JEE (ADVANCED)—2017
MATHEMATICS PAPER-I

SECTION I

- This section contains **SEVEN** questions.
- Each question has **FOUR** options (a), (b), (c) and (d). **One** or **More Than One** of these four option(s) is/are correct.
- For each question, marks will be awarded in one of the following categories:

Full marks	:	+ 4	If only the bubble(s) corresponding to all the correct option(s) is (are) darkened
Partial marks	:	+ 1	For darkening a bubble corresponding **to each correct option,** provided NO incorrect option is darkened
Zero marks	:	0	If none of the bubbles is darkened
Negative Marks	:	– 2	In all other cases

- For example, if (a), (c) and (d) are all the correct options for a question, darkening all these three will get +4 marks; darkening only (a) and (d) will get +2 marks; and darkening (a) and (b) will get –2 marks, as a wrong option is also darkened.

1. Let X and Y be two events such that $P(X) = \frac{1}{3}$, $P(X|Y) = \frac{1}{2}$ and $P(Y|X) = \frac{2}{5}$. Then

(a) $P(Y) = \frac{4}{15}$ (b) $P(X'|Y) = \frac{1}{2}$

(c) $P(X \cup Y) = \frac{2}{5}$ (d) $P(X \cap Y) = \frac{1}{5}$

2. Let $f : \mathbf{R} \to (0, 1)$ be a continuous function. Then, which of the following function(s) has(have) the value zero at some point in the interval (0, 1)?

(a) $e^x - \int_0^x f(t)\sin t\,dt$

(b) $f(x) + \int_0^{\pi/2} f(t)\sin t\,dt$

(c) $x - \int_0^{\pi/2-x} f(t)\cos t\,dt$

(d) $x^9 - f(x)$

3. Let a, b, x and y be real numbers such that $a - b = 1$ and $y \neq 0$. If the complex number $z = x + iy$ satisfies $\text{Im}\left(\frac{az+b}{z+1}\right) = y$, then which of the following is(are) possible value(s) of x?

(a) $1 - \sqrt{1+y^2}$ (b) $-1 - \sqrt{1-y^2}$

(c) $1 + \sqrt{1+y^2}$ (d) $-1 + \sqrt{1-y^2}$

4. If $2x - y + 1 = 0$ is a tangent to the hyperbola $\frac{x^2}{a^2} - \frac{y^2}{16} = 1$, then which of the following CANNOT be sides of a right angled triangle?

(a) a, 4, 1 (b) $2a$, 4, 1

(c) a, 4, 2 (d) $2a$, 8, 1

5. Let $[x]$ be the greatest integer less than or equal to x. Then, at which of the following point(s) the function $f(x) = x\cos(\pi(x + [x]))$ is discontinuous?

(a) $x = -1$ (b) $x = 1$

(c) $x = 0$ (d) $x = 2$

6. Which of the following is(are) NOT the square of a 3×3 matrix with real entries?

(a) $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -1 \end{bmatrix}$ (b) $\begin{bmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & -1 \end{bmatrix}$

(c) $\begin{bmatrix} -1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & -1 \end{bmatrix}$ (d) $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

7. If a chord, which is not a tangent, of the parabola $y^2 = 16x$ has the equation $2x + y = p$, and midpoint (h, k), then which of the following is(are) possible value(s) of p, h and k?

(a) $p = -1, h = 1, k = -3$

(b) $p = 2, h = 3, k = -4$

(c) $p = -2, h = 2, k = -4$

(d) $p = 5, h = 4, k = -3$

SECTION II

- This section contains **Five** questions
- The answer to each question is a **SINGLE DIGIT INTEGER** ranging from 0 to 9, both inclusive
- For each question, marks will be awarded in one of the following categories:
  Full Marks : +3 If only the bubble corresponding to the correct answer is darkened
  Zero Marks : 0 In all other cases

8. For a real number α, if the system

$$\begin{bmatrix} 1 & \alpha & \alpha^2 \\ \alpha & 1 & \alpha \\ \alpha^2 & \alpha & 1 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1 \\ -1 \\ 1 \end{bmatrix}$$

of linear equations, has infinitely many solutions, then $1 + \alpha + \alpha^2 =$

9. The sides of a right angled triangle are in arithmetic progression. If the triangle has area 24, then what is the length of its smallest side?

