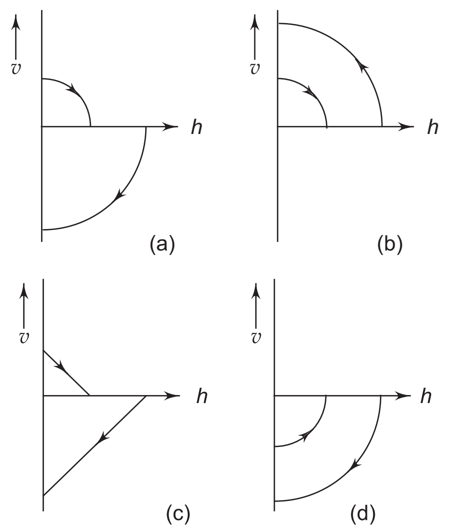
Practice Test Paper – II

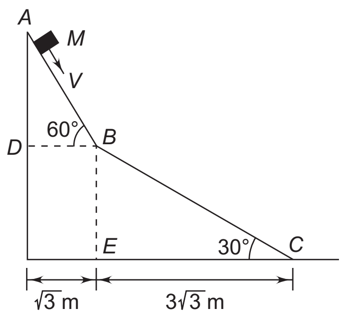
1. A vernier calliper in which 10 divisions of vernier scale coincide with 9 divisions of main scale (each division of the main scale is 1 mm apart) is used to measure the edge length of a cube of mass 4.832 g. If the main scale reading is 12 mm and the 2nd division of vernier scale coincides with main scale, the density of the cube (in g cm^{-3}) up to appropriate significant figure is
   (a) 2.661 (b) 2.66
   (c) 2.67 (d) 2.660
2. A ball is dropped vertically from a height h above the ground. It hits the ground and bounces up vertically to a height $h/2$. Neglecting subsequent motion and air resistance, its velocity v varies with the height h as



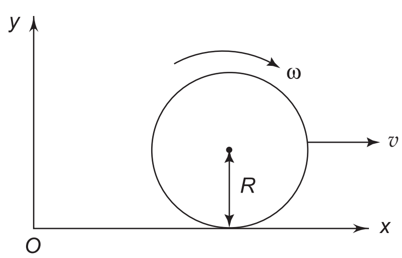
3. The coordinates of a particle moving in a plane are given by
   $$x = a\cos(pt)$$
   and $$y = b\sin(pt)$$
   where a, b and p are positive constants and $b < a$. Then
   (a) the path of the particle is a parabola
   (b) the velocity and acceleration of the particle are perpendicular to each other at $t = \pi/2p$.
   (c) the acceleration of the particle is always directed towards a focus
   (d) the distance travelled by the particle in time interval $t = 0$ to $t = \pi/2p$ is a.
4. When a bicycle is in motion and the pedalling is stopped, the force of friction exerted by the ground acts.
   (a) in the backward direction on both the front and the rear wheels
   (b) in the forward direction on both the front and the rear wheels
   (c) in the forward direction on the front wheel and in the backward direction on the rear wheel
   (d) in the backward direction on the front wheel and in the forward direction on the rear wheel
5. A piece of uniform string hangs vertically so that its free end just touches the horizontal surface of a table. The upper end of the string is now released. At any time during the falling of the string, the total force on the surface of the table is n times the weight of the part of the string lying on the surface. The value of n is
   (a) 1 (b) 2
   (c) 3 (d) 4

Questions 6 and 7 are based on the following paragraph.

A small block of mass M moves on a frictionless surface of an inclined plane, as shown in figure. The angle of the incline suddenly change from 60° to 30° at point B. The block is initially at rest at A. Assume that collisions between the block and the incline are totally inelastic (g = 10 m/s^2)



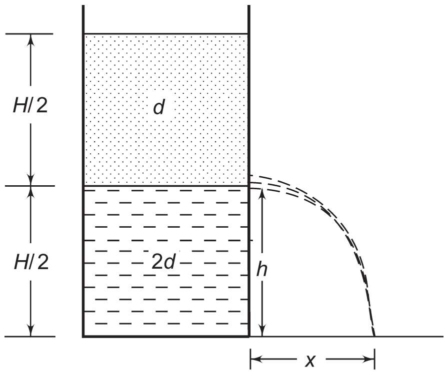
6. The speed of the block at point B immediately after it strikes the second incline is
   (a) $\sqrt{60}$ ms^{-1} (b) $\sqrt{45}$ ms^{-1}
   (c) $\sqrt{30}$ ms$^{\pm 1}$ (d) $\sqrt{15}$ ms$^{\pm 1}$
7. The speed of the block at point C, immediately before it leaves the second incline is
   (a) $\sqrt{120}$ ms^{-1} (b) $\sqrt{105}$ ms^{-1}
   (c) $\sqrt{90}$ ms^{-1} (d) $\sqrt{75}$ ms^{-1}
8. A point mass of 1 kg collides elastically with a stationary point mass of 5 kg. After their collision, the 1 kg mass reverses its direction and moves with a speed of 2 ms^{-1}. Which of the following statements is are correct for the system of these two masses?
   (a) Total momentum of the system is 12 kg ms^{-1}
   (b) Momentum of 5 kg mass after collision is 4 kg ms^{-1}
   (c) Kinetic energy of the centre of mass is 0.75 J
   (d) Total kinetic energy of the system is 4 J
9. A disc of mass M and radius R is rolling with angular speed ω on a horizontal surface as shown in the figure. The magnitude of angular momentum of the disc about the origin O is



