Model Test Paper — II

SECTION I

(Single Correct Answer Type)

This section contains 13 multiple choice questions. Each question has four choices (a), (b), (c) and (d) out of which **ONLY ONE** is correct.

1. In the determination of Young's modulus $\left(Y = \dfrac{4MLg}{\pi \ell d^2}\right)$ by using Searle's method, a wire of length $L = 2$ m and diameter $d = 0.5$ mm is used. For a load $M = 2.5$ kg, an extension $\ell = 0.25$ mm in the length of the wire is observed. Quantities d and ℓ are measured using a screw gauge and a micrometer, respectively. They have same pitch of 0.5 mm. The number of divisions on their circular scale is 100. The contributions to the maximum probable error of the Y measurement.

(a) due to the errors in the measurement of d and ℓ are the same.
(b) due to the error in the measurement of d is twice that due to the error in the measurement of ℓ.
(c) due to the error in the measurement of ℓ is twice that due to the error in the measurement of d.
(d) due to the error in the measurement of d is four times that due to the error in the measurement of ℓ.

2. A mixture of 2 moles of helium gas (atomic mass = 4 amu) and 1 mole of argon gas (atomic mass = 40 amu) is kept at 300 K in a container. The ratio of the rms speeds $\left(\dfrac{v_{rms}(\text{helium})}{v_{rms}(\text{argon})}\right)$ is

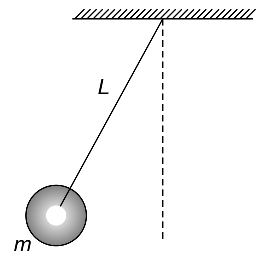
(a) 0.32 (b) 0.45
(c) 2.24 (d) 3.16

3. Two large vertical and parallel plates having a separation d are connected to a battery of voltage V. A particle of charge q is released at rest between the two plates. It is found to move at an angle θ to the vertical just after release. Then V is given by

(a) $\dfrac{mgd \sin\theta}{q}$ (b) $\dfrac{mgd \cos\theta}{q}$

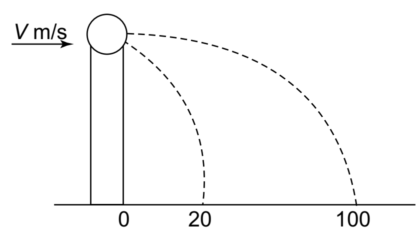
(c) $\dfrac{mgd \tan\theta}{q}$ (d) $\dfrac{mgd}{q}$

4. A ball of mass (m) 0.5 kg is attached to the end of string having (L) 0.5 m. The ball is rotated on a horizontal circular path about vertical axis. The maximum tension that the string can bear is 324 N. The maximum possible value of angular velocity of ball (in radian/s) is



(a) 9 (b) 18
(c) 27 (d) 36

5. A ball of mass 0.2 kg rests on a vertical post of height 5 m. A bullet of mass 0.01 kg, travelling with a velocity V m/s in a horizontal direction, hits the centre of the ball. After the collision, the ball and bullet travel independently. The ball hits the ground at a distance of 20 m and the bullet at a distance of 100 m from the foot of the post. The velocity V of the bullet is

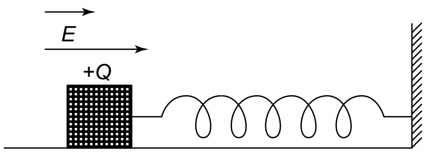


(a) 250 m/s (b) 250 m/s
(c) 400 m/s (d) 500 m/s

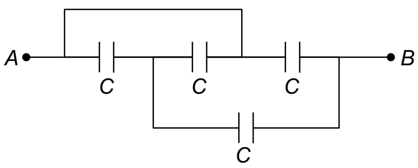
6. A wooden block performs SHM on a frictionless surface with frequency, v_0. The block carries a charge $+Q$ on its surface. If now a uniform electric field $\vec{E}$ is switched-on as shown in the figure then the SHM of the block will be

(a) of the same frequency and with shifted mean position.

(b) of the same frequency and with the same mean position.
(c) of changed frequency and with shifted mean position.
(d) of changed frequency and with the same mean position.



