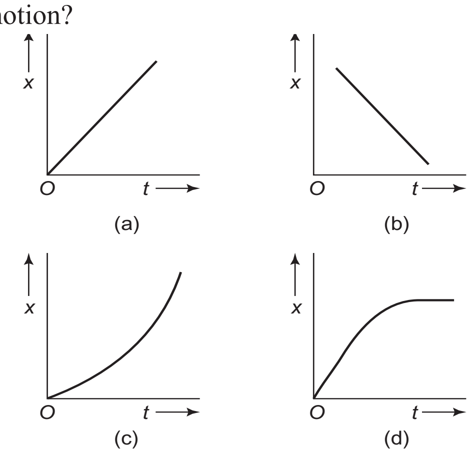
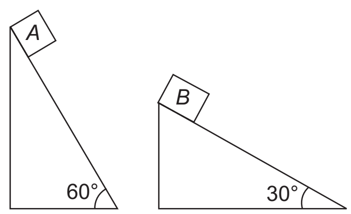
Practice Test Paper – I

1. In an experiment the angles are required to be measured using an instrument. 29 divisions of the main scale exactly coincide with the 30 divisions of the vernier scale. If the smallest division of the main scale is half-a-degree (= 0.5°), then the least count of the instrument is :
   (a) one degree (b) half degree
   (c) one minute (d) half minute
2. If L, R, C and V respectively represent inductance, resistance, capacitance and potential difference, then the dimensions of $\frac{L}{RCV}$ are the same as those of
   (a) current (b) $\frac{1}{\text{current}}$
   (c) charge (d) $\frac{1}{\text{charge}}$
3. The velocity (v) of a body moving along the positive x-direction varies with displacement (x) from the origin as $v = k\sqrt{x}$, where k is a constant. Which of the graphs shown in the following figure correctly represents the displacement-time ($x – t$) graph of the motion?



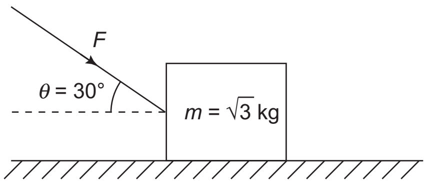
4. A stone tied to a string of length L is whirled in a vertical circle with the other end of the string at the centre. At a certain instant of time, the stone is at its lowest position and has a speed u. The magnitude of the change in its velocity as it reaches a position where the string is horizontal is
   (a) $\sqrt{u^2 - 2gL}$ (b) $\sqrt{2gL}$
   (c) $\sqrt{u^2 - gL}$ (d) $\sqrt{2(u^2 - gL)}$
5. A particle located at $x = 0$ at time $t = 0$, starts moving along the positive x-direction with a velocity v that varies as $v = \alpha\sqrt{x}$ where α is a constant. The displacement x of the particle varies wih time t as
   (a) $t^{1/2}$ (b) t^3
   (c) t^2 (d) t
6. A particle is moving with velocity $v = K(y\hat{\mathbf{i}} + x\hat{\mathbf{j}})$, where K is a constant. The general equation for its path is
   (a) $y = x^2$ + constant (b) $y^2 = x$ + constant
   (c) xy = constant (d) $y^2 = x^2$ + constant
7. Two fixed frictionless inclined planes making an angle 30° and 60° with the horizontal are shown in the following figure. Two block A and B are place on the two planes. What is the relative vertical acceleration of A with respect to B?



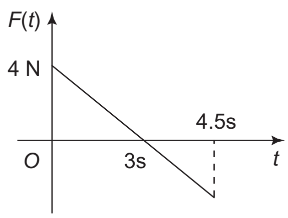
(a) 4.9 ms^{-2} in horizontal direction
(b) 9.8 ms^{-2} in vertical direction
(c) zero
(d) 4.9 ms^{-2} in vertical direction

8. What is the maximum value of the force F such that the block shown in the following figure does not move? The coefficient of friction between the block and the horizontal surface is 0.5. (Take $g = 10$ ms^{-2})

(a) 20 N (b) 10 N
(c) 12 N (d) 15 N



9. A block of mass 2 kg is free to move along the x-axis. It is at rest and from $t = 0$ onwards it is subjected to a time-dependent force $F(t)$ in the x direction. The force $F(t)$ varies with t as shown in the following figure. The kinetic energy of the block after 4.5 seconds is



(a) 4.50 J (b) 7.50 J
(c) 5.06 J (d) 14.06 J

10. The potential energy function for the force between two atoms in a diatomic molecule is approximately given by $U(x) = \frac{a}{x^{12}} - \frac{b}{x^6}$, where a and b are constants and x is the distance between the atoms. If the dissocation energy of the molecule is D $[U(x = \infty) - U_{\text{at equilibrium}}]$, D is

(a) $\frac{b^2}{6a}$ (b) $\frac{b^2}{2a}$
(c) $\frac{b^2}{12a}$ (d) $\frac{b^2}{4a}$

11. A thin uniform rod of length l and mass m is swinging freely about a horizontal axis passing through its end. Its maximum angular speed is ω. Its centre of mass rises to a maximum height of :

(a) $\frac{1}{2}\frac{l^2\omega^2}{g}$ (b) $\frac{1}{6}\frac{l^2\omega^2}{g}$
(c) $\frac{1}{3}\frac{l^2\omega^2}{g}$ (d) $\frac{1}{6}\frac{l\omega}{g}$

12. A block of base 10 cm × 10 cm and height 15 cm is kept on an inclined plane. The coefficient of friction between them is $\sqrt{3}$. The inclination θ of this inclined plane from the horizontal plane is gradually increased from 0°. Then

(a) at $\theta = 30°$, the block will start sliding down the plane
(b) the block will remain at rest on the plane up to certain θ and then it will topple
(c) at $\theta = 60°$, the block will start sliding down the plane and continue to do so at higher angles
(d) at $\theta = 60°$, the block will start sliding down the plane and on further increasing θ, it will topple at certain θ.