   (a) $\frac{1}{2} MR^2 \omega$
   (b) $MR^2 \omega$
   (c) $\frac{3}{2} MR^2 \omega$
   (d) $2 MR^2 \omega$
10. **Statement-1 :** Two cylinders, one hollow (metal) and the other solid (wood) with the same mass and identical dimensions are simultaneously allowed to roll without slipping down an inclined plane from the same height. The hollow cylinder will reach the bottom of the inclined plane first.
    Statement-2 : By the principle of conservation of energy, the total kinetic energies of both the cylinders are identical when they reach the bottom of the incline.
    (a) Statement-1 and 2 are both true and Statement-2 is the correct explanation of Statement-1.
    (b) Statement-1 and 2 are both true but Statement-2 is not the correct explanation of Statement-1.
    (c) Statement-1 is true and Statement-2 is false.
    (d) Statement-2 is true and Statement-1 is false.
11. A satellite of mass m is moving in a circular orbit of radius r around a planet of mass M.
    (a) The magnitude of angular momentum with respect to the centre of the orbit is $m\sqrt{GMr}$, where G is the gravitation constant.
    (b) The magnitude of the angular momentum is $mR\sqrt{2gr}$ where g is the acceleration due to gravity on the surface of the planet.
    (c) The direction of angular momentum is parallel to the plane of the orbit.
    (d) The direction of angular momentum is inclined at 45° to the plane of the orbit.
12. A spring balance A reads 2 kg with a block suspended from it. Another balance B reads 5 kg when a beaker with liquid is put on the pan of the balance. When the block is immersed in water
    (a) Balance A will read 2 kg
    (b) Balance A will read more than 2 kg
    (c) Balance B will read 5 kg
    (d) Balance B will read more than 5 kg.

Questions 13 and 14 are based on the following passage.

A container of large uniform cross-sectional area A, resting on a horizontal surface, holds two immiscible non-viscous and incompressible liquids of densities d and $2d$, each of a height $H/2$ as shown in the figure. The lower density liquid is open to the atmosphere having pressure P_0. A tiny hole of area s ($s << A$) is punched on the vertical side of the container at a height h ($h = H/2$).



13. The initial speed of efflux of the liquid at the hole is

(a) $\sqrt{(3H-2h)\frac{g}{2}}$ (b) $\sqrt{(2H-h)g}$

(c) $\sqrt{(3H-4h)\frac{g}{2}}$ (d) $\sqrt{2gh}$

14. The horizontal distance x travelled by the liquid is

(a) $\sqrt{h(3H-4h)}$ (b) $\sqrt{h(2H-3h)}$

(c) $\sqrt{H(3H-2h)}$ (d) $\sqrt{H(3H-h)}$

Questions 15 and 16 are based on the following passage.

A uniform cylinder of length L and mass M having cross-sectional area A is suspended, with its length vertical, from a fixed point by a massless spring, such that it is half submerged in a liquid of density σ at equilibrium position. When the cylinder is given a small downward push and released, it starts oscillating vertically with a small amplitude.

15. If the cylinder is given a small downward displacement x from the equilibrium position and released, the restoring force F acting on it is

(a) $-Mgx$ (b) $-(k+A\sigma g)x$

(c) $=-\left(k-\frac{1}{2}A\sigma g\right)x$ (d) $-\left(k+\frac{1}{2}A\sigma g\right)x$

16. The time period T of the vertical oscillations of the cylinder is

(a) $2\pi\sqrt{\frac{M}{k}}$

(b) $2\pi\left[\frac{M}{\left(k+\frac{1}{2}A\sigma g\right)}\right]^{1/2}$

(c) $2\pi\left[\frac{M}{(k-A\sigma g)}\right]^{1/2}$

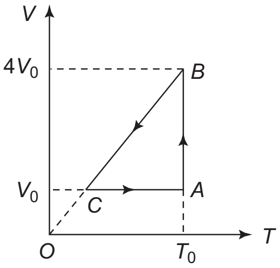
(d) $2\pi\left[\frac{M}{k+A\sigma g}\right]^{1/2}$

17. A stationary source is emitting sound at a fixed frequency f_0, which is reflected by two cars approaching the source. The difference between the frequencies of sound reflected from the cars is 1.2% of f_0. What is the difference in the speeds of the cars? The cars are moving at constant speeds much smaller than the speed of sound which is 330 ms^{-1}.

(a) 0.98 ms^{-1} (b) 1.98 ms^{-1}

(c) 2.98 ms^{-1} (d) 3.98 ms^{-1}

18. One mole of an ideal gas in initial state A undergoes a cyclic process $ABCA$, as shown in the figure. Its pressure at A is P_0.



(a) Internal energy at B is greater than that at A.

(b) Work done by the gas in process $A \rightarrow B$ is $P_0 V_0$

(c) Pressure at C is $\frac{P_0}{4}$

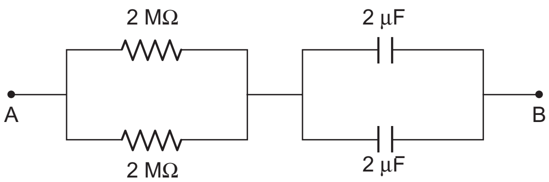
(d) Temperature at C is $\frac{T_0}{2}$

19. Two spherical bodies A (radius 6 cm) and B (radius 18 cm) are at temperature T_1 and T_2, respectively. The maximum intensity in the emission spectrum of A is at 500 nm and in that of B is at 1500 nm. Considering them to be black bodies, what will be the ratio of the rate of total energy radiated by A to that of B?