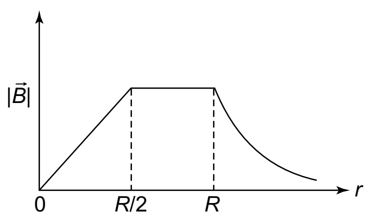
7. In the network shown in the figure, each capacitor has capacitance C. The equivalent capacitance between A and B is

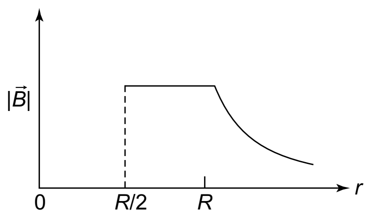


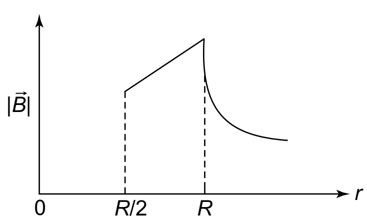
(a) C (b) $\frac{2C}{3}$

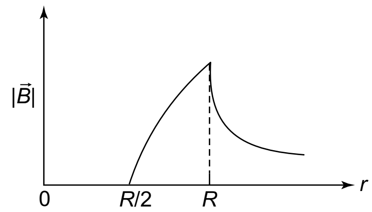
(c) $\frac{4C}{3}$ (d) $\frac{5C}{3}$

8. An infinitely long hollow conducting cylinder with inner radius $R/2$ and other radius R carries a uniform current density along its length. The magnitude of the magnetic field, $|\vec{B}|$ as a function of the radial distance r from the axis is best represented by

(a)


(b)


(c)


(d)


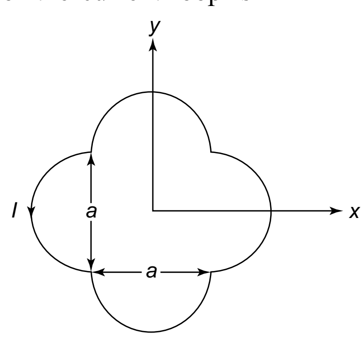
9. A student is performing the experiment of resonance column. The diameter of the column tube is 4 cm. The frequency of the tuning fork is 512 Hz. The air temperature is 38°C in which the speed of sound is 336 m/s. The zero of the meter scale coincides with the top end of the Resonance Column tube. When the first resonance occurs, the reading of the water level in the column is

(a) 14.0 cm (b) 15.2 cm
(c) 16.4 cm (d) 17.6 cm

10. Young's double slit experiment is carried out by using green, red and blue light, one colour at a time. The fringe widths recorded are β_G, β_R and β_B respectively. Then,

(a) $\beta_G > \beta_B > \beta_R$ (b) $\beta_B > \beta_G > \beta_R$
(c) $\beta_R > \beta_B > \beta_G$ (d) $\beta_R > \beta_G > \beta_B$

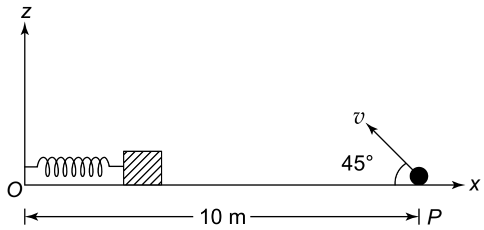
11. A loop carrying current I lies in the x-y plane as shown in the figure. The unit vector $\hat{k}$ is coming out of the plane of the paper. The magnetic moment of the current loop is



(a) $a^2 I\hat{k}$ (b) $\left(\frac{\pi}{2}+1\right)a^2 I\hat{k}$

(c) $-\left(\frac{\pi}{2}+1\right)a^2 I\hat{k}$ (d) $(2\pi+1)a^2 I\hat{k}$

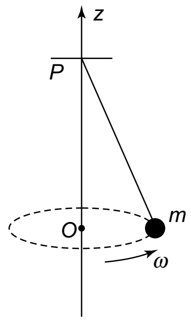
12. A small block is connected to one end of a massless spring of un-stretched length 4.9 m. The other end of the spring (see the figure) is fixed. The system lies on a horizontal frictionless surface. The block is stretched by 0.2 m and released from rest at $t = 0$. It then executes simple harmonic motion with angular frequency $\omega = \pi/3$ rad/s. Simultaneously at $t = 0$, a small pebble is projected with speed v from point P at an angle of 45° as shown in the figure. Point P is at a horizontal distance of 10 m from O. If the pebble hits the block at $t = 1$ s, the value of v is (take g = 10 m/s^2)