13. An ideal spring with spring-constant k is hung from the ceiling and a block of mass M is attached to its lower end. The mass is released with the spring initially unstretched. Then the maximum extension in the spring is

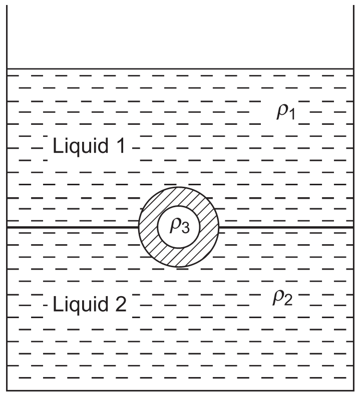
(a) 4 Mg/k (b) 2 Mg/k
(c) Mg/k (d) $Mg/2k$

14. A planet in a distant solar system is 10 times more massive than the earth and its radius is 10 times smaller. Given that the escape velocity from the earth is 11 km s^{-1}, the escape velocity from the surface of the planet would be

(a) 0.11 km s^{-1}
(b) 1.1 km s^{-1}
(c) 11 km s^{-1}
(d) 110 km s^{-1}

15. A jar is filled with two non-mixing liquids 1 and 2 having densities ρ_1 and ρ_2 respectively. A solid ball, made of a material of density ρ_3 is dropped in the jar. It comes to equilibrium in the position shown in figure given on next page. Which of the following is true for ρ_1, ρ_2 and ρ_3?

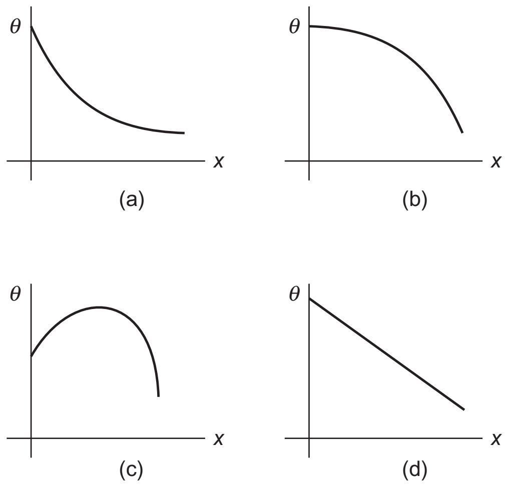
(a) $\rho_1 < \rho_3 < \rho_2$
(b) $\rho_3 < \rho_1 < \rho_2$
(c) $\rho_1 > \rho_3 > \rho_2$
(d) $\rho_1 < \rho_2 < \rho_3$



16. A small metal sphere of radius r and density ρ falls from rest in a viscous liquid of density σ and coefficient of viscosity η. Due to friction, heat is produced. The rate of production of heat when the sphere has acquired the terminal velocity is proportional to

    (a) r^2 (b) r^3
    (c) r^4 (d) r^5

17. A long metallic bar is carrying heat from one of its ends to the other end under steady-state. The variation of temperature θ along the length x of the bar from its hot end is best described by which of the following figures?



18. An ideal gas is expanding such that PT^2 = constant. The coefficient of volume expansion of the gas is

    (a) $\frac{1}{T}$ (b) $\frac{2}{T}$
    (c) $\frac{3}{T}$ (d) $\frac{4}{T}$

19. The displacement y of a particle of a medium can be expressed as
$$y = \tan^{-6} \sin\left(100t + 20x + \frac{\pi}{4}\right)$$
where y and x are in metre and t in second. The speed of the wave (in ms^{-1}) is

    (a) 2000 (b) 5
    (c) 20 (d) 5π

20. The equation of a wave on a string of linear mass density 0.04 kg m^{-1} is given by
$y = 0.02\ (\text{m})\ \sin\left[2\pi\left(\frac{t}{0.04(\text{s})} - \frac{x}{0.50(\text{m})}\right)\right]$. The tension in the string is

    (a) 4.0 N (b) 12.5 N
    (c) 0.5 N (d) 6.25 N

21. A simple pendulum attached to the ceiling of a stationary lift has a time period T. The distance y covered by the lift moving upwards varies with time t as $y = t^2$ where y is in metre and t in second. If g = 10 ms^{-2}, the time period of the pendulum will be

    (a) $\sqrt{\frac{4}{5}}\,T$ (b) $\sqrt{\frac{5}{6}}\,T$
    (c) $\sqrt{\frac{5}{4}}\,T$ (d) $\sqrt{\frac{6}{5}}\,T$

22. A hollow pipe of length 0.8 m is closed at one end. At its open end a 0.5 m long uniform string is vibrating in its second harmonic and it resonates with the fundamental frequency of the pipe. If the tension in the wire is 50 N and the speed of sound is 320 ms^{-1}, the mass of the string is

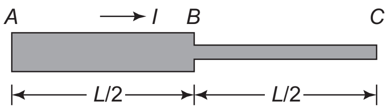
    (a) 5 grams (b) 10 grams
    (c) 20 grams (d) 40 grams

23. This question contains Statement-1 and Statement-2. Of the four choices given after the statements, choose the one that best describes the two statements.
Statement-1: For a charged particle moving from point P to point Q, the net work done by an electrostatic field on the particle is independent of the path connecting point P to point Q.
Statement-2: The net work done by a conservative force on an object moving along a closed loop is zero.