(a) 3 (b) 6

(c) 9 (d) 12

20. At time $t = 0$, a battery of 10 V is connected across points A and B in the given circuit. If the capacitors have no charge initially, the time t (in seconds) when the voltage across the capacitors become 4V is



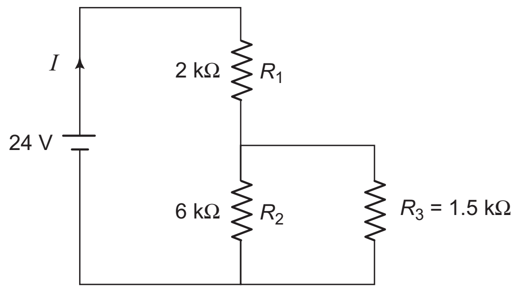
(a) $3 \ln\left(\frac{5}{2}\right)$ (b) $4 \ln\left(\frac{5}{3}\right)$

(c) $5 \ln\left(\frac{4}{3}\right)$ (d) 10 ln (4)

21. In the circuit shown in the figure, the ratio of powers dissipated in R_1 and R_2 is

(a) $\frac{27}{4}$ (b) $\frac{25}{3}$

(c) $\frac{23}{2}$ (d) 9



22. **Statement 1 :** Two particles having equal charges and masses m_1 and m_2, after being accelerated by the same potential difference (V), enter a region of uniform magnetic field and describe circular paths of radii r_1 and r_2 respectively. Then

$$\frac{m_1}{m_2} = \sqrt{\frac{r_1}{r_2}}$$

Statement 2 : Gain in kinetic energy = work done to accelerate the charged particle through potential difference V.

(a) Both statements are true
(b) Both statements are false
(c) Statement-1 is true, and Statement-2 is false.
(d) Statement-1 is false, and Statement-2 is true.

Questions 23 and 24 are based on the following passage.

An *LCR* series circuit with 100 Ω resistance is connected to an a.c. source of 200 V and angular frequency 300 rad/sec. When only the capacitance is removed, the current leads the voltage by 60°. When only the inductance is removed, the current leads the voltage by 60°.

23. The current in the circuit is

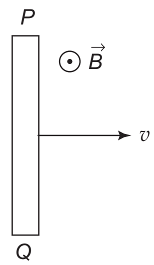
(a) $\sqrt{2}$ A (b) 2 A
(c) $2\sqrt{2}$ A (d) 1 A

24. The power dissipated in the circuit is

(a) 200 W (b) 400 W
(c) 800 W (d) 100 W

25. A metal rod *PQ* moves at a constant vlocity v in a direction perpendicular to its length. A constant uniform magnetic field $\vec{B}$ exists in a direction perpendicular to the rod as well as its velocity as shown in the figure.

(a) There is no electric field in the rod
(b) The electric potential is the same at every point on the rod
(c) There is no induced current in the rod
(d) The induced current flows from P at Q.



26. The radius of curvature of the curved face of a thin planoconvex lens is 10 cm and it is made of glass of refractive index 1.5. A small object is approaching the lens with a speed of 1 cm s^{-1} moving along the principal axis. When the object is at a distance of 30 cm from the lens, the megnitude of the speed of the image is

(a) 1 cm s^{-1} (b) 2 cm s^{-1}
(c) 3 cm s^{-1} (d) 4 cm s^{-1}

27. A glass plate of refractive index μ_3 = 1.5 is coated with a thin layer of thickness t and refractive index μ_2 = 1.8. Light of wavelength λ travelling in air is incident normally on the layer. It is partly reflected at the upper and the lower surfaces of the layer and the two reflected rays interfere. If λ = 648 nm, the least value of t for which the waves interface constructively is

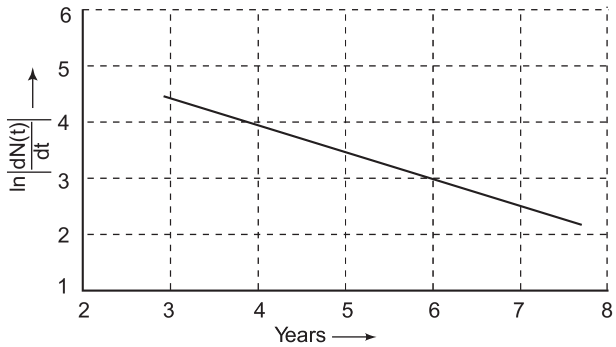
(a) 90 nm (b) 180 nm
(c) 108 nm (d) 216 nm

28. The de Broglie wavelength of an electron moving with a velocity of 1.5×10^8 ms^{-1} is equal to that of a photon. The ratio of the kinetic energy of the photon to that of the electron is

(a) 1 (b) 2
(c) 3 (d) 4

29. To determine the half of radioactive element, a student plots a graph of ln $\left|\frac{dN(t)}{dt}\right|$ versus t. Hence $\frac{dN(t)}{dt}$ is the rate of radioactive decay at time t. If the number of radioactive nuclei of this element decreases by a factor of p after 4.16 years, find the value of p.

(a) 2 (b) 4
(c) 8 (d) 16



30. An electron and a proton are moving on straight parallel paths with same velocity. They enter a semi-infinite region of uniform magnetic field perpendicular to the velocity. Which of the following statement is true ?
    (a) They will never come out of the magnetic field region.
    (b) They will come out travelling along different inclined paths.
    (c) They will come out at the same time.
    (d) They will come out at different times.