(a) $\sqrt{50}$ m/st (b) $\sqrt{51}$ m/s

(c) $\sqrt{52}$ m/s (d) $\sqrt{53}$ m/s

13. A small mass m is attached to a massless string whose other end is fixed at P as shown in the figure. The mass is undergoing circular motion in the x-y plane with centre at O and constant angular speed ω. If the angular momentum of the system, calculated about O and P are denoted by $\vec{L}_O$ and $\vec{L}_P$ respectively, then



(a) $\vec{L}_O$ and $\vec{L}_P$ do not very with time.
(b) $\vec{L}_O$ varies with time while $\vec{L}_P$ remains constant.
(c) $\vec{L}_O$ remains constant while $\vec{L}_P$ varies with time.
(d) $\vec{L}_O$ and $\vec{L}_P$ both vary with time.

SECTION II

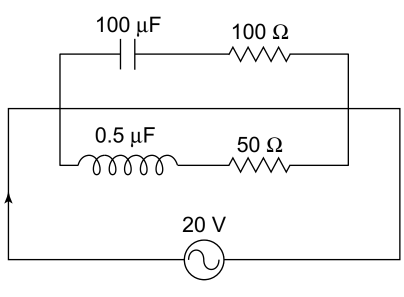
(Multiple Correct Answer Type)

This section contains 7 multiple choice questions. Each question has four choices (a), (b), (c) and (d) out of which **ONE or MORE** are correct.

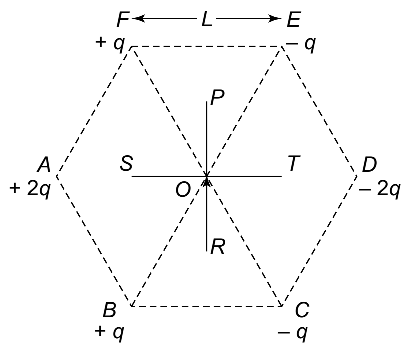
14. Consider the motion of a positive point charge in a region where there are simultaneous uniform electric and magnetic fields $\vec{E} = E_0\hat{j}$ and $\vec{B} = B_0\hat{j}$. At time $t = 0$, this charge has velocity $\vec{v}$ in x-y plane, making an angle θ with the x-axis. Which of the following option(s) is(are) correct for time $t > 0$?
(a) If $\theta = 0°$, the charge moves in a circular path in the x-z plane.
(b) If $\theta = 0°$, the charge undergoes helical motion with constant pitch along the y-axis.
(c) If $\theta = 10°$, the charge undergoes helical motion with its pitch increasing with time, along the y-axis.
(d) If $\theta = 90°$, the charge undergoes linear but accelerated motion along the y-axis.

15. A person blows into open-end of a long pipe. As a result, a high pressure pulse of air travels down the pipe. When this pulse reaches the other end of the pipe,
(a) a high-pressure pulse starts travelling up the pipe, if the other end of the pipe is open.
(b) a low-pressure pulse starts travelling up the pipe, if the other end of the pipe is open.
(c) a low-pressure pulse starts travelling up the pipe, if the other end of the pipe is closed.
(d) a high-pressure pulse starts travelling up the pipe, if the other end of the pipe is closed.

16. In the given circuit, the AC source has ω = 100 rad/s. Considering the inductor and capacitor to be ideal, the correct choice(s) is (are)
(a) The current through the circuit, I is 0.3 A.
(b) The current through the circuit, I is $0.3\sqrt{2}$A.
(c) The voltage across 100 Ω resistor = $10\sqrt{2}$V.
(d) The voltage across 50 Ω resistor = 10 V.



17. Six point charges are kept at the vertices of a regular hexagon of side L and centre O, as shown in the figure, Given that $K = \frac{1}{4\pi\varepsilon_0}\frac{q}{L^2}$, which of the following statement(s), is (are) correct?



(a) The electric field at O is $6K$ along OD.
(b) The potential at O is zero.
(c) The potential at all points on the line PR is same.
(d) The potential at all points on the line ST is same.

18. Two solid cylinders P and Q of same mass and same radius start rolling down a fixed inclined plane from the same height at the same time. Cylinder P has most of its mass concentrated near its surface, while Q has most of its mass concentrated near the axis. Which statement(s) is (are) correct?
(a) Both cylinders P and Q reach the ground at the same time.
(b) Cylinders P has larger acceleration than cylinder Q.
(c) Both cylinders reach the ground with same translational kinetic energy.
(d) Cylinder Q reaches the ground with larger angular speed.

