Solutions of Physics JEE Main—2016

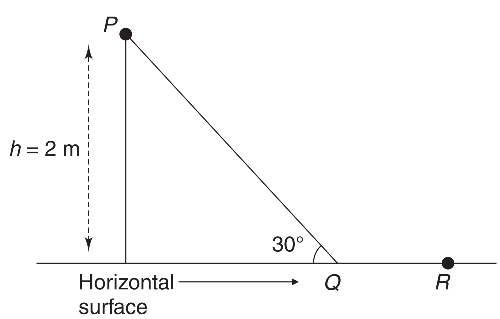
1. A uniform string of length 20 m is suspended from a rigid support. A short wave pulse is introduced at its lowest end. It starts moving up the string. The time taken to reach the support is: (Take $g = 10\ \text{ms}^{-2}$)

   (a) 2 s (b) $2\sqrt{2}$ s
   (c) $\sqrt{2}$ s (d) $2\pi\sqrt{2}$ s

2. A person trying to lose weight by burning fat lifts a mass of 10 kg upto a height of 1 m 1000 times. Assume that the potential energy lost each time he lowers the mass is dissipated. How much fat will he use up considering the work done only when the weight is lifted up? Fat supplies 3.8×10^7 J of energy per kg which is converted to mechanical energy with a 20% efficiency rate. Take $g = 9.8\ \text{ms}^{-2}$

   (a) 6.45×10^{-3} kg (b) 9.89×10^{-3} kg
   (c) 12.89×10^{-3} kg (d) 2.45×10^{-3} kg

3. A point particle of mass m, moves along the uniformly rough track PQR as shown in the figure. The coefficient of friction, between the particle and the rough track equals μ. The particle is released, from rest, from the point P and it comes to rest at a point R. The energies, lost by the ball, over the parts, PQ and QR, of the track, are equal to each other, and no energy is lost when particle changes direction from PQ and QR. The values of the coefficient of friction μ and the distance $x\ (= QR)$, are, respectively close to:



   (a) 0.2 and 3.5 m (b) 0.29 and 3.5 m
   (c) 0.29 and 6.5 m (d) 0.2 and 6.5 m

4. Two identical wires A and B, each of length 'l' carry the same current I. Wire A is bent into a circle of radius R and wire B is bent to form a square of side 'a'. If B_A and B_B are the values of magnetic field at the centres of the circle and square respectively, then the ratio $\frac{B_A}{B_B}$ is:

   (a) $\frac{\pi^2}{16\sqrt{2}}$ (b) $\frac{\pi^2}{16}$
   (c) $\frac{\pi^2}{8\sqrt{2}}$ (d) $\frac{\pi^2}{8}$

5. A galvanometer having a coil resistance of 100 Ω gives a full scale deflection, when a current of 1 mA is passed through it. The value of the resistance, which can convert this galvanometer into ammeter giving a full scale deflection for a current of 10 A, is:

   (a) 2 Ω (b) 0.1 Ω
   (c) 3 Ω (d) 0.01 Ω

6. An observer looks at a distant tree of height 10 m with a telescope of magnifying power of 20. To the observer the tree appears:

   (a) 10 times nearer (b) 20 times taller
   (c) 20 times nearer (d) 10 times taller

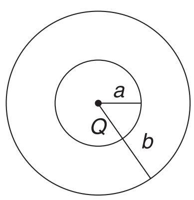
7. The temperature dependence of resistances of Cu and undoped Si in the temperature range 300-400K, is best described by:

   (a) Linear increase for Cu, exponential increase for Si
   (b) Linear increase for Cu, exponential decrease for Si

(a) A for electromagnets and B for electric generators.
(b) A for transformers and B for electric generators.
(c) B for electromagnets and transformers.
(d) A for electric generators and transformers.