Answers

1. (b)	2. (a)	3. (b)	4. (a)
5. (c)	6. (b)	7. (b)	8. (c)
9. (c)	10. (d)	11. (a)	12. (d)
13. (c)	14. (a)	15. (b)	16. (d)
17. (b)	18. (c)	19. (c)	20. (b)
21. (b)	22. (d)	23. (b)	24. (b)
25. (d)	26. (d)	27. (a)	28. (d)
29. (c)	30. (d)		

Solutions

1. Vernier constant (V.C.) = 1 mm – $\left(\frac{9}{10}\right)$ mm. = 0.1 mm.

$$\therefore \quad l = 12 \text{ mm} + 2 \times \text{V.C.} = 12 \text{ mm} + 2 \times 0.1 \text{ mm} = 12.2 \text{ mm} = 1.22 \text{ cm}$$

$$\text{Density } (\rho) = \frac{4.832g}{(1.22\,\text{cm})^3} = 2.661 \text{ g cm}^{-3}$$

Since the least accurate quantity (namely l) has 3 significant figures, the value of ρ is rounded off to 3 significant figures. Hence $\rho = 2.66$ g cm^{-3}

2. The velocity at a height h is given by $v^2 = u^2 + 2gh$. For downward motion, $u = 0$ and the value of g is negative and h becomes more and more negative. Hence v^2 increases with h. Since the velocity vector is directed downwards, v becomes more and more negative. Since $v^2 \propto h$, the graph of v versus h is parabolic. Hence graphs (c) and (d) are wrong. For upward motion, $v^2 = u^2 + 2gh$. Here g is directed downwards and h is positive. Consequently, v^2 decreases with h. Since the direction of the velocity vector is positive, v becomes less and less positive. Here also the variation of v with h is parabolic. Since v becomes less and less positive, graph (b) is not correct. Hence the correct choice is (a).

3. Given $\quad x = a\cos(pt) \quad (1)$

$\qquad y = b\sin(pt) \quad (2)$

From (1) and (2) $\frac{x^2}{a^2} + \frac{y^2}{b^2} = \cos^2(pt) + \sin^2(pt) = 1$

Hence the path of the particle is an ellipse.

Let the position vector of the particle at time t be

$$\vec{r} = x\hat{i} + y\hat{j}$$

$$\therefore \text{ Velocity } \vec{v} = \frac{d\vec{r}}{dt} = \frac{d\vec{x}}{dt}\hat{i} + \frac{d\vec{y}}{dt}\hat{j}$$

$$\Rightarrow \quad \vec{v} = -ap\sin(pt)\hat{i} + bp\cos(pt)\hat{j}$$

$$\text{At } t = \pi/2p, \; \vec{v} = -ap\sin\left(\frac{\pi}{2}\right)\hat{i} + bp\cos\left(\frac{\pi}{2}\right)\hat{j} = -ap\hat{i}$$

$$\text{Acceleration } \vec{a} = -ap^2\cos(pt)\hat{i} - bp^2\sin(pt)\hat{j}$$

$$\text{At } \quad t = \pi/2p, \; \vec{a} = -bp^2\hat{j}$$

$$\therefore \text{ At } \quad t = \pi/2p, \; \vec{v}\cdot\vec{a} = abp^3\left(\hat{i}\cdot\hat{j}\right) = 0$$

Hence $\vec{v} \perp \vec{a}$

It is easy to see that choice (c) and (d) are incorrect.

4. When an external force is applied to move a body, the force of friction acts in the opposite direction. But when a body itself applies a force in order to move, the force of friction acts in the direction of motion. While pedalling, the external force is applied to the rear wheel and as a result the front wheel moves by itself. So, while pedalling, the force of friction by the ground acts in the backward direction on the forntwheel and in the forward direction on the rear wheel. When pedalling is stopped, the force of friction by the ground acts in the backward direction on both the front and the rear wheels as long as bicycle remains in motion. So the correct choice is (a).

5. Let x be the length of the string lying on the surface on the table at an instant of time t. If an additional length dx of the string falls on the surface in time dt, the velocity v of this element when it strikes the surface is given by ($\because u = 0$)

$$v^2 = u^2 + 2gx = 0 + 2gx$$

or $$v^2 = 2gx \qquad (1)$$

The total force on the surface is

F = rate of change of momentum of element of length dx + weight of a length x of the string lying on the table.

If m is the mass per unit length of the string, then

$$F = \frac{d}{dt}(mdxv) + mxg = mv\frac{dx}{dt}$$
$$+ mxg = mv^2 + mxg \qquad (2)$$
$$\left(\because v = \frac{dx}{dt}\right)$$

Using (1) in (2) we get

$$F = 2\,mxg + mxg = 3\,mxg$$

But $mx = M$, the mass of the string lying on the table. Hence

$$F = 3\,Mg$$

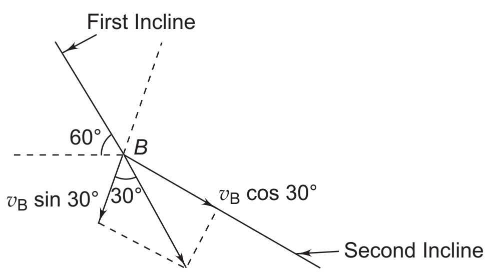
Thus $n = 3$.