10. Let $f : \mathbf{R} \to \mathbf{R}$ be a differentiable function such that $f(0) = 0$, $f\left(\frac{\pi}{2}\right) = 3$ and $f'(0) = 1$. If

$$g(x) = \int_x^{\frac{\pi}{2}} [f'(t) \operatorname{cosec} t - \cot t \operatorname{cosec} t\, f(t)]\, dt$$

For $x \in (0, \frac{\pi}{2}]$, then $\lim_{x\to 0} g(x) =$

11. For how many values of p, the circle $x^2 + y^2 + 2x + 4y - p = 0$ and the coordinate axes have exactly three common points?

12. Words of length 10 are formed using the letters A, B, C, D, E, F, G, H, I, J. Let x be the number of such words where no letter is repeated; and let y be the number of such words where exactly one letter is repeated twice and no other letter is repeated. Then, $\frac{y}{9x} =$

SECTION III

- This section contains **Six** questions of matching type
- This section contains **Two** tables (each having 3 columns and 4 rows)
- Based on each table, there are **Three** questions
- Each question has **Four** options (a), (b), (c) and (d). **Only One** of theses four options is correct
- For each question, marks will be awarded in one of the following categories:
  Full marks : + 3 If only the bubble corresponding to the correct option is darkened
  Zero marks : 0 If none of the bubbles is darkened
  Negative marks : –1 In all other cases

Answer Q. 13, 14 and 15 by appropriately matching the information given in the three columns of the following
Columns 1, 2 and 3 contain conics, equations of tangents to the conics and points of contact, respectively

Column 1	Column 2	Column 3
(I) $x^2 + y^2 = a^2$	(i) $my = m^2x + a$	(P) $\left(\frac{a}{m^2}, \frac{2a}{m}\right)$
(II) $x^2 + a^2y^2 = a^2$	(ii) $y = mx + a\sqrt{m^2+1}$	(Q) $\left(\frac{-ma}{\sqrt{m^2+1}}, \frac{a}{\sqrt{m^2+1}}\right)$
(III) $y^2 = 4ax$	(iii) $y = mx + \sqrt{a^2m^2-1}$	(R) $\left(\frac{-a^2m}{\sqrt{a^2m^2+1}}, \frac{1}{\sqrt{a^2m^2+1}}\right)$
(IV) $x^2 - a^2y^2 = a^2$	(iv) $y = mx + \sqrt{a^2m^2+1}$	(S) $\left(\frac{-a^2m}{\sqrt{a^2m^2-1}}, \frac{-1}{\sqrt{a^2m^2-1}}\right)$

13. For $a = \sqrt{2}$, if a tangent is drawn to a suitable conic (Column 1) at the point of contact (–1, 1), then which of the following options is the only CORRECT combination for obtaining its equation?

(a) (I) (ii) (Q) (b) (I) (i) (P)
(c) (III) (i) (P) (d) (II) (ii) (Q)

14. The tangent to a suitable conic (Column 1) at $\left(\sqrt{3}, \frac{1}{2}\right)$ is found to be $\sqrt{3}x + 2y = 4$, then which of the following options is the only CORRECT combination?

(a) (IV) (iv) (S) (b) (II) (iv) (R)
(c) (IV) (iii) (S) (d) (II) (iii) (R)

15. If a tangent to a suitable conic (Column 1) is found to be $y = x + 8$ and its point of contact is (8, 16), then which of the following options is the only CORRECT combination?

(a) (III) (i) (P) (b) (I) (ii) (Q)
(c) (II) (iv) (R) (d) (III) (ii) (Q)

Answer Q. 16, Q. 17 and Q. 18 by appropriately matching the information given in the three columns of the following table.

Let $f(x) = x + \log_e x - x \log_e x$, $x \in (0, \infty)$,

- Column 1 contains information about zeros of $f(x)$, $f'(x)$ and $f''(x)$.
- Column 2 contains information about the limiting behaviour of $f(x)$, $f'(x)$ and $f''(x)$ at infinity.
- Column 3 contains information about increasing/decreasing nature of $f(x)$ and $f'(x)$.