17. A pendulum clock loses 12 s a day if the temperature is 40°C and gains 4 s a day if the temperature is 20°C. The temperature at which the clock will show correct time, and the co-efficient of linear expansion (α) of the metal of the pendulum shaft are respectively:
(a) 60°C; $\alpha = 1.85 \times 10^{-4}$/°C
(b) 30°C; $\alpha = 1.85 \times 10^{-3}$/°C
(c) 55°C; $\alpha = 1.85 \times 10^{-2}$/°C
(d) 25°C; $\alpha = 1.85 \times 10^{-5}$/°C

18. The region between two concentric spheres of radii '*a*' and '*b*', respectively (see figure), has volume charge density $\rho = \frac{A}{r}$, where A is a constant and r is the distance from the centre. At the centre of the spheres is a point charge Q. The value of A such that the electric field in the region between the spheres will be constant, is:

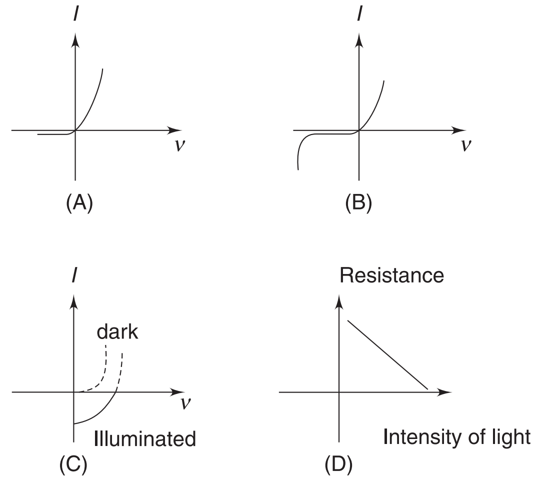


(a) $\frac{Q}{2\pi(b^2 - a^2)}$ (b) $\frac{2Q}{\pi(b^2 - a^2)}$
(c) $\frac{2Q}{\pi a^2}$ (d) $\frac{Q}{2\pi a^2}$

19. In an experiment for determination of refractive index of glass of a prism by $i - \delta$, plot, it was found that a ray incident at angle 35°, suffers a deviation of 40° and that it emerges at angle 79°. In the case which of the following is closest to the maximum possible value of the refractive index?
(a) 1.6 (b) 1.7
(c) 1.8 (d) 1.5

20. A student measures the time period of 100 oscillations of a simple pendulum four times. The data set is 90 s, 91 s, 95 s and 92 s. If the minimum division in the measuring clock is 1 s, then the reported mean time should be:
(a) 92 ± 5.0 s (b) 92 ± 1.8 s
(c) 92 ± 3 s (d) 92 ± 2 s

21. Identify the semiconductor devices whose characteristics are given below, in the order (a), (b), (c), (d)



(a) Zener diode, Simple diode, Light dependent resistance, Solar cell
(b) Solar cell, Light dependent resistance, Zener diode, Simple diode
(c) Zener diode, solar cell, Simple diode, Light dependent resistance
(d) Simple diode, Zener diode, Solar cell, Light dependent resistance

22. Radiation of wavelength λ, is incident on a photocell. The fastest emitted electron has speed v If the wavelength is changed to $\frac{3\lambda}{4}$, the speed of the fastest emitted electron will be:
(a) $< v\left(\frac{4}{3}\right)^{\frac{1}{2}}$ (b) $= v\left(\frac{4}{3}\right)^{\frac{1}{2}}$
(c) $= v\left(\frac{3}{4}\right)^{\frac{1}{2}}$ (d) $> v\left(\frac{3}{4}\right)^{\frac{1}{2}}$

23. A particle performs simple harmonic motion with amplitude A. Its speed is trebled at the instant that it is at a distance $\frac{2A}{3}$ from equilibrium position. The new amplitude of the motion is:
(a) $3A$ (b) $A\sqrt{3}$
(c) $\frac{7A}{3}$ (d) $\frac{A}{3}\sqrt{41}$

24. A particle of mass m is moving along the side of square of side '*a*' with a uniform speed v in the *x-y* plane as shown in the figure:

Given $E_1 = E_2$. Equating (1) and (2), we get $x = 2\sqrt{3}$ m $\simeq 3.5$ m

Total P.E. lost is $E = E_1 + E_2 = 2\sqrt{3}\,\mu mg + \mu mg\,x$

$$= \mu mg\,(2\sqrt{3} + x)$$
$$= \mu mg\,(2\sqrt{3} + 2\sqrt{3})$$
$$= 4\sqrt{3}\ \mu mg$$

Total P.E. lost is also equal to

$$E' = mgh = mg \times 2 = 2\,mg$$

Since $E = E'$, we get

$$4\sqrt{3}\ \mu mg = 2\,mg \quad \Rightarrow \mu = \frac{1}{2\sqrt{3}} \simeq 0.29$$

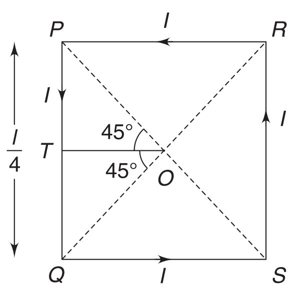
So the correct choice is (b).