6. Let v_B be the speed of the block just before it strikes the second incline.

$$\frac{1}{2}mv_B^2 = mg \times AD = mg \times BD \tan 60°$$

$$\Rightarrow v_B = (2 \times 10 \times \sqrt{3}\tan 60°)^{1/2} = \sqrt{60}\ \text{ms}^{-1}$$

This velocity can be resolved into two components. $v_B \cos 30°$ along the second incline and $v_B \sin 30°$ perpendicular to it (see the following figure)



In an inelastic collision, the perpendicular component becomes zero after the collision. Hence the speed of the block at point B immediately after the collision is $v_B \cos 30° = \sqrt{60} \times \frac{\sqrt{3}}{2} = \sqrt{45}\ \text{ms}^{-1}$.

7. Let v_C be the speed at C. From conservation of energy, gain in K.E. = loss in P.E., i.e.

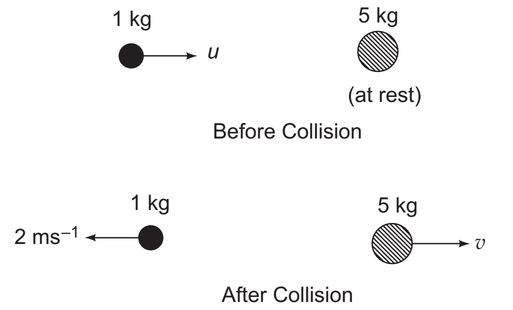
$$\frac{1}{2}mv_C^2 - \frac{1}{2}mv_B^2 = mg \times BE = mg \times EC \tan 30°$$

$$\Rightarrow \quad v_C^2 = v_B^2 + 2g \times 3\sqrt{3}\ \tan 30°$$

$$= 45 + 2 \times 10 \times 3\sqrt{3} \times \frac{1}{\sqrt{3}} = 105$$

$$\Rightarrow \quad v_C = \sqrt{105}\ \text{ms}^{-1}.$$

8.



From conservation of linear momentum,

$$u = 5v - 2$$
$$\Rightarrow \quad u + 2 = 5v \qquad (1)$$

From conservation of kinetic energy,

$$u^2 = 4 + 5v^2$$
$$\Rightarrow \quad (u - 2)(u + 2) = 5v^2 \qquad (2)$$

From Eqs. (1) and (2), we get

$$u - 2 = v \qquad (3)$$

Solving Eqs. (1) and (3), we get $u = 3\ \text{ms}^{-1}$ and $v = 1\ \text{ms}^{-1}$.

(a) Total momentum = 1 kg × 3 ms^{-1} = 3 kg ms^{-1}

(b) Momentum of 5 kg mass after collision = 5 kg × 1 ms^{-1} = 5 kg ms^{-1}

(c) Velocity of centre of mass is

$$v_{CM} = \frac{1 \times u + 5 \times 0}{1 + 5} = \frac{u}{6} = \frac{1}{2}\ \text{ms}^{-1}$$

Kinetic energy of centre of mass is

$$K_{CM} = \frac{1}{2}(1 + 5) \times \left(\frac{1}{2}\right)^2 = 0.75\ \text{J}$$

(d) Total kinetic energy of the system is

$$K = \frac{1}{2} \times 1 \times (3)^2 + 0 = 4.5\ \text{J}$$

So the correct choice is (c).

9. The angular momentum about O is

$$\vec{L}_O = \vec{L}_{CM} + M(\vec{R} \times \vec{v})$$

Its magnitude is $(\because \vec{R} \perp \vec{v})$ and $L_{CM} = I\omega$

$$L_O = I\omega + MRv$$

$$= \left(\frac{1}{2}MR^2\right)\omega + MR \times R\omega \qquad (\because v = R\omega)$$

$$= \frac{3}{2}MR^2\omega$$

10. The acceleration of the centre of mass is

$$a = \frac{g\sin\theta}{\left(1+\dfrac{I}{mR^2}\right)}$$

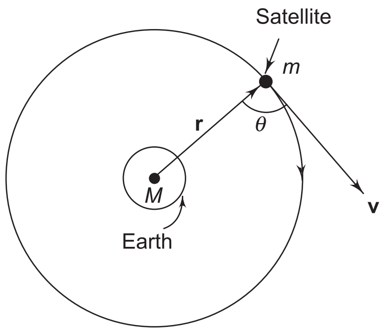
For a hollow cylinder $I_h = mR^2$

For a solid cylinder $I_s = \frac{1}{2}mR^2$

Since $I_h > I_s$, $a_h < a_s$. Hence the solid cylinder will reach the bottom before the hollow cylinder. From the principle of conservation of energy, kinetic energy at the bottom = potential energy (= mgh) which is the same for both the cylinders.

11. At a certain instant of time let r be the radius vector of the satellite from the centre of its circular orbit. If the velocity of the satellite is **v** as shown in the figure, its angular momentum is given by

$$\mathbf{L} = \mathbf{r} \times (m\mathbf{v})$$



The magnitude of angular momentum is

$$L = mrv \sin\theta$$

where θ is the angle between vectors **r** and **v**. For a circular orbit, $\theta = 90°$. Therefore

$$L = mrv \qquad (1)$$

The gravitational force of attraction on the satellite is GMm/r^2 which provides the necessary centripetal force mv^2/r, i.e.

$$\frac{GMm}{r^2} = \frac{mv^2}{r}$$

or $\quad (mrv) = m\sqrt{GMr} \qquad (2)$

From Eqs. (1) and (2), we have

$$L = m\sqrt{GMr}$$

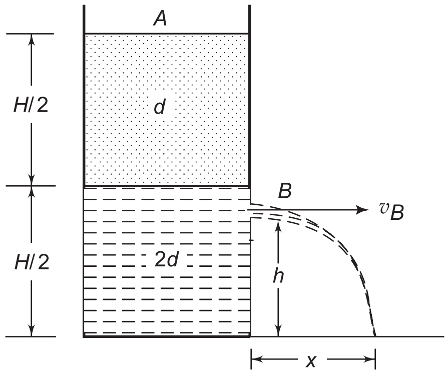
This gives the magnitude of angular momentum. The direction of angular momentum is perpendicular to the plane of the orbit.

