the charge in given by

$$Q = Q_0 \cos(\omega t)$$

The presence of resistance R is the circuit damps these oscillatons. For a damped oscillator, we have

$$Q = Q_0\, e^{-kt} \cos(\omega t)$$

where k is the damping coefficient which depends on the value of R. Therefore,

$$Q^2 = Q_0^2\, e^{-2kt} \cos^2(\omega t)$$

$$\Rightarrow \qquad Q^2 = \text{Q}^2{}_0 - e^{-2kt} \cos^2\left(\frac{\omega}{\sqrt{LC}}\right) \qquad (1)$$

Form Eq. (1) it follows that Q^2 decays exponentially with t. Also, since $L_1 > L_2$, $\cos^2\left(\frac{\omega}{\sqrt{L_1 C}}\right)$ will be greater than $\cos^2\left(\frac{\omega}{\sqrt{L_2 C}}\right)$ at a given time t. Hence the correct choice is (a).