4. $2\pi R = l \quad \Rightarrow R = \dfrac{l}{2\pi}$

The magnetic field at the centre of the circular coil A of radius R is

$$B_A = \frac{\mu_0 I}{2R} = \frac{\mu_0 I \pi}{l} \quad (1)$$

Refer to the following figure.



Magnetic field at O due to current I in PQ is

$$B_{PQ} = \frac{\mu_0 I}{4\pi a}\,(\sin 45° + \sin 45°)$$

where $a = OT = PT = \dfrac{l}{8}$ ($\because PT = OT \tan 45° = OT$)

$$B_{PQ} = \frac{4\mu_0 I}{\sqrt{2}\,\pi l}$$

Therefore, the magnetic field at O due to the complete sqs loop $PQRS$ is

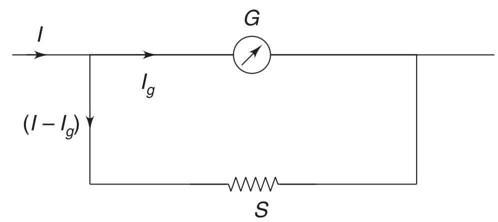
$$B_B = 4 \times \frac{4\mu_0 I}{\sqrt{2}\,\pi l} = \frac{8\sqrt{2}\,\mu_0 I}{\pi l} \quad (2)$$

Dividing (1) by (2)we get

$$\frac{B_A}{B_B} = \frac{\pi^2}{8\sqrt{2}}$$

So the correct choice is (c).

5.



$$I_g G = (I - I_g)S$$
$$S = \frac{I_g G}{I - I_g}$$

Given $I = 10$ A, $I_g = 1$ mA $= 0.001$ A and $G = 100\ \Omega$.

$$S = \frac{0.001 \times 100}{(1 - 0.001)}$$
$$= \frac{0.01}{0.999}$$
$$\simeq 0.01\ \Omega$$

So the choice is (d).

6. A person looking through a telescope observes angular magnification (which is called magnifying power). If a distant tree subtends an angle α at his unaided eye, then the angle subtended at his eye (when he looks through the telescope) by the image will be 20 α. Hence the image will appear 20 times taller, so the correct choice is (b).

7. In the given temperature range, the resistance of a metal (such as copper) increases linearly with temperature but the resistance of a semi-conductor (such as silicon) decreases exponentially with temperature. So the correct choice is (b).

8. The correct choice is (d).

9. Let N_o be the radioactive nuclei present is each sample at $t = 0$.

For sample A: No. of half lives in 80 minutes $= \dfrac{80}{20} = 4$

Number of nuclei decayed in 4 half lives $= \dfrac{N_o}{2^4} = \dfrac{N_o}{16}$

Number of nuclei left undecayed is

$$N_A = N_o - \frac{N_o}{16} = \frac{15N_o}{16}$$

For sample B: No. of half lives in 80 minutes $= \dfrac{80}{20} = 2$

Number nuclei decayed in 2 half lives $= \dfrac{N_o}{2^2} = \dfrac{N_o}{4}$

Number of nuclei left undecayed is

$$N_B = N_o - \frac{N_o}{4} = \frac{3N_o}{4}$$

$$\frac{N_A}{N_B} = \frac{15N_o}{16} \times \frac{4}{3N_o} = \frac{5}{4}$$

So the correct choice is (c).

10. The equation of straight line AB is

$$P = mV + C \quad (1)$$

where m is the slope and C is the intercept. From the given figure,

$$\text{Slope} \quad m = \frac{2P_0 - P_0}{V_0 - 2V_0} = -\frac{P_0}{V_0} \quad (2)$$