Column 1	Column 2	Column 3
(I) $f(x) = 0$ for some $x \in (1, e^2)$	(i) $\lim_{x\to\infty} f(x) = 0$	(P) f is increasing in (0, 1)
(II) $f'(x) = 0$ for some $x \in (1, e)$	(ii) $\lim_{x\to\infty} f(x) = -\infty$	(Q) f is decreasing in (e, e^2)
(III) $f'(x) = 0$ for some $x \in (0, 1)$	(iii) $\lim_{x\to\infty} f'(x) = -\infty$	(R) f' is increasing in (0, 1)
(IV) $f''(x) = 0$ for some $x \in (1, e)$	(iv) $\lim_{x\to\infty} f''(x) = 0$	(S) f' is decreasing in (e, e^2)

16. Which of the following options is the only INCORRECT combination?

(a) (I) (iii) (P) (b) (II) (iv) (Q)
(c) (II) (iii) (P) (d) (III) (i) (R)

17. Which of the following options is the only CORRECT combination?

(a) (I) (ii) (R) (b) (III) (iv) (P)
(c) (II) (iii) (S) (d) (IV) (i) (S)

18. Which of the following options is the only CORRECT combination?

(a) (III) (iii) (R) (b) (IV) (iv) (S)
(c) (II) (ii) (Q) (d) (I) (i) (P)

Answers

Section-I

1. (a), (b) **2.** (c), (d) **3.** (b), (d)
4. (a), (c), (d) **5.** (a), (b), (d) **6.** (a), (c)
7. (b)

Section-II

8. 1 **9.** 6 **10.** 2
11. 2 **12.** 5

Section-III

13. (a) **14.** (b) **15.** (a)
16. (d) **17.** (c) **18.** (c)

Hints and Solutions

1. $P(X|Y) = \dfrac{P(X \cap Y)}{P(Y)}$ and $P(Y|X) = \dfrac{P(X \cap Y)}{P(X)}$

$\therefore \dfrac{2}{5} = \dfrac{P(X \cap Y)}{1/3} \Rightarrow P(X \cap Y) = \dfrac{2}{15}$

Now, $P(Y) = \dfrac{P(X \cap Y)}{P(X|Y)} = \dfrac{2/15}{1/2} = \dfrac{4}{15}$

$P(X'|Y) = 1 - P(X|Y) = 1 - \dfrac{1}{2} = \dfrac{1}{2}$

and $P(X \cup Y) = P(X) + P(Y) - P(X \cap Y)$

$$= \frac{1}{3} + \frac{4}{15} - \frac{2}{15} = \frac{7}{15}.$$

2. Let $g_1(x) = e^x - \int_0^x f(t) \sin t \, dt$,

then $g_1'(x) = e^x - f(x) \sin x$

As $0 < f(x) < 1$ and $0 < \sin x < 1 \ \forall \ x \in (0, 1)$

$g_1'(x) > 0 \ \forall \ x \in (0, 1)$

Also, $\lim_{x \to 0+} g_1(x) = 1$

$\Rightarrow \quad g_1(x) > 1 \ \forall \ x \in (0, 1)$

Let $g_2(x) = f(x) + \int_0^{\pi/2} f(t) \sin t \, dt$,

Note that $g_2(x) > f(x) > 0 \ \forall \ x \in (0, 1)$

Next, let

$$g_3(x) = x - \int_0^{\pi/2 - x} f(t) \cos t \, dt$$

$$g_3(0) = -\int_0^{\pi/2} f(t) \cos t \, dt < 0$$

$[\because \ 0 < f(t), \cos t < 1]$

Also,

$$g_3(1) = 1 - \int_0^{\pi/2 - 1} f(t) \cos t \, dt > 0$$

$[\because \ 0 < f(t), \cos t < 1]$

As $g_3(x)$ continuous on **R**, we get $g_3(x) = 0$ at least for one value of $x \in (0, 1)$