12. Due to the upward force of buoyancy on the block exerted by the liquid, the apparent weight of the block will be less than 2 kg. The hanging block exerts a downward force on the liquid (and the beaker) equal in magnitude to the upward buoyant force. Therefore, balance B will read more than 5 kg.

13. The initial speed of efflux of the liquid at the hole at point B can be determined by applying Bernoulli's theorem at the top (point A) of the liquids and at point B where the hole is punched. If v_A and v_B are the speeds of the liquid at points A and B respectively, then from the equation of continuity, we have

$$Av_A = sv_B$$

or $\quad v_A = \left(\frac{s}{A}\right) v_B \approx 0 \; (\because s << A)$



Applying Bernoulli' theorem at points A and B, we get

$$P_A + \frac{1}{2} + dv_A^2 + dg\left(\frac{H}{2}\right) + (2d)g\left(\frac{H}{2}\right)$$

$$= P_B + \frac{1}{2}(2d)v_B^2 + (2d)gh$$

Now $P_A = P_B$ = atmospheric pressure (P_0) and $v_A \approx 0$. Hence, we get

$$P_0 + 0 + \frac{1}{2}dgH + dgH = P_0 + dv_B^2 + 2dgh$$

Which gives $v_B = \left[\left(\frac{3}{2}H - 2h\right)g\right]^{1/2}$

14. The time taken by the liquid to fall through a vertical height h is given by

$$t = \sqrt{\frac{2h}{g}}$$

The horizontal distance x travelled by the liquid in time t moving with a constant horizontal velocity v_B is

$$x = v_B t = \left[\left(\frac{3}{2}H - 2h\right)g\right]^{1/2} \times \left(\frac{2h}{g}\right)^{\frac{1}{2}}$$

$$= [h\,(3H - 4h]^{1/2} \qquad (1)$$

15. The upthrust on the cylinder with half its length submerged in the liquid is given by

$$U = \text{weight of the liquid displaced by a length } L/2 \text{ of the cylinder}$$

$$= A \times \frac{L}{2} \times \sigma \times g = \frac{L}{2}(A\sigma g)$$

Let x_0 be the extension of the spring when it is in equilibrium. Then

$$kx_0 = Mg - \frac{L}{2}(A\sigma g) \quad (1)$$

Let x be the small downward displacement given to the cylinder so that the submerged length of the cylinder is now $\left(\frac{L}{2}+x\right)$ and the extension of the spring is now $(x_0 + x)$. The upthrust now is $\left(\frac{L}{2}+x\right)$ $A\sigma g$ and the force in the spring is $k(x_0 + x)$. Hence, the restoring force on the cylinder is

$$F = -\left[k(x_0 + x) - \left\{Mg - \left(\frac{L}{2}+x\right)A\sigma g\right\}\right] \quad (2)$$

Using Eq. (1) in Eq. (2), we have

$$F = -(kx + A\sigma g x)$$

or $$F = -(k + A\sigma g)x$$

16. The acceleration of the cylinder is

$$a = \frac{F}{M} = -\left(\frac{k + A\sigma g}{M}\right)x \quad (3)$$

Comparing Eq. (3) with $a = -\omega^2 x$, where $\omega = \frac{2\pi}{T}$, we find that the correct choice is (d).

17. If v is the speed of sound and u_1 and u_2 are the speeds of the cars, the difference between the frequencies of sound reflected from the cars is

$$\Delta f = f_0\left(\frac{v+u_1}{v-u_1}\right) - f_0\left(\frac{v+u_2}{v-u_2}\right)$$

$$= f_0\left[\left(1+\frac{u_1}{v}\right)\left(1-\frac{u_1}{v}\right)^{-1} - \left(1+\frac{u_2}{v}\right)\left(1-\frac{u_2}{v}\right)^{-1}\right]$$

$$= f_0\left[\left(1+\frac{2u_1}{v}\right) - \left(1+\frac{2u_2}{v}\right)\right] \quad (\because u_1, u_2 << v)$$

$$= \frac{2f_0}{v}(u_1 - u_2)$$

Given $\Delta f = \frac{1.2 f_0}{100}$. Thus

$$\frac{1.2 f_0}{100} = \frac{2f_0}{330}(u_1 - u_2)$$

$$\Rightarrow (u_1 - u_2) = \frac{330 \times 1.2}{200} = 1.98 \text{ ms}^{-1}$$

18. Process $A \rightarrow B$ is isothermal. Hence internal energy at A = internal enrgy at B

$$\therefore W_{A\rightarrow B} = mRT_0 \ln\left(\frac{4V_0}{V_0}\right) = P_0 V_0 \ln(4)$$

Also $P_A V_A = P_B V_B \Rightarrow P_B = P_A\left(\frac{V_A}{V_B}\right) = \frac{P_0}{4}$

Since the line BC passes through the origin of V-T graph, $V \propto T$. So process $B \rightarrow C$ is isobaric. Hence

$$P_C = P_B = \frac{P_0}{4}$$

Also $\frac{V_C}{T_C} = \frac{V_B}{T_B} \Rightarrow T_C = \frac{T_0}{4}$

Thus the only correct choice is (c).