    (a) Statement-1 is true, Statement-2 is true, Statement-2 is not the correct explanation of Statement-1.
    (b) Statement-1 is false, Statement-2 is true.
    (c) Statement-1 is true, Statement-2 is false.
    (d) Statement-1 is true, Statement-2 is true, Statement-2 is the correct explanation of Statement-1.

24. A tiny spherical oil drop carrying a net charge q is balanced in still air with a vertical uniform electric field of strength $\frac{81\pi}{7} \times 10^5$ Vm^{-1}. When the field is switched off, the drop is observed to fall with terminal velocity 2×10^{-3} ms^{-1}. Given $g = 9.8$ ms^{-2}, viscosity of the air = 1.8×10^{-5} Ns m^{-2} and the density of oil = 900 kg m^{-3}, the magnitude of q is
    (a) 1.6×10^{-19} C (b) 3.2×10^{-19} C
    (c) 4.8×19^{-19} C (d) 8.0×10^{-19} C

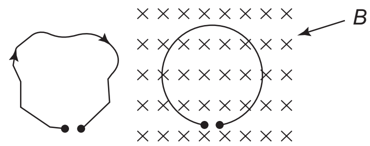
25. Two wires AB and BC, each of length $L/2$ are made of the same material. The radius of wire AB is $2r$ and of wire BC is r. The current I flows through the composite wire (see figure). Choose the correct statement from the following.
    (a) Potential difference across BC is twice that across AB.
    (b) Power dissipated in BC is four times the power dissipated in AB.
    (c) Current densities in AB and BC are equal.
    (d) Electric fields in AB and BC are equal.



26. Two conductors have the same resistance at 0°C but their temperature coefficients of resistance are α_1 and α_2, the respective temperature coefficients of their series and parallel combinations are nearly
    (a) $\frac{\alpha_1+\alpha_2}{2}, \frac{\alpha_1+\alpha_2}{2}$ (b) $\frac{\alpha_1+\alpha_2}{2}, \alpha_1+\alpha_2$
    (c) $\alpha_1+\alpha_2, \frac{\alpha_1+\alpha_2}{2}$ (b) $\alpha_1+\alpha_2, \frac{\alpha_1\alpha_2}{\alpha_1+\alpha_2}$

27. A charged particle moves through a magnetic field perpendicular to its direction. Then
    (a) the momentum changes but the kinetic energy is constant
    (b) both momentum and kinetic energy of the particle are not constant
    (c) both, momentum and kinetic energy of the particle are constant
    (d) kinetic energy changes but the momentum is constant.

28. A thin flexible wire of length L is connected to two adjacent fixed points and carries a current I in the clockwise direction, as shown in the following figure. When the system is put in a uniform magnetic field of strength B going into the plane of the paper, the wire takes the shape of a circle. The tension in the wire is



    (a) IBL (b) $\frac{IBL}{\pi}$
    (c) $\frac{IBL}{2\pi}$ (d) $\frac{IBL}{4\pi}$

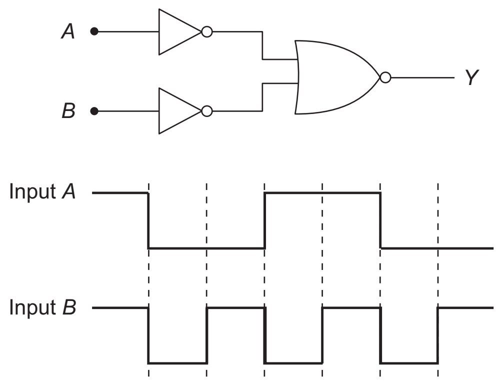
29. An AC voltage source of variable angular frequency ω and fixed amplitude V_0 is connected in series with a capacitance C and an electric bulb of resistance R (inductance zero). When ω is increased
    (a) the bulb glows dimmer
    (b) the bulb glows brighter
    (c) total impedance of the circuit is unchanged
    (d) total impedance of the circuit increases

30. In a series LCR circuit $R = 200\ \Omega$ and the voltage and the frequency of the main supply is 220 V and 50 Hz respectively. On taking out the capacitance from the circuit, the current lags behind the voltage by 30°. On taking out the inductor from the circuit the current leads the voltage by 30°. The power dissipated in the LCR circuit is
    (a) 242 W (b) 305 W
    (c) 210 W (d) Zero W

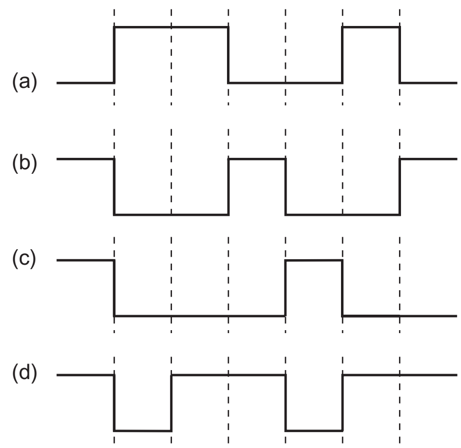
31. The rms value of the electric field of the light comming from the Sun is 720 N/C. The average total energy density of the electromagnetic wave is
    (a) 81.35×10^{-12} J/m^3
    (b) 3.3×10^{-3} J/m^3
    (c) 4.58×10^{-6} J/m^3
    (d) 6.37×10^{-9} J/m^3