Finally, let

$$g_4(x) = x^9 - f(x)$$
$$g_4(0) = -f(0) < 0$$
$$g_4(1) = 1 - f(1) > 0$$

$\therefore \quad g_4(x) = 0$ at least for one value of $x \in (0, 1)$

3. $\dfrac{az + b}{z + 1} = \dfrac{a(z + 1) + b - a}{z + 1} = a - \dfrac{1}{z + 1}$

Now,

$$y = \text{Im}\left(\frac{az + b}{z + 1}\right) = -\text{Im}\left(\frac{1}{z + 1}\right)$$

$$= -\text{Im}\left(\frac{\bar{z} + 1}{z\bar{z} + z + \bar{z} + 1}\right)$$

$$y = \frac{y}{z\bar{z} + z + \bar{z} + 1}$$

$\Rightarrow \quad x^2 + y^2 + 2x + 1 = 1 \quad [\because y \neq 0]$

$\Rightarrow \quad (x + 1)^2 = 1 - y^2$

$\Rightarrow \quad x = -1 \pm \sqrt{1 - y^2}$

4. $y = 2x + 1$ is a tangent to the hyperbola $\dfrac{x^2}{a^2} - \dfrac{y^2}{16} = 1$

if $c^2 = a^2 m^2 - b^2$

i.e. $1 = a^2 (2^2) - 16$

$\Rightarrow \quad a^2 = 17/4$

As, $4^2 + 1^2 = 17 > a^2$; a, 4, 1 cannot form a right triangle.

Next, $(2a)^2 = 4a^2 = 17 = 4^2 + 1$, therefore $2a$, 4, 1 are sides of a right triangle.

Since, $a^2 = 17/4 < 4^2 + 2^2$

$\Rightarrow a$, 4, 2 cannot be sides of a right triangle.

Lastly $1 + (2a)^2 = 1 + 4a^2 = 1 + 17 = 18 < 8^2$

$\Rightarrow 2a$, 8, 1 cannot be sides of a right triangle.

5. $f(x) = x \cos(\pi(x + [x]))$

$$= \begin{cases} x\cos(\pi(x - 2)), & -2 \le x < -1 \\ x\cos(\pi(x - 1)), & -1 \le x < 0 \\ x\cos(\pi x), & 0 \le x < 1 \\ x\cos(\pi(x + 1)), & 1 \le x < 2 \\ x\cos(\pi(x + 2)), & 2 \le x < 3 \end{cases}$$

$$= \begin{cases} x\cos(\pi x) & \text{if } -2 \le x < -1 \\ -x\cos(\pi x) & \text{if } -1 \le x < 0 \\ x\cos(\pi x) & \text{if } 0 \le x < 1 \\ -x\cos(\pi x) & \text{if } 1 \le x < 2 \\ x\cos(\pi x) & \text{if } 2 \le x < 3 \end{cases}$$

$$\lim_{x \to -1-} f(x) = (-1)\cos(-\pi) = 1$$

$$\lim_{x \to -1+} f(x) = -(-1)\cos(\pi) = -1$$

$$\lim_{x \to 0^-} f(x) = 0 = \lim_{x \to 0+} f(x)$$

$$\lim_{x \to 1-} f(x) = (1)\cos(\pi) = -1$$

$$\lim_{x \to 1+} f(x) = -(1)\cos\pi = 1$$

$$\lim_{x \to 2-} f(x) = -2\cos(2\pi) = -2$$

and $\lim_{x \to 2+} f(x) = 2\cos(2\pi) = 2$

Thus, f is discontinuous at $x = -1$, 1 and 2.

6. If A is a 3×3 matrix, then

$$\det(A^2) = (\det(A))^2 \ge 0$$

As $\begin{vmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -1 \end{vmatrix} = -1 = \begin{vmatrix} -1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & -1 \end{vmatrix}$

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -1 \end{bmatrix}, \begin{bmatrix} -1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & -1 \end{bmatrix}$$

cannot be square of a 3×3 matrix.