To find C, we have for point A,

$$2P_0 = mV_0 + C = -\frac{P_0}{V_0} \times V_0 + C$$

$$\Rightarrow \quad C = 3P_0 \quad (3)$$

Using (2) and (3) in (1) we have

$$P = -\left(\frac{P_0}{V_0}\right)V + 3P_0 \quad (4)$$

Equation of state is

$$PV = nRT$$

$$T = \frac{PV}{nR} \quad (5)$$

Using (4) in (5) we have

$$T = \frac{1}{nR}\left(-\frac{P_0}{V_0}V^2 + 3P_0V\right) \quad (6)$$

T will be maximum if $\frac{dT}{dV} = 0$ and $\frac{d^2T}{dV^2} < 0$. From (6) we find that

$$\frac{dT}{dV} = \frac{1}{nR}\left(-\frac{2P_0}{V_0} \times V + 3P_0\right)$$

Setting $\frac{dT}{dV} = 0$, we have

$$0 = \frac{1}{nR}\left(-\frac{2P_0V}{V_0} \times 3P_0\right)$$

$$\Rightarrow \quad V = \frac{3V_0}{2}$$

Putting this value in (6), we get

$$T_{\max} = \frac{1}{nR}\left[-\frac{P_0}{V_0} \times \left(\frac{3V_0}{2}\right)^2 + 3P_0 \times \frac{3V_0}{2}\right]$$

$$= \frac{9P_0V_0}{4nR}$$

It is easy to check that for $V = \frac{3V_0}{2}, \frac{d^2T}{dV^2}$ is negative.

Hence the correct choice is (d).

11. Resistance of the lamp is

$$R = \frac{80\text{ V}}{10\text{ A}} = 8\ \Omega$$

Given $I_{\text{rms}} = 10$ A, $V_{\text{rms}} = 220$ V and $\omega = 2\pi\nu = 100\pi$ rad s^{-1}.

$$I_{\text{rms}} = \frac{V_{\text{rms}}}{\sqrt{R^2 + X_L^2}}$$

$$\Rightarrow \quad 10 = \frac{220}{\sqrt{8^2 + X_L^2}}$$

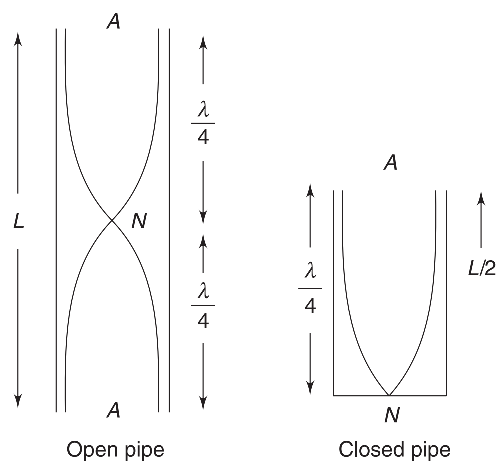
$$\Rightarrow 64 + X_L^2 = \left(\frac{220}{10}\right)^2 = 484$$

$$\Rightarrow \quad X_L = \sqrt{420} = 20.5\ \Omega$$

$$\Rightarrow \quad \omega L = 20.5$$

$$\Rightarrow \quad L = \frac{20.5}{2\pi \times 50} \simeq 0.065 \text{ H, which is choice (c).}$$

12.



Fundamental frequency of open pipe is $\left(\because\ L = \frac{\lambda}{4} + \frac{\lambda}{4} = \frac{\lambda}{2}\right)$ and hence $\lambda = 2L$.

$$f = \frac{v}{\lambda} = \frac{v}{2L}$$

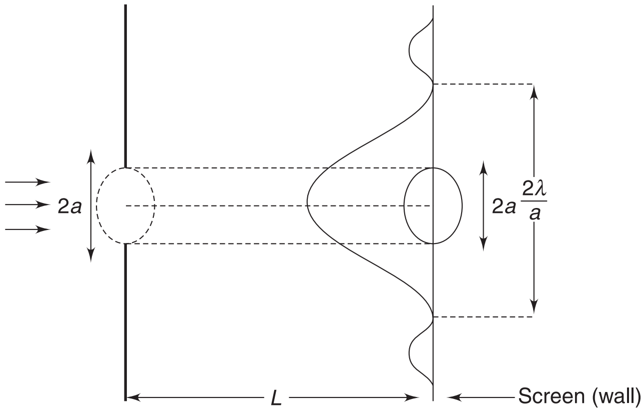
Fundamental frequency of the closed pipe is

$$\left(\because\ \frac{L}{2} = \frac{\lambda}{4} \text{ or } \lambda = 2L\right)$$

$$f' = \frac{v}{\lambda} = \frac{v}{2L} = f$$

So the correct choice is (c).

13. Refer to the following figure.



Linear width of the circular spot for undiffracted beam is $2a$.