32. In an optics experiment, with the position of the object fixed, a student varies the position of a convex lens and for each position, the screen is adjusted to get a clear image of the object. A graph between the object distance u and the image distance v, from the lens, is plotted using the same scale for the two axes. A straight line passing through the origin and making an angle of 45° with the x-axis meets the experimental curve at P. The coordinates of P will be:
    (a) (f, f) (b) $(4f, 4f)$
    (c) $(2f, 2f)$ (d) $\left(\frac{f}{2}, \frac{f}{2}\right)$

33. An astronomical telescope has a large aperture to
    (a) reduce spherical aberration
    (b) have high resolution
    (c) increase span of observation
    (d) have low dispersion
34. A biconvex lens of focal length 15 cm is in front of a plane mirror. The distance between the lens and the mirror is 10 cm. A small object is kept at a distance of 30 cm from the lens. The final image is
    (a) virtual and at a distance of 16 cm from the mirror
    (b) real and at a distance of 16 cm from the mirror
    (c) virtual and at a distance of 20 cm from the mirror
    (d) real and at a distance of 20 cm from the mirror
35. A parallel beam of fast moving electrons is incident normally on a narrow slit. A screen is placed at a large distance from the slit. If the speed of the electrons is increased, which of the following statements is correct?
    (a) Diffraction pattern is not observed on the screen in the case of electrons
    (b) The angular width of the central maximum of the diffraction pattern will increase
    (c) The angular width of the central maximum will decrease
    (d) The angular width of the central maximum will remain the same.
36. The work function of a substance is 4.0 eV. The longest wavelength of light that can cause photoelectron emission from this substance is approximately
    (a) 540 nm (b) 400 nm
    (c) 310 nm (d) 220 nm
37. The half-life of ^{215}At is 100 μs. The time taken for the radioactivity of a sample of ^{215}At to decay to 1/16th of its initial value is
    (a) 400 μs (b) 6.3 μs
    (c) 40 μs (d) 300 μs
38. If a source of power 4 kW produces 10^{20} photons/second, the radiation belongs to a part of the spectrum called
    (a) X-rays
    (b) ultraviolet rays
    (c) microwaves
    (d) γ-rays
39. Suppose an electron is attracted towards the origin by a force $\frac{k}{r}$ where 'k' is a constant and 'r' is the distance of the electron from the origin. By applying Bohr model to this system, the radius of the nth orbital of the electron is found to be 'r_n' and the kinetic energy of the electron to be 'T_n'. Then which of the following is true?
    (a) $T_n \propto \frac{1}{n}, r_n \propto n^2$
    (b) $T_n \propto \frac{1}{n^2}, r_n \propto n^2$
    (c) T_n independent of n, $r_n \propto n$
    (d) $T_n \propto \frac{1}{n}, r_n \propto n$
40. The logic circuit shown in the following figure has the input waveforms 'A' and 'B' as shown. Pick out the correct output waveform from the next figure.



Output is :



Answers

1. (c)	2. (b)	3. (c)	4. (c)
5. (d)	6. (d)	7. (a)	8. (b)
9. (c)	10. (d)	11. (b)	12. (b)
13. (b)	14. (d)	15. (a)	16. (d)

17. (d)	18. (c)	19. (b)	20. (d)
21. (b)	22. (b)	23. (d)	24. (d)
25. (b)	26. (a)	27. (a)	28. (c)
29. (b)	30. (a)	31. (c)	32. (c)
33. (b)	34. (b)	35. (c)	36. (c)
37. (a)	38. (a)	39. (c)	40. (c)

Solutions

1. Value of 1 main scale division = 0.5°
   30 vernier scale divisions
   $= 29 \times$ main scale divisions
   $= 29 \times 0.5°$

   $\therefore$ Value of 1 vernier scale division = $\frac{29}{30} \times 0.5°$

   Least count = value of 1 m.s.d – value of 1 v.s.d.

   $= 0.5° - \frac{29}{30} \times 0.5° = \frac{0.5°}{30} = \frac{0.5}{30} \times 60$ min

   = 1 min

2. RC has dimension of time and V has the dimensions of $L\frac{dI}{dt}$. Hence

   $$\left[\frac{L}{RCV}\right] = \left[\frac{1}{T} \times \frac{T}{A}\right] = \frac{1}{\text{current}}$$

3. Given $v = k\sqrt{x} \Rightarrow v^2 = k^2x$. Differentiating, we have

   $$2v\frac{dv}{dt} = k^2\frac{dx}{dt} = k^2 v \qquad \left(\because v = \frac{dx}{dt}\right)$$

   $$\Rightarrow \quad \frac{dv}{dt} = \frac{k^2}{2}$$

   $$\Rightarrow \quad \int dv = \frac{k^2}{2}\int dt$$

   $$\Rightarrow \quad v = \frac{k^2 t}{2}$$

   $$\Rightarrow \quad \frac{dx}{dt} = \frac{k^2 t}{2}$$

   $$\Rightarrow \quad \int dx = \frac{k^2}{2}\int t\, dt$$

   $$\Rightarrow \quad x = \frac{k^2}{4}t^2$$

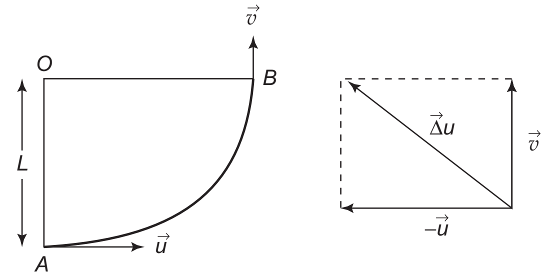
   Thus $x \propto t^2$.