Also, $\begin{bmatrix}1 & 0 & 0\\0 & 0 & -1\\0 & 1 & 0\end{bmatrix}^2 = \begin{bmatrix}1 & 0 & 0\\0 & -1 & 0\\0 & 0 & -1\end{bmatrix}$

and $\begin{bmatrix}1 & 0 & 0\\0 & 1 & 0\\0 & 0 & 1\end{bmatrix}^2 = \begin{bmatrix}1 & 0 & 0\\0 & 1 & 0\\0 & 0 & 1\end{bmatrix}$

7. Equation of chord of $y^2 = 16x$ which has (h, k) as mid point is

$$ky - 8(x + h) = k^2 - 16h$$
$$8x - ky = 8h - k^2$$

Comparing it with $2x + y = p$, we get

$$\frac{8}{2} = \frac{-k}{1} = \frac{8h - k^2}{p}$$

$\Rightarrow$ $k = -4,\ 4p = 8h - k^2$

$\Rightarrow$ $k = -4,\ 8h - 4p = 16$

$\Rightarrow$ $k = -4,\ 2h - p = 4$ ...(i)

So only possible choices are (b) and (c).

For $p = 2$, $h = 3$, $k = -4$, (i) is satisfied and $(-4)^2 \neq 16(3)$

For $p = -2$, $h = 2$, $k = -4$, (1) is not satisfied.

$\therefore$ only choice is (b).

8. As the system of equations has infinite number of solutions

$$\Delta = \begin{vmatrix}1 & \alpha & \alpha^2\\ \alpha & 1 & \alpha\\ \alpha^2 & 1 & 1\end{vmatrix} = 0$$

Using $R_1 \to R_1 - \alpha R_2$, we get

$$\Delta = \begin{vmatrix}1-\alpha^2 & 0 & 0\\ \alpha & 1 & \alpha\\ \alpha^2 & 1 & 1\end{vmatrix} = 0$$

$\Rightarrow$ $(1 - \alpha^2)(1 - \alpha) = 0$

$\Rightarrow$ $(1 + \alpha)(1 - \alpha)^2 = 0$

$\Rightarrow$ $\alpha = -1, 1$

For $\alpha = 1$, first two equation become

$$x + y + z = 1$$
$$x + y + z = -1$$

which is not possible

$\therefore$ $\alpha = -1$

Thus, $1 + \alpha + \alpha^2 = 1$

9. Let sides of triangle be $a - d$, a, $a + d$, where $a > d > 0$

We have

$$(a + d)^2 = a^2 + (a - d)^2$$

$\Rightarrow$ $4ad = a^2 \Rightarrow a = 4d$

Area of triangle $= \frac{1}{2} a\,(a - d)$

$\Rightarrow$ $24 = \frac{1}{2}(4d)(4d - d)$

$\Rightarrow$ $24 = 6d^2$

$\Rightarrow$ $d = 2$ $[\because d > 0]$

$\therefore$ smallest side $= a - d = 3d = 6$.

10. $g(x) = f(t)\operatorname{cosec} t\Big]_x^{\pi/2} - \int_x^{\pi/2} f(t)(-\operatorname{cosec} t \cot t)\,dt$

$$-\int_x^{\pi/2} (\operatorname{cosec} t \cot t) f(t)\,dt$$

$= f\left(\frac{\pi}{2}\right)$ cosesc $\left(\frac{\pi}{2}\right) - f(x)$ cosec x

$= 3 - \frac{f(x)}{\sin x}$

$$\lim_{x\to 0} g(x) = \lim_{x\to 0+} g(x) = 3 - \lim_{x\to 0+}\frac{f(x)}{\sin x}$$

$$= 3 - \lim_{x\to 0+}\frac{f'(x)}{\cos x} = 3 - \frac{f'(0)}{1}$$

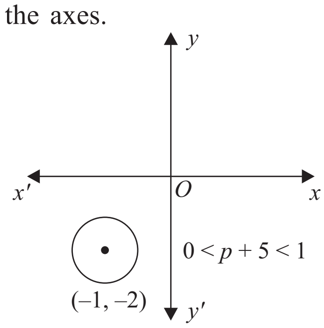
$= 3 - 1 = 2.$

11. Write the equation as

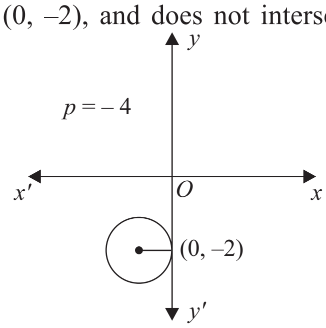
$(x + 1)^2 + (y + 2)^2 = p + 5$ ...(i)

We must have $p + 5 > 0$.