Half angular width of the first maximum is

$$\theta_1 = \frac{\lambda}{a}$$

Angular width of the first maximum is

$$2\theta_1 = \frac{2\lambda}{a}$$

$\therefore$ Linear width of the first maximum $= \dfrac{2\lambda L}{a}$

Sum of two linear widths is

$$b = 2a + \frac{2\lambda L}{a} \quad (1)$$

b will be minimum if $\dfrac{db}{da} = 0$ and $\dfrac{d^2b}{da^2} > 0$.

Now $\dfrac{db}{da} = \dfrac{d}{da}\left(2a + \dfrac{2\lambda L}{a}\right)$

$$= 2 - \frac{2\lambda L}{a^2}$$

Putting $\dfrac{db}{da} = 0$ we set

$$0 = 2 - \frac{2\lambda L}{a^2} \Rightarrow a = \sqrt{\lambda L}$$

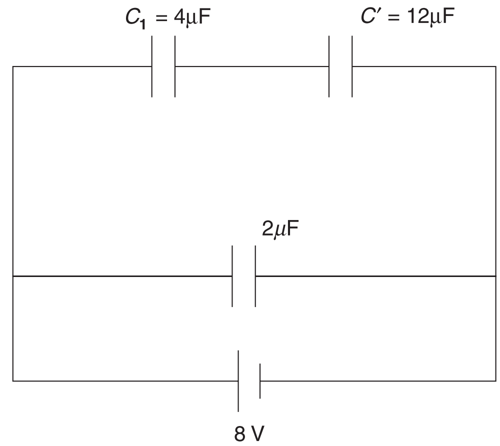
Putting this value of a in (1) we have

$$b_{\min} = 2\sqrt{\lambda L} + \frac{2\lambda L}{\sqrt{\lambda L}}$$

$$= 2\sqrt{\lambda L} + 2\sqrt{\lambda L} = 4\sqrt{\lambda L}$$

It is easy to check that when $a = \sqrt{\lambda L}$, $\dfrac{d^2b}{da^2}$ is positive. So the correct choice is (b).

14. The given circuit can be redrawn as follows.



Potential difference across the series combination of C_1 and C' is 8 V. If V_1 and V' are the potential differences across C_1 and C' respectively, then

$$C_1V_1 = C'V'$$

$$\Rightarrow \quad 4 \times 10^{-6} \times V_1 = 12 \times 10^{-6} \times V'$$

$$\Rightarrow \quad V_1 = 3\,V' \quad (1)$$

Also $\quad V_1 + V' = 8 \quad (2)$

Equations (1) and (2) give $V_1 = 6$ V and $V' = 2$ V. Therefore, charges on $4\,\mu$F capacitor and $9\,\mu$F capacitor is

$$Q_1 = (4 \times 10^{-6}) \times 6 = 24 \times 10^{-6}\text{ C}$$

and $\quad Q_2 = (9 \times 10^{-6}) \times 2 = 18 \times 10^{-6}$ C

Total charge $Q = Q_1 + Q_2 = 42 \times 10^{-6}$ C

The electric field at $r = 30$ m is

$$E = \frac{Q}{4\pi\epsilon_0 r^2} = \frac{42\times10^{-6}\times9\times10^9}{30\times30}$$

$$= 420\text{ NC}^{-1}, \text{ which is option (c).}$$

15. Energy $E = h\nu = \dfrac{hc}{\lambda}$. The wavelength of radiowaves is the longest and of X-rays is the shortest, In fact $\lambda_{\text{radio}} > \lambda_{\text{yellow}} > \lambda_{\text{blue}} > \lambda_{\text{x-rays}}$.

Hence $\quad \nu_D < \nu_B < \nu_A < \nu_C$

So $\quad E_D < E_B < E_A < E_C$

Thus the correct choice is (d).

16. The area enclosed by the hysteresis loop gives the energy dissipated. It is clear that material *B* dissipates less energy than material *A*. Hence, if the core of a transformer is made of material B, it will have a higher efficiency. Also the core of an electromagnet

should be made of a material which has a small retentivity and a small coercivity. It follows from the given loops that material B satisfies both these conditions. Hence the correct choice is (c).

17. $T = 2\pi\sqrt{\frac{L}{g}}$. Therefore,

$$\frac{\Delta T}{T} = \frac{1}{2}\frac{\Delta L}{L} = \frac{1}{2}\times\frac{L\alpha\Delta\theta}{L} = \frac{1}{2}\alpha\Delta\theta$$

$\therefore$ Loss of time per second $= \frac{1}{2}\alpha\Delta\theta$

Loss of time per day $= \left(\frac{1}{2}\alpha\Delta\theta\right)\times(24\times 60\times 60)$ s.