19. A current carrying infinitely long wire is kept along the diameter of a circular wire loop, without touching it, the correct statement(s) is (are)
(a) The emf induced in the loop is zero if the current is constant.
(b) The emf induced in the loop is finite if the current is constant.
(c) The emf induced in the loop is zero if the current decreases at a steady rate.
(d) The emf induced in the loop is infinite if the current decreases at a steady rate.

20. Two spherical planets P and Q have the same uniform density ρ, masses M_P and M_Q and surface areas A and $4A$ respectively. A spherical planet R also has uniform density ρ and its mass is $(M_P + M_Q)$. The escape velocities from the planets P, Q and R are V_P, V_Q and V_R, respectively. Then
(a) $V_Q > V_R > V_P$ (b) $V_R > V_Q > V_P$
(c) $V_R/V_P = 3$ (d) $V_P/V_Q = \frac{1}{2}$

SECTION III
(Paragraph Type)

This section contains 4 multiple choice questions based on two paragraphs. Each question has four choices (a), (b), (c) and (d) out of which **only ONE** is correct.

Questions 21 and 22 are based on the following paragraph.

The β-decay process, discovered around 1900, is basically the decay of a neutron (n). In the laboratory, a proton (p) and an electron (e^-) are observed as the decay product of the neutron. Therefore, considering the decay of a neutron as a two-body decay process, it was predicted theoretically that the kinetic energy of the electron should be a constant. But experimentally, it was observed that the electron kinetic energy has continuous spectrum. Considering a three-body decay process, i.e. $n \rightarrow p + e^- + \overline{\nu}_e$, around 1930, Pauli explained the observed electron energy spectrum. Assuming the anti-neutrino ($\overline{\nu}_e$) to be massless and possessing negligible energy, and the neutron to be at rest, momentum and energy conservation principles are applied. From this calculation, the maximum kinetic energy for the electron is 0.8×10^6 eV, The kinetic energy carried by the proton is only the recoil energy.

21. What is the maximum energy of the anti-neutrino?
(a) Zero
(b) Much less than 0.8×10^6 eV
(c) Nearly 0.8×10^6 eV
(d) Much larger than 0.8×10^6 eV

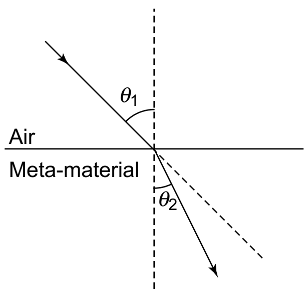
22. If the anti-neutrino had a mass of 3 eV/c^2 (where c is the speed of light) instead of zero mass, what should be the range of the kinetic energy, K, of the electron?
(a) $0 \le K \le 0.8 \times 10^6$ eV
(b) $3.0 \text{ eV} \le K \le 0.8 \times 10^6$ eV
(c) $3.0 \text{ eV} < K \le 0.8 \times 10^6$ eV
(d) $0 < K \le 0.8 \times 10^6$ eV

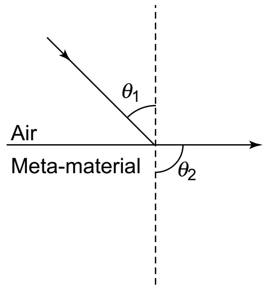
Questions 23 and 24 are based on the following paragraph.

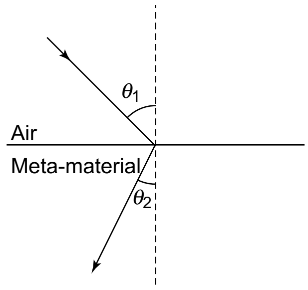
Most materials have the refractive index, $n > 1$. So, when a light ray from air enters a naturally occurring material, then by Snell's law, $\dfrac{\sin\theta_1}{\sin\theta_2} = \dfrac{n_2}{n_1}$, it is understood that the refracted ray bends towards the normal. But it never emerges on the same side of the normal as the incident ray. According to electromagnetism, the refractive index of the medium is given by the relation, $n = \left(\dfrac{c}{v}\right) = \pm\sqrt{\varepsilon_r \mu_r}$ where c is the speed of electromagnetic waves in vacuum, v its speed in the medium, ε_r, and μ_r, are relative permittivity and permeability of the medium respectively.