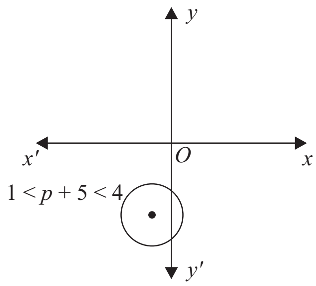
(i) If $0 < p + 5 < 1$, (i) does not intersect any of the axes.



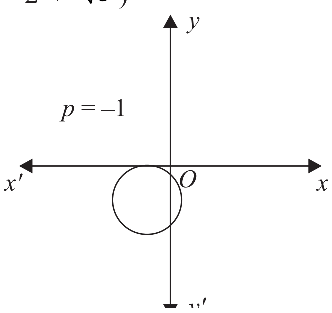
(ii) If $p + 5 = 1$, (i) meets the y-axis exactly once at $(0, -2)$, and does not intersect x-axis



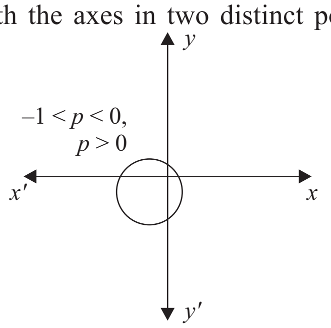
(iii) If $1 < p + 5 < 4$, (i) meets the y-axis exactly twice but does not intersect the x-axis.



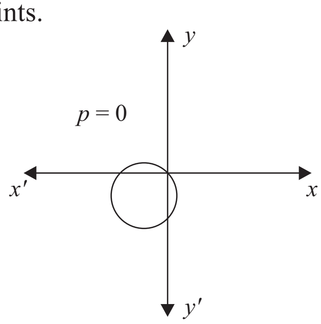
(iv) If $p + 5 = 4$ or $p = -1$, (i) meets the x-axis at $(-1, 0)$ and y-axis at $(0, -2-\sqrt{3})$ and $(0, -2+\sqrt{3})$



(v) If $4 < p + 5 < 5$ or $p + 5 > 5$, circle (i) does not pass through the origin and meets both the axes in two distinct points.



(vi) For $p = 0$, (i) meets the axes in exactly three points.



12. $x = {}^{10}P_{10} = 10!$

$$y = ({}^{10}C_1)\,({}^{9}C_8)\left(\frac{10!}{2!}\right)$$

$$= (10)\,(9)\left(\frac{10!}{2!}\right)$$

$$\Rightarrow \quad \frac{y}{9x} = \frac{(10)(9)(10!)/2}{9(10!)} = 5$$

13. For $a = \sqrt{2}$, $(-1, 1)$ lies only on $x^2 + y^2 = a^2$

Equation of tangent to this circle is

$$y = mx + a\sqrt{m^2+1} \qquad \text{[column 2, (ii)]}$$

For $\quad a = \sqrt{2}$

$$y = mx + \sqrt{2}\sqrt{m^2+1}$$

It will pass through $(-1, 1)$

if $\quad 1 = -m + \sqrt{2}\sqrt{m^2+1}$

$\Rightarrow \quad (m+1)^2 = 2(m^2+1) \Rightarrow (m-1)^2 = 0$

$\Rightarrow \quad m = 1$

Point of contact $\left(\dfrac{-\sqrt{2}}{\sqrt{1+1}}, \dfrac{\sqrt{2}}{\sqrt{1+1}}\right) = (-1, 1)$

Thus, correct option is (a).