If θ is the temperature at which the clock gives correct time,

$$\frac{1}{2}\alpha(40-\theta)\times(24\times 60\times 60) = 12 \quad (1)$$

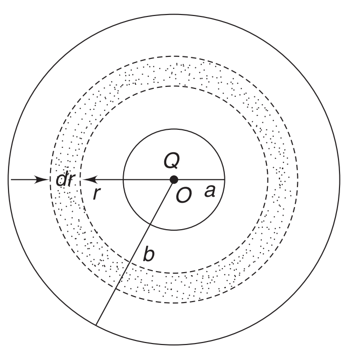
and $$\frac{1}{2}\alpha(\theta-20)\times(24\times 60\times 60) = 4 \quad (2)$$

Dividing (1) and (2) we have

$$\frac{40-\theta}{\theta-20} = 3 \quad \Rightarrow \quad \theta = 25°\text{C}$$

Using θ = 25°C in (1) or (2) we get $\alpha = 1.85 \times 10^{-5}$ per °C so the correct choice is (d).

18. Divide the region between the two spheres into a large number of concentric spherical elements each of a very small width *dr*. Consider one such element at a distance *r* from the centre *O* of the spheres as shown in the figure.



Volume of element $= 4\pi r^2\, dr$. Volume charge density is $\rho = \frac{A}{r}$ (given). Therefore, charge on the element is

$$dq = 4\pi r^2\, dr \times \frac{A}{r} = 4\pi A\, r dr$$

Total charge in the region between $r = a$ and $r = r$ is

$$q = \int_a^r dq = 4\pi A\int_a^r r dr$$
$$= 2\pi A\,(r^2 - a^2)$$

This charge can be assumed to be concentrated at the centre *O*. So the total charge at *O* is

$$Q' = Q + q = Q + 2\pi A\,(r^2 - a^2)$$

The electric field at a distance *r* from Q' is

$$E = \frac{Q'}{4\pi\epsilon_0 r^2}$$
$$= \frac{1}{4\pi\epsilon_0 r^2}\,[Q + 2\pi A\,(r^2 - a^2)]$$
$$= \frac{Q}{4\pi\epsilon_0 r^2} + \frac{2\pi A}{4\pi\epsilon_0 r^2}\,(r^2 - a^2)$$

$$\Rightarrow \quad E = \frac{A}{2\epsilon_0} + \frac{Q}{4\pi\epsilon_0 r^2} - \frac{Aa^2}{2\epsilon_0 r^2}$$

E will be constant (i.e., *E* will be independent of *r*) if the last two terms in the above equation cancel each other, i.e., if

$$\frac{Q}{4\pi\epsilon_0 r^2} = \frac{Aa^2}{2\epsilon_0 r^2}$$

$$\Rightarrow \quad A = \frac{Q}{2\pi a^2}$$

So the correct choice is (d).

19. Given $i = 35°$, $\delta = 40°$ and $e = 79°$. Now

$$\delta = i + e - A$$
$$\Rightarrow \quad A = i + e - \delta$$
$$= 35° + 79° - 40° = 74°$$

If δ_m is the angle of minimum deviation, the refractive index of the prism is given by

$$\mu = \frac{\sin\left(\frac{A+\delta_m}{2}\right)}{\sin\left(\frac{A}{2}\right)} \quad (1)$$

$$\Rightarrow \quad \mu\sin\left(\frac{A}{2}\right) = \sin\left(\frac{A}{2} + \frac{\delta_m}{2}\right)$$
$$= \sin\left(\frac{A}{2}\right)\cos\left(\frac{\delta_m}{2}\right) + \cos\left(\frac{A}{2}\right)\sin\left(\frac{\delta_m}{2}\right)$$

Partial differentiation of this equation gives (since μ and *A* are constants),

$$0 = -\frac{1}{2}\sin\left(\frac{A}{2}\right)\sin(\delta_m) + \frac{1}{2}\cos\left(\frac{A}{2}\right)\cos(\delta_m)$$

$$\Rightarrow \tan \delta_m = \frac{1}{\tan \frac{A}{2}} = \frac{1}{\tan 37°} \qquad (\because \quad A = 74°)$$

Since tan 37° is slightly greater than tan 30° $\left(\text{which is } \frac{1}{\sqrt{3}}\right)$, It follows that δ_m is slightly less than 60°. The actual calculation (using trignometric tables given δ_m = 53°. This angle is greater than the given value of deviation (which is δ = 40°). Now, with δ_m = 53° and A = 74°,

$$\mu = \frac{\sin\left(\frac{74° + 53°}{2}\right)}{\sin\left(\frac{74°}{2}\right)} = \frac{\sin 63.5°}{\sin 37°}$$

Using tables μ turns out to be nearly equal to 1.5. So the closest option is (d).