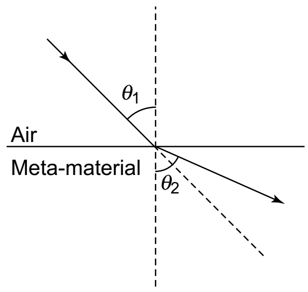
In normal materials, ε_r and μ_r, are positive, implying positive n for the medium. When both ε_r and μ_r, are negative, one must choose the negative root of n. Such negative refractive index materials can now be artificially prepared and are called meta-materials. They exhibit significantly different optical behaviour, without violating any physical laws. Since n is negative, it results in a change in the direction of propagation of the refracted light. However, similar to normal materials, the frequency of light remains unchanged upon refraction even in meta-materials.

23. For light incident from air on a meta-material. the appropriate ray diagram is

(a) 

(b) 

(c) 

(d) 

24. Choose the correct statement.

(a) The speed of light in the meta-material is $v = c|n|$

(b) The speed of light in the meta-material is $v = \dfrac{c}{|n|}$

(c) The speed of light in the meta-material is $v = c$.

(d) The wavelength of the light in the meta-material (λ_m) is given by $\lambda_m = \lambda_{\text{air}}\,|n|$. where λ_{air} is wave1ength of the light in air.

SECTION IV

(Assertion-Reason Type)

This section contains 2 questions. Each question has statement-1 followed by statement-2. Only one of the following four choices (a), (b), (c) and (d) is correct.

(a) Statement-1 is true, statement-2 is true and is the correct explanation of statement-1.

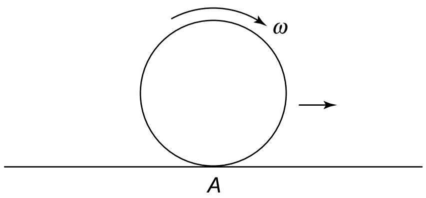
(b) Statement-1 is true and statement-2 is true but is not the correct explanation of statement-1.

(c) Statement-1 is true but statement-2 is false.

(d) Statement-1 is false but statement-2 is true.

25. Statement-1

A sphere is rolling on a rough surface in the direction of the arrow as shown in the figure. The force of friction at the point of contact will be in the direction of the arrow.



Statement-2
Friction opposes motion.

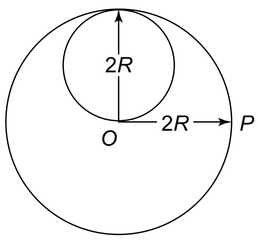
26. **Statement-1**
No induced emf is developed across the ends of a conductor if it is moved parallel to a magnetic field.
Statement-2
No force acts on the free electrons of the conductor.

SECTION V
(Integer Answer Type)

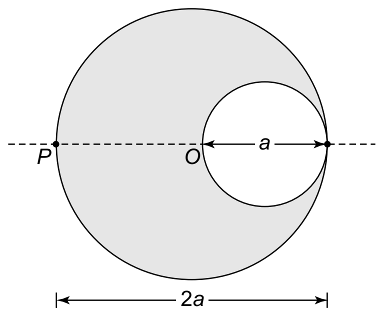
This section contains 3 questions. The answer to each question is single digit integer ranging from 0 to 9 (both inclusive)

27. A proton is fired from very far away towards a nucleus with charge $Q = 120e$, where e is the electronic charge. It makes a closest approach of 10 fm to the nucleus. The de Broglie wavelength (in units of fm) of the proton at its start is: (take the proton mass, $m_p = (5/3) \times 10^{-27}$ kg: $h/e = 4.2 \times 10^{-15}$ J.s/C; $\frac{1}{4\pi\varepsilon_0} = 9 \times 10^9$ m/F: 1 fm = 10^{-15} m)

28. A lamina is made by removing a small disc of diameter $2R$ from a bigger disc of uniform mass density and radius $2R$, as shown in the figure. The moment of inertia of this lamina about axes passing though O and P is I_O and I_P respectively. Both these axes are perpendicular to the plane of the lamina. The ratio I_P/I_O to the nearest integer is



29. A cylindrical cavity of diameter a exists inside a cylinder of diameter $2a$ shown in the figure. Both the cylinder and the cavity are infinitely long. A uniform current density J flows along the length. If the magnitude of the magnetic field at the point P is given by $\frac{N}{12}\mu_0 aJ$, the value of N is



SECTION VI
(Matrix Match Type)

30. In Column I are listed some charged bodies and current carrying conductors. Match them with the effects they produce listed in column II