4. From energy conservation

   $$\frac{1}{2}mu^2 = \frac{1}{2}mv^2 + mgL$$

   $$\Rightarrow \quad v = \sqrt{u^2 - 2gL}$$

   $$\Delta v = \sqrt{v^2 + (-u)^2} = \sqrt{2(u^2 - gL)}$$



5. Given $v = \alpha\sqrt{x}$. Since $v = \frac{dx}{dt}$, we have

   $$\frac{dx}{dt} = \alpha\sqrt{x}$$

   or $$\frac{dx}{\sqrt{x}} = \alpha dt$$

   Integrating, we have

   $$\int_0^x \frac{dx}{\sqrt{x}} = \alpha\int_0^t dt$$

   which gives $-2\sqrt{x} = \alpha\, t$ or $x = \frac{\alpha^2 t^2}{4}$. Hence x is proportional to t^2.

6. Given $\vec{v} = K(y\hat{\mathbf{i}} + x\hat{\mathbf{j}})$. Hence

   $v_x = Ky$ and $v_y = Kx$. Therefore,

   $\frac{dx}{dt} = Ky$ and $\frac{dy}{dt} = Kx$.

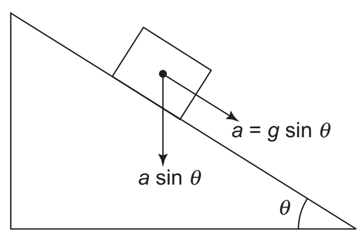
   Now $$\frac{dy}{dx} = \frac{dy/dt}{dx/dt} = \frac{Kx}{Ky} = \frac{x}{y}$$

   $$\Rightarrow \quad y\,dy = x\,dx$$

   Integrating we get $\frac{y^2}{2} = \frac{x^2}{2} + c$

   where c = constant of integration.
   Hence $y^2 = x^2 + 2c$

7. Acceleration along the inclined plane is $a = g\sin\theta$. Vertical component of a is $a\cos(90° - \theta) = a\sin\theta = g\sin^2\theta$. [Refer figure]



For block A, vertical acceleration is $g\sin^2(60°) = \frac{3g}{4}$ and for block B the vertical acceleration is $g\sin^2(30°) = \frac{g}{4}$. Therefore the relative vertical acceleration of A with respect to $B = \frac{3g}{4} - \frac{g}{4} = \frac{g}{2}$ $= 4.9\ \text{ms}^{-2}$.

8. The horizontal component of F parallel to the surface is $F\sin\theta$. Hence maximum value of F is given by

$$F\sin\theta = \mu mg$$

or $$F\sin 60° = 0.5\times\sqrt{3}\times 10$$

or $$F\frac{\sqrt{3}}{2} = 0.5\times\sqrt{3}\times 10$$

which gives $F = 10$ N.

9. Slope of graph $= -\frac{4\text{N}}{3\text{s}} = -\frac{4}{3}\text{Ns}^{-1}$. Therefore,

$$F = -\frac{4}{3}t + 4.$$

Now change in momentum $= \int F\,dt$

$$\Rightarrow \quad mv - 0 = \int_0^{4.5\text{s}}\left(-\frac{4}{3}t + 4\right)dt$$

or $$2v = \left|-\frac{2t^2}{3} + 4t\right|_0^{4.5\text{s}} = 4.5$$

which gives $v = 2.25\ \text{ms}^{-1}$. Therefore,

$$\text{K.E.} = \frac{1}{2}mv^2 = \frac{1}{2}\times 2\times(2.25)^2$$

$$\simeq 5.06\ \text{J}.$$

10. $$U(x) = \frac{a}{x^{12}} - \frac{b}{x^6} \qquad (1)$$

$\therefore \quad U(x=\infty) = 0$

Force $$F = -\frac{dU}{dx} = -\frac{d}{dx}\left(\frac{a}{x^{12}} - \frac{b}{x^6}\right)$$

$$\Rightarrow \quad F = -\left(\frac{12a}{x^{13}} + \frac{6b}{x^7}\right) \qquad (2)$$

At equilibrium $F = 0$. Putting $F = 0$ in Eq. (2) we get

$$x^6 = \frac{2a}{b}$$

Putting $x^6 = \frac{2a}{b}$ in Eq. (1) we get

$$U \text{ at equilibrium} = \frac{a}{\left(\frac{2a}{b}\right)^2} - \frac{b}{\frac{2a}{b}} = -\frac{b^2}{4a}$$

$$\therefore \quad D = [U \text{ at } x = \infty - U \text{ at equilibrium}]$$

$$= 0 - \left(-\frac{b^2}{4a}\right) = \frac{b^2}{4a}.$$

11. Use $mgh = \frac{1}{2}I\omega^2$ and $I = \frac{ml^2}{3}$.

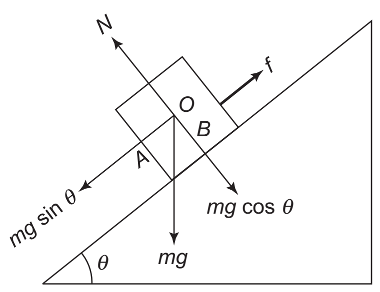
12. The block will just begin to slide if the downward force $mg\sin\theta$ just overcomes the frictional force, i.e. if $mg\sin\theta = \mu N = \mu\, mg\cos\theta \Rightarrow \tan\theta = \mu = \sqrt{3}$ $\Rightarrow \theta = 60°$ [Refer figure]
The block will topple if the torque due to normal reaction N about O just exceeds the torque due to $mg\sin\theta$ about 0, i.e.

$$N\times OA = mg\sin\theta\times OB$$

$$\Rightarrow mg\cos\theta\times 5\text{ cm} = mg\sin\theta\times\frac{15}{2}\text{ cm}$$

$$\Rightarrow \tan\theta = \frac{2}{3} \Rightarrow \theta \simeq 34°.$$

Since θ for toppling is less than θ for sliding, the correct choice is (b).