20. The mean value of the four measurements of time for 100 oscillations is

$$\bar{t} = \frac{t_1 + t_2 + t_3 + t_4}{4}$$
$$= \frac{90 + 91 + 95 + 92}{4} = 92 \text{ s}$$

Deviations (or errors) of values of t from the mean value are

$$|\bar{t} - t_1| = |92 - 90| = 2\text{s}$$
$$|\bar{t} - t_2| = |92 - 91| = 1\text{s}$$
$$|\bar{t} - t_3| = |92 - 95| = 3\text{s}$$
$$|\bar{t} - t_4| = |92 - 92| = 0\text{s}$$

$$\text{Average error} = \frac{2 + 1 + 3 + 0}{4} = 1.5\text{s}$$

Since the least count of the clock is 1 s, average error rounded off to appropriate significant figure is either 1s or 2s. So the reported mean time should be either (92 ± 1) s or (92 ± 2) s. So the correct choice is (d).

21. It is a question based on the experimental observation as written in NCERT book which states that the correct option is (d).

22. $h\nu = \frac{1}{2}mv^2 + w_0$. Since $\nu = \frac{c}{\lambda}$

$$v = \sqrt{\frac{2}{m}\left[\frac{hc}{\lambda} - w_0\right]} \qquad (1)$$

For wavelength $\frac{3\lambda}{4}$, we have

$$v' = \sqrt{\frac{2}{m}\left[\frac{4hc}{3\lambda} - w_0\right]} \qquad (2)$$

Dividing (2) by (1)

$$\frac{v'}{v} = \left[\frac{\frac{4hc}{3\lambda} - w_0}{\frac{hc}{\lambda} - w_0}\right]^{1/2}$$

$$\text{or} \quad \left(\frac{v'}{v}\right)^2 = \frac{\frac{4hc}{3\lambda} - w_0}{\frac{hc}{\lambda} - w_0}$$

$$= \frac{\left(\frac{4hc}{3\lambda} - w_0\right) + \left(\frac{hc}{\lambda} + w_0\right)}{\left(\frac{hc}{\lambda} - w_0\right) + \left(\frac{hc}{\lambda} + w_0\right)}$$

$$= \frac{\frac{4}{3} + 1}{2} = \frac{7}{6}$$

$\Rightarrow \quad \frac{v'}{v} = \sqrt{\frac{7}{6}} = 1.08$. So choices (b) and (c) are incorrect. Choice (a) gives $\frac{v'}{v} < \sqrt{\frac{4}{3}}$ or $\frac{v'}{v} < 1.15$ and choice (d) gives $\frac{v'}{v} > \sqrt{\frac{3}{4}}$ or $\frac{v'}{v} > 0.87$. But $\frac{v'}{v} = 1.08$, *i.e.* $v' > v$. So choice (a) also wrong. Hence the correct choice is (d).

23. $$v = \omega\sqrt{A^2 - x^2} \qquad (1)$$

At $x = \frac{2A}{3}$,

$$v' = \omega\sqrt{A^2 - \frac{4A^2}{9}} = \omega\sqrt{\frac{5A^2}{9}} = \omega A \times \frac{\sqrt{5}}{3}$$

Given $v' = 3v$. Hence

$$v' = \sqrt{5}\ \omega A$$

Let A_n be the new amplitude. Then

$$v' = \omega(A_n^2 - x^2)^{1/2}$$

$$\Rightarrow \quad \sqrt{5}\ \omega A = \omega(A_n^2 - x^2)^{1/2}$$

$$\Rightarrow \quad 5A^2 = A_n^2 - x^2$$

$$\Rightarrow \quad A_n^2 = 5A^2 + x^2 = 5A^2 + \frac{4A^2}{9} \qquad \left(\because x = \frac{2A}{3}\right)$$