19. From Wien's displacement law $\lambda_m T$ = constant, we have

$$\lambda_A T_A = \lambda_B T_B \Rightarrow \frac{T_A}{T_B} = \frac{\lambda_B}{\lambda_A}$$

From Stefan's law, $E = \sigma A T^4 = \sigma(4\pi R^2)T^4$, we have

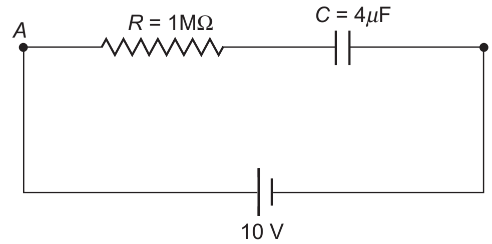
$$\frac{E_A}{E_B} = \left(\frac{R_A}{R_B}\right)^2 \times \left(\frac{T_A}{T_B}\right)^4$$

$$= \left(\frac{R_A}{R_B}\right)^2 \times \left(\frac{\lambda_B}{\lambda_A}\right)^4$$

$$= \left(\frac{6}{18}\right)^2 \times \left(\frac{1500}{50}\right)^4$$

$$= 9$$

20. Equivalent resistance is $R = 1\ \text{M}\Omega = 10^6\ \Omega$ and equivalent capacitance is C = 4 μF = 4×10^{-6} F.



$$Q = Q_0(1 - e^{-t/RC})$$

$$CV = CV_0(1 - e^{-t/RC})$$

$$\Rightarrow V = V_0(1 - e^{t/4}) \quad (\because RC = 4)$$

Given V = 4 V and V_0 = 10 V. Therefore

$$4 = 10\,(1 - e^{-t/4})$$

$$e^{-t/4} = \frac{3}{5} \Rightarrow e^{t/4} = \frac{5}{3}$$

$$\Rightarrow \frac{t}{4} = \ln\left(\frac{5}{3}\right) \Rightarrow t = 4\ln\left(\frac{5}{3}\right)$$

21. Equivalent resistance $R = \dfrac{6 \times 1.5}{6 + 1.5} + 2$

$$= 3.2\text{ k}\,\Omega$$

$$= 3.2 \times 10^3\,\Omega$$

Current $I = \dfrac{24}{3.2 \times 10^3} = 7.5 \times 10^{-3}\text{ A} = 7.5\text{ mA}$

Potential difference across

$R_1 = (2 \times 10^3) \times (7.5 \times 10^{-3}) = 15\text{ V}$

$\therefore$ Potential difference across

$R_2 =$ p.d. across $R_L = 24 - 15 = 9\text{V}$

$$\frac{\text{Power dissipated in } R_1}{\text{Power dissipated in } R_2} = \frac{(15)^2/2 \times 10^3}{(9)^2/6 \times 10^3} = \frac{25}{3}$$

22. Kinetic energy $K = qV$.

Therefore $r_1 = \dfrac{\sqrt{2m_1qV}}{qB}$ and $r_2 = \dfrac{\sqrt{2m_2qV}}{qB}$

Hence $\dfrac{r_1}{r_2} = \sqrt{\dfrac{m_1}{m_2}} \Rightarrow \dfrac{m_1}{m_2} = \left(\dfrac{r_1}{r_2}\right)^2$.

Statement-1 is false and Statement-2 is true.

23. When capacitance is removed, the circuit contains only inductance and resistance. Phase difference θ between the current and voltage is then given by

$$\tan\theta = \frac{\omega L}{R} \text{ or } \omega L = R\tan\theta$$

$$= 100\tan 60°$$

When the circuit contains only capacitance and resistance, the phase difference between the voltage and current is given by

$$\tan\phi = \frac{1}{RC\omega}$$

$$\therefore \quad \frac{1}{C\omega} = R\tan\phi = 100\tan 60°$$

The impedance of the LCR circuit is given by

$$Z = \sqrt{R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2}$$

$$= \sqrt{R^2 + (100\tan 60° - 100\tan 60°)^2}$$

$$= R = 100\,\Omega.$$

The current is given by

$$I = \frac{V}{R} = \frac{200}{100} = 2\text{ A}$$

24. The power dissipated in the circuit is

$$P = I^2R = 4 \times 100 = 400\text{ W}$$

25. From Fleming‘s left hand rule, the free electrons experience a force from Q to P. As a result a current flows from P to Q. Also end Q acquires a positive charge (due to deficiency of electrons) and end P acquires a negative charge (due to gain of electrons). Hence an electric field exists in the rod in the direction Q to P.

26. $\dfrac{1}{f} = (\mu - 1) \times \dfrac{1}{R} = (1.5 - 1) \times \dfrac{1}{10}$ gives $f = 20$ cm.