13. Let x be the extension in the spring when it is loaded with mass M. The change in gravitational potential energy $= Mgx$. This must be the energy stored in the spring which is given by $\frac{1}{2}kx^2$. Thus

$$\frac{1}{2}kx^2 = Mg\,x \text{ or } x = \frac{2Mg}{k}$$

14. $v_e = \sqrt{\frac{GM}{R}}$. The correct choice is (d).

15. Since liquid 1 is above liquid 2, $\rho_1 < \rho_2$. Since the ball sinks in liquid 1, $\rho_3 > \rho_1$. Since the ball floats in liquid 2, $\rho_3 > \rho_2$. Hence $\rho_1 < \rho_3 < \rho_2$, which is choice (a).

16. The terminal velocity is

$$v_t = \frac{2}{9}\frac{(\rho-\sigma)r^2 g}{\eta}$$

The rate of production of heat (or power dissipated) is given by

$$P = fv_t, \text{ where } f = 6\pi\eta r v_t$$

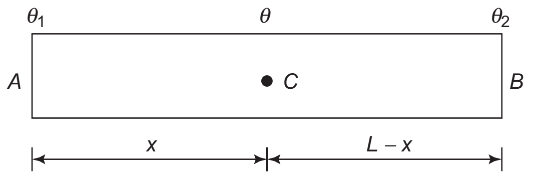
$\therefore$ $$P = 6\pi\eta\, r\, v_t \times v_t$$

$$= 6\,\pi\eta\, r\left[\frac{2}{9}\frac{(\rho-\sigma)r^2 g}{\eta}\right]^2$$

$$= \left[\frac{8\pi g^2}{27\eta}(\rho-\sigma)^2\right] r^5$$

i.e. $P \propto r^5$.

17. Let θ be the temperature at point C at a distance x from end A of the bar whose ends are kept at temperatures θ_1 and θ_2 with $\theta_1 > \theta_2$. L = length of the bar (see following figuer)



In the steady state, the rate of flow of heat from A to C = rate of flow of heat from C to B, i.e.

$$\frac{KA(\theta_1-\theta)}{x} = \frac{KA(\theta-\theta_2)}{(L-x)}$$

$\Rightarrow$ $$\theta = \theta_1 - \left(\frac{\theta_1-\theta_2}{L}\right)x$$

Thus, the graph of θ versus x is a straight line with a positive intercept and a negative slope.

18. Given $PT^2 = k$ (constant). From $PV = nRT$, we have

$P = \dfrac{nRT}{V}$. Hence

$$nRT^3 = kV$$

Differentiating we have

$$3nRT^2\Delta T = k\Delta V \Rightarrow \frac{\Delta V}{\Delta T} = \frac{3nRT^2}{k}$$

Coefficient of volume expansion is

$$\gamma = \frac{\Delta V}{V\Delta T} = \frac{3nRT^2}{kV} \quad (1)$$

Using $V = \dfrac{nRT}{P}$ and $PT^2 = k$ in Eq. (1),

we get $\gamma = \dfrac{3}{T}$.

19. Compare the given equation with

$y = a\sin(\omega t + kx + \phi)$

Wave velocity $v = \dfrac{\omega}{k}$. The correct choice is (b).

20. $v = \sqrt{\dfrac{T}{\mu}} \Rightarrow T = \mu v^2 = \mu\left(\dfrac{\omega}{k}\right)^2$,

μ = mass per unit length of the string. The correct choice is (d).

21. Given $y = t^2$. The velocity of the lift varies with t as

$$v = \frac{dy}{dt} = 2t$$

$\therefore$ Acceleration $a = \dfrac{dv}{dt} = 2\text{ ms}^{-2}$, directed upwards,

Hence

$$T' = 2\pi\sqrt{\frac{l}{g+a}}$$

and $$T = 2\pi\sqrt{\frac{l}{g}}$$

$\therefore$ $$\frac{T'}{T} = \sqrt{\frac{g}{g+a}} = \sqrt{\frac{10}{(10+2)}} = \sqrt{\frac{5}{6}}$$

22. Let l be the length of the string and L the length of the pipe. The frequency of the string vibrating in the second harmonic is

$\nu = \dfrac{2}{2l}\sqrt{\dfrac{T}{\mu}}$; μ = mass per unit length of the string

The fundamental frequency of the closed pipe is

$$\nu' = \frac{v}{4L}\text{ ; } v = \text{speed of sound}$$

For resonance, $\nu = \nu'$, i.e.

$$\frac{1}{l}\sqrt{\frac{T}{\mu}} = \frac{v}{4L}$$

Substituting the values of l, T, v and L, we get

$$\mu = \frac{1}{50}\text{ kg m}^{-1}$$

$\therefore$ Mass of string $= \mu l = \dfrac{1}{50} \times 0.5 = 10^{-2}$ kg = 10 grams. So the correct choice is (b).

23. Electrostatic field is conservative. For a conservative field, the work done to take a charged particle from a point P to another point Q is independent of the path followed by the particle to go from P to Q. In addition, work done to take the particle from Q to P is equal and opposite in sign. Hence the net work done in moving the particle from P to Q and then back to P is zero. So the correct choice is (d).