14. Write $\sqrt{3}\,x + 2y = 4$ as

$$y = -\frac{\sqrt{3}}{2}x + 2$$

$\therefore \quad m = -\sqrt{3}/2$

For column 2, (iii) option

$\sqrt{a^2m^2-1} = 2$

In this case option(S), column 3 is not possible as

$$\frac{1}{2} = \frac{-1}{\sqrt{a^2m^2-1}} \text{ does not hold.}$$

For column 2, (iv) option

$$m = -\sqrt{3}/2,\ 2 = \sqrt{a^2m^2+1}$$

In this for case option (R)

$$= \frac{-a^2m}{\sqrt{a^2m^2+1}} = \sqrt{3}\,,\ \frac{1}{\sqrt{a^2m^2+1}} = \frac{1}{2}$$

$$\Rightarrow \frac{-a^2(-\sqrt{3}/2)}{2} = \sqrt{3} \Rightarrow a^2 = 4 \Rightarrow a = 2$$

Thus, correct option is (b), that is, (II), (iv), (R)

15. When $y = x + 8$, $m = 1$

For option Q, set

$$\frac{a}{\sqrt{m^2+1}} = 16 \Rightarrow a = 16\sqrt{2}$$

But then $\dfrac{-ma}{\sqrt{m^2+1}} \neq 8$

For option R, $\dfrac{1}{\sqrt{a^2m^2+1}} = 16$

$\Rightarrow \sqrt{a^2+1} = \frac{1}{16}$. Not possible.

$\therefore$ only option left is P.

Setting $\frac{a}{m^2} = 8 \Rightarrow a = 8,\ \frac{2a}{m} = 16$

Thus, correct option is (a).

For Q 16, Q 17, Q 18

$$f(x) = x + \log_e x - x \log_e x$$
$$= x + (1 - x) \log_e x,\ x \in (0, \infty)$$
$$f'(x) = 1 + \frac{1-x}{x} - \log_e x$$
$$= \frac{1}{x} - \log_e x,\ x \in (0, \infty)$$
$$f''(x) = -\frac{1}{x^2} - \frac{1}{x},\ x \in (0, \infty)$$

As $f(1) = 1 + (1 - 1) \log_e(1) = 1 > 0$

and $f(e^2) = e^2 + (1 - e^2) \log_e (e^2)$

$= e^2 + (1 - e^2)(2) = 2 - e^2 < 0,$

$f(x) = 0$ for some $x \in (1, e^2)$

Next, $f'(1) = 1 > 0$

and $f'(e) = \frac{1}{e} - 1 < 0$

$\therefore$ $f'(x) = 0$ for some $x \in (1, e)$

Also, $f''(x) > 0\ \forall\ x \in (0, 1)$

$\therefore$ $f'(x) = 0$ for some $x \in (0, 1)$ is incorrect

Lastly, $f''(x) < 0$ V $x \in (1, e)$

$\therefore f''(x) = 0$ for some $x \in (1, e)$ is incorrect.

We have, $\lim_{x\to\infty} f(x) = \lim_{x\to\infty} [1 - (1 - x)(1 - \log_e x)]$

$= \lim_{x\to\infty} [1 - (x - 1)(\log_e x - 1)] = -\infty$

$$\lim_{x\to\infty} f'(x) = \lim_{x\to\infty} \left(\frac{1}{x} - \log_e x\right) = -\infty$$

$$\lim_{x\to\infty} f''(x) = \lim_{x\to\infty} \left(-\frac{1}{x} - \frac{1}{x^2}\right) = 0.$$

For column 3,

$$f'(x) > 0\ \forall\ x \in (0, 1)$$

$\Rightarrow f$ increases on $(0, 1)$

Next, $f'(x) = \frac{1 - x\log_e x}{x} < 0 \quad \forall\ x \in (e, e^2)$

$\Rightarrow f(x)$ decreases on (e, e^2)

We have

$$f''(x) = -\frac{1}{x}\left(1 + \frac{1}{x}\right) < 0 \quad \forall\ x \in (0, 1)$$

$\Rightarrow f'$ decreases on $(0, 1)$

Also, $f''(x) < 0 \quad \forall\ x \in (e, e^2)$

$\Rightarrow f'$ decreases on (e, e^2)

16. In option (d), (III), (i), (R) is only incorrect option.

17. In option (c), (II), (iii), (S) is the only correct combination

18. In option (c), (II), (ii), (Q) is the only correct combination.