Lens formula is (here $u = -30$ cm and $f = +20$ cm)

$$\frac{1}{v} - \frac{1}{u} = \frac{1}{f} \quad (1)$$

or $\dfrac{1}{v} - \dfrac{1}{-30} = \dfrac{1}{20}$ which gives $v = 60$ cm

Differentiating (1) with respect to time t, we get

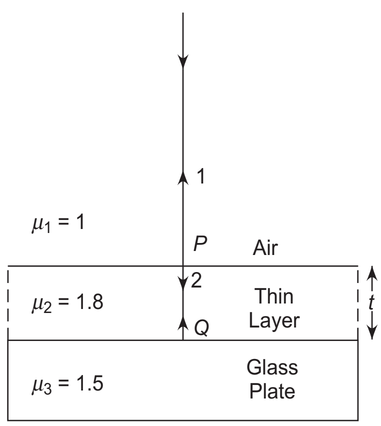
$$-\frac{1}{v^2}\frac{dv}{dt} + \frac{1}{u^2}\frac{du}{dt} = 0$$

or $\dfrac{dv}{dt} = \left(\dfrac{v^2}{u^2}\right)\dfrac{du}{dt}$

or Speed of image $= \left(\dfrac{v^2}{u^2}\right) \times$ speed of object

$$= \left(\frac{60}{30}\right)^2 \times 1 = 4\text{ cm s}^{-1},$$

27. Refer to the following figure. A ray of light travelling in air ($\mu_1 = 1$) falls normally on a thin layer ($\mu_2 = 1.8$) of thickness t. It is partly reflected at point P as wave1 and partly refracted as wave 2. Wave 2 on meeting the surface of the glass plate ($\mu_3 = 1.5$) is reflected at point Q and travels along QP.



Waves 1 and 2 meet at point P where they interfere. We know that when a wave is travelling in a rarer medium and gets reflected at the boundary of a denser medium, it undergoes a phase change of π or a path change of $\lambda/2$. Thus wave 1 has an optical path of $\Delta_1 = \lambda/2$. Wave 2 travelling from P to Q in the layer of refractive index 1.8 gets reflected at Q from the boundary of glass of refractive index 1.5. Thus wave 2 travelling in a denser medium is reflected from the boundary of a rarer medium undergoes no phase change due to reflection. Therefore, Optical path for wave 2 from P to Q and from Q to P in the layer is

$$\Delta_2 = \text{refractive index of layer} \times 2(PQ)$$
$$= \mu_2 \times 2t = 2\mu_2 t$$

$\therefore$ Optical path difference between waves 1 and 2 at point p is

$$\Delta = \Delta_2 - \Delta_1 = 2\mu_2 t - \frac{\lambda}{2}$$

Now, for constructive interference, $\Delta = n\lambda$; $n = 0, 1, 2, \ldots$

or $\quad 2\mu_2 t - \dfrac{\lambda}{2} = n\lambda$ or $2\mu_2 t = \left(n + \dfrac{1}{2}\right)\lambda$

or $\quad t = \dfrac{\left(n + \frac{1}{2}\right)\lambda}{2\mu_2}$

The minimum value of t corresponds to $n = 0$. Hence

$$t_{\min} = \frac{\lambda}{4\mu_2} = \frac{648 \text{ nm}}{4 \times 1.8} = 90 \text{ nm}.$$

28. Speed of photon (c) = 3×10^8 ms^{-1}. Let λ be the wavelength of the photon. The de Broglie wavelength of the electron = $\dfrac{h}{mv}$

Given $\quad \lambda = \dfrac{h}{mv}$. Now

$$\frac{\text{K.E.of photon}}{\text{K.E.of electron}} = \frac{h\nu}{\frac{1}{2}mv^2} = \frac{2hc}{mv^2\lambda} \qquad \left(\because \nu = \frac{c}{\lambda}\right)$$

$$= \frac{2c}{v} \qquad \left(\because \lambda = \frac{h}{mv}\right)$$

$$= \frac{2 \times 3 \times 10^8}{1.5 \times 10^8} = 4$$

29. $N = N_0 e^{-\lambda t} \Rightarrow \dfrac{dN}{dt} = -\lambda N_0 e^{-\lambda t}$.

Therefore $\left|\dfrac{dN}{dt}\right| = \lambda N_0 e^{-\lambda t}$

$\Rightarrow \quad \ln \left|\dfrac{dN}{dt}\right| = \ln(\lambda N_0) - \lambda t$

The slope of the graph is = $\dfrac{4-3}{4-6} = -\dfrac{1}{2}$

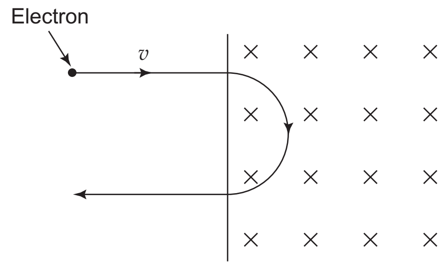
$\therefore \quad \lambda = \dfrac{1}{2}$ per year

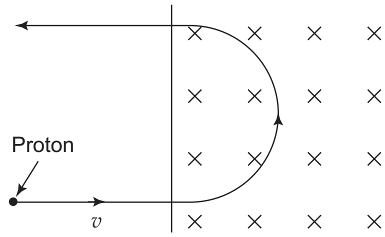
Half left $T_{1/2} = \dfrac{0.693}{1/2} = 1.386$ year

$\therefore \quad$ Number of half lives = $\dfrac{4.16}{1.386} = 3$

Hence $p = (2)^3 = 8$

30. The electron and the proton will come out travelling along parallel paths after completing their semicircles as shown in the following figures.





Since $\quad r = \dfrac{mv}{qB}$, $r_p > r_e$

The time after which a charged particle comes out is given by

$$t = \frac{\pi m}{qB}$$

Since $m_p > m_e$ and q and B are the same, $t_p > t_e$.