24. Terminal velocity v is given by

$$6\pi\eta r v = mg = \left(\frac{4\pi r^3}{3}\right)\rho g$$

Substituting the values of η, v, ρ and g, we get $r = \frac{3}{7}\times 10^{-5}$ m. The oil drop will be balanced in air if

$$qE = mg = \left(\frac{4\pi r^3}{3}\right)\rho g$$

Substituting the values of E, r, ρ and g, we get $q = 8.0\times 10^{-19}$ C.

25. $R = \frac{\rho l}{\pi r^2}$. Since the two wires are made of the same material, resistivity ρ is the same for wires AB and BC. Since the wires have equal lengths, it follows that $R \propto 1/r^2$. Hence

$$\frac{R_{AB}}{R_{BC}} = \frac{1}{4}, \text{ i.e } R_{BC} = 4R_{AB}$$

Since the current, is the same in the two wires, it follows from Ohm's law ($V = IR$) that $V_{BC} = 4\ V_{AB}$. Hence choice (a) is wrong. Now power dissipated is $P = I^2 R$. Since I is the same, $P \propto R$. Hence

$$\frac{P_{BC}}{P_{AB}} = \frac{R_{AB}}{R_{BC}} = 4$$

Hence chioce (b) is correct. Choice (c) is wrong because current density (i.e. current per unit area) is different in wires AB and BC because their cross-sectional areas are different. The electric field in a wire is $E = V/l$. Since the two wires have the same length (l), E is proportional to potential difference (V). Since $V_{BC} = 4\ V_{AB}$, $E_{BC} = 4E_{AB}$. Hence choice (d) is also incorrect.

26. The resistance of a conductor at temperature t°C is given by

$$R = R_0(1 + \alpha t)$$

where R_0 is the resistance at 0°C

For series combination

$$R_s = R_1 + R_2$$

At 0°C, $R_s = R_0 + R_0 = 2R_0$

$\therefore\quad 2R_0(1 + \alpha_s t) = R_0(1 + \alpha_1 t), + R_0(1 + \alpha_2 t)$

$\Rightarrow\quad \alpha_s = \frac{1}{2}(\alpha_1 + \alpha_2)$

For parallel combination

$$\frac{1}{R_p} = \frac{1}{R_1} + \frac{1}{R_2}$$

At 0°C, $\frac{1}{R_p} = \frac{1}{R_0} + \frac{1}{R_0} = \frac{2}{R_0} \Rightarrow R_p = \frac{R_0}{2}$

$$\therefore\quad \frac{1}{\frac{R_0}{2}(1+\alpha_p t)} = \frac{1}{R_0(1+\alpha_1 t)} + \frac{1}{R_0(1+\alpha_2 t)}$$

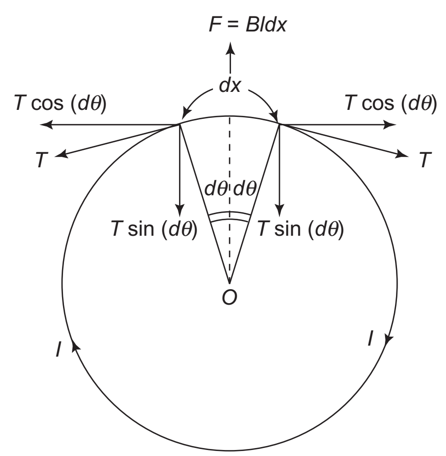
$\Rightarrow\quad 2(1 + \alpha_p t)^{-1} = (1 + \alpha_1 t)^{-1} + (1 + \alpha_2 t)^{-1}$

$\Rightarrow\quad 2(1 - \alpha_p t) = (1 - \alpha_1 t) + (1 - \alpha_2 t)$ $\quad(\because\ \alpha t << 1)$

$\Rightarrow\quad \alpha_p = \frac{1}{2}(\alpha_1 + \alpha_2)$

27. The magnetic force experienced by the charged particle is perpendicular to its velocity. Hence the force does no work on the particle. Therefore, the speed and hence the kinetic energy of the particle remain unchanged. Since the velocity of the particle is perpendicular to the magnetic field, it will move along a circular path in the region of the magnetic field. Therefore, its velocity and hence linear momentum will continuously change due to change in the direction of motion of the particle moving a circle. Hence the correct choice is (a).

28. Consider a small element of the wire of length dx. The horizontal components $T\cos(d\theta)$ cancel each other. The vertical components add up to $2T\sin(d\theta)$ because in the limit $d\theta \to 0$, they are collinear. The magnetic force an element dx is $F = BIdx$ vertically upward. [see following figure]



For equilibrium,

$$2T\sin(d\theta) = BIdx = BIR(2\,d\theta)$$

where R is the radius of the ring. For small $d\theta$, $\sin d\theta = d\theta$.

Hence $\quad 2T\,d\theta = 2BIR\,d\theta$

$\Rightarrow\quad T = BIR = \frac{BIL}{2\pi}\qquad(\because\ L = 2\pi R)$

29. Impedance $Z = \left[R^2 + \frac{1}{(\omega C)^2}\right]^{1/2}$, $I_0 = \frac{V}{Z}$. As ω increases, Z decreases. Hence the current I_0 increases. Therefore, the bulb glows brighter, which is choice (b).

30. In a series LCR circuit, the phase angle ϕ between voltage and current is given by

$$\tan\phi = \frac{\omega L - \frac{1}{\omega C}}{R}$$

If capacitor is taken out, $\tan\phi = \frac{\omega L}{R}$

$\Rightarrow \tan 30° = \frac{\omega L}{R}$. If inductor is taken out,

$\tan\phi = -\frac{1}{\omega CR}$

$\Rightarrow -\tan 30° = -\frac{1}{\omega CR}$. Hence

$$\frac{\omega L}{R} = \frac{1}{\omega CR}$$

$$\Rightarrow \quad \omega L = \frac{1}{\omega C}$$

This is the resonance condition. At resonance, impedance $Z = R$. Hence

$$P = \frac{V^2}{R} = \frac{(220)^2}{200} = 242 \text{ W}$$

31. Total energy density $= \frac{1}{2}\varepsilon_0 E_0^2$

$$= \varepsilon_0 E_{rms}^2 \quad \left[\because E_{rms} = \frac{E_0}{\sqrt{2}}\right]$$

$$= (8.85 \times 10^{-12}) \times (720)^2$$

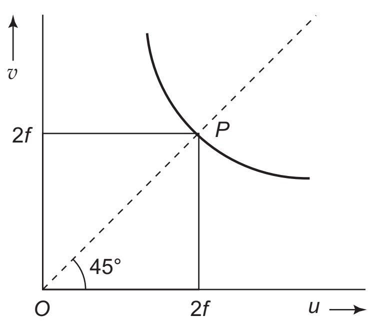
$$= 4.58 \times 10^{-6} \text{ Jm}^{-3}$$

32. For a convex lens forming a real image

$$\frac{1}{v} + \frac{1}{u} = \frac{1}{f} \quad (1)$$

The graph of v against u is as given in the following figure.

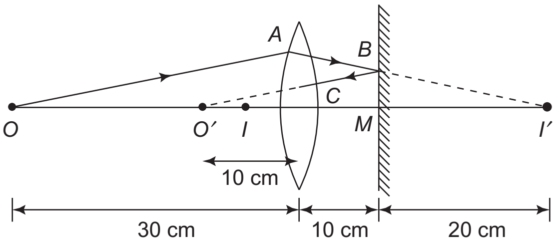
Since the u and v scales are the same, $u = v$ at point P. Putting $u = v$ in Eq. (1) gives $u = 2f$. Also $v = 2f$. Hence the correct choice is (c).



33. Resolving power $= \frac{D}{1.22\lambda}$, where D = diameter (aperture) of the objective of the telescope. Hence the correct choice is (b).

34. For refraction of ray OA by the lens, [see following figure]

$\frac{1}{v'} - \frac{1}{-30} = \frac{1}{15} \Rightarrow v' = 30$ cm



So the image I' is 20 cm from the mirror. After reflection from the mirror, the virtual image O' is such that $O'M = MI'$. So O' is 10 cm to the left of the lens. O' serves as the virtual object for the refraction of ray BC by the lens. For this refraction, $u = +10$ cm. Hence

$$\frac{1}{v} - \frac{1}{u} = \frac{1}{f}$$

$$\Rightarrow \quad \frac{1}{v} - \frac{1}{10} = \frac{1}{15} \Rightarrow v = 6 \text{ cm}$$

Hence the final real image I will be at a distance of $10 + 6 = 16$ cm from the mirror.

35. de Broglie wavelength of electron is

$$\lambda = \frac{h}{p} = \frac{h}{mv}$$

If speed v of electron is increased, momentum $p(= mv)$ will increase. Hence wavelength λ will decrease. Now, the angular width of the central maximum of the diffraction pattern is 2θ where θ is given by

$$\sin\theta = \frac{\lambda}{a}$$

where a is the width of the slit. Thus, if λ decreases, θ and hence 2θ will decrease. Therefore, the correct statement is (c).

36. Use $\lambda_{max} = \frac{hc}{W_0}$. The correct choice is (c).

37. Since $\frac{1}{16} = \frac{1}{2^4}$, the time taken for the sample to decay to $\frac{1}{16}$th of initial value = 4 half lives = $4 \times 100\ \mu s = 400\ \mu s$

38. Power $P = 4 \times 10^3$ W $= 4 \times 10^3$ Js^{-1}
Since the energy of photon is $h\nu$, the power of 10^{20} photons $= 10^{20} \times h\nu$ Js^{-1}. Hence
$10^{20}\, h\nu = 4 \times 10^3$

$$\Rightarrow \qquad \nu = \frac{4\times10^3}{10^{20}\times6.6\times10^{-34}} \simeq 6 \times 10^{16} \text{ Hz}$$

The corresponding wavelength is

$$\lambda = \frac{c}{\nu} = \frac{3\times10^8}{6\times10^{16}} = 5 \times 10^{-9} \text{ m} = 50 \text{ Å}$$

This is the wavelength corresponding to X-rays.

39. If v_n is the speed of the electron in the nth orbit, then

$$\frac{m v_n^2}{r_n} = \frac{k}{r_n}$$

$$\Rightarrow \qquad v_n^2 = \frac{k}{m}$$

$$\therefore \qquad T_n = \frac{1}{2} m v_n^2 = \frac{k}{2},$$ which is independent of n.

Also, angular momentum, $L_n = \dfrac{nh}{2\pi}$

or $\qquad m\, v_n\, r_n = \dfrac{nh}{2\pi}$ which gives

$$r_n = \frac{nh}{2\pi m v_n} = \frac{nh}{2\pi m}\sqrt{\frac{m}{k}} = \frac{nh}{2\pi\sqrt{mk}}$$

Thus $r_n \propto n$. Hence the correct choice is (c).

40. From Figures, it follows that when $A = 0$ and $B = 0$, $Y = 0$, when $A = 0$ and $B = 1$; $Y = 1$, when $A = 1$ and $B = 0$, $Y = 1$ and when $A = B = 1$ then $Y = 1$. This is the truth table of OR gate.