Column I	Column II
(a) A uniformly charged stationary ring	(p) Electric field
(b) A uniformly charged ring rotating	(q) Magnetic field
(c) A coil carrying a current $I = I_0 \sin \omega$	(r) Magnetic moment
(d) A wire carrying a constant current	(s) Induced electric field

Answers

Section-I

1. (a)	**2.** (d)	**3.** (a)
4. (d)	**5.** (d)	**6.** (a)
7. (d)	**8.** (d)	**9.** (b)
10. (d)	**11.** (b)	**12.** (a)
13. (c)		

Section-II

14. (c), (d)	**15.** (b), (d)	
16. (a), (c)	**17.** (a), (b), (c)	
18. (d)	**19.** (a), (c)	**20.** (b), (d)

Section-III

21. (c)	**22.** (d)	**23.** (c)
24. (b)		

Section-IV

25. (a)	**26.** (a)

Section-V

27. (7)	**28.** (3)	**29.** (5)

Section-VI

30. (a) → (p), (b) → (p), (q), (r),
(c) → (p), (r), (s), (d) → (q)

Solutions

Section-I

1. Least count of screw gauge = $\dfrac{0.5 \text{ mm}}{100} = 0.005$ mm.

Since M and L are fixed and g is constant, $\Delta M = 0$, $\Delta L = 0$ and $\Delta g = 0$. Hence

$$\frac{\Delta Y}{Y} = \frac{\Delta \ell}{\ell} + \frac{2\Delta d}{d}$$

$$\frac{\Delta \ell}{\ell} = \frac{0.005 \text{ mm}}{0.25 \text{ mm}} = 0.02$$

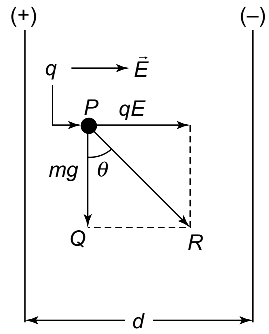
$$\frac{2\Delta d}{d} = \frac{2 \times 0.005 \text{ mm}}{0.5 \text{ mm}} = 0.02$$

2. $v_{\text{rms}} = \sqrt{\dfrac{3RT}{M}}$

$$\frac{v_{\text{rms}}(\text{helium})}{v_{\text{rms}}(\text{argon})} = \sqrt{\frac{M_{\text{Ar}}}{M_{\text{He}}}} = \sqrt{\frac{40}{4}} = \sqrt{10} = 3.16$$

3. $\tan\theta = \dfrac{QR}{PQ} = \dfrac{qE}{mg}$

$$\Rightarrow E = \frac{mg\tan\theta}{q}$$

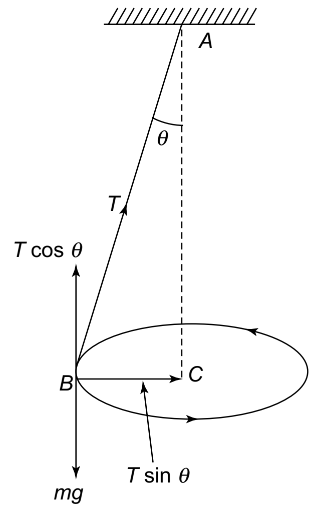


Also $E = \dfrac{V}{d}$. Hence

$$\frac{V}{d} = \frac{mg\tan\theta}{q}$$

$$\Rightarrow V = \frac{mgd\tan\theta}{q}$$

4. Radius of the circular path is $BC = r = L\sin\theta$, where $L = AB$ is the length of the string. The vertical component $T\cos\theta$ of tension T balances with the weight mg and the horizontal component $T\sin\theta$ provides the necessary centripetal force for circular motion. Hence



$$T\sin\theta = mr\omega^2 = m(L\sin\theta)\omega^2$$

$$\Rightarrow \quad T = mL\omega^2$$

$$\therefore \quad T_{\max} = mL\,\omega^2_{\max}$$

$$\Rightarrow \quad 324 = 0.5 \times 0.5 \times \omega^2_{\max}$$

$$\Rightarrow \quad \omega_{\max} = 36 \text{ rad s}^{-1}$$

5. Time of flight $(t_f) = \sqrt{\dfrac{2h}{g}} = \sqrt{\dfrac{2\times 5}{10}} = 1$s

Horizontal range (R) = horizontal velocity × time of flight

∴ Horizontal velocities of the bullet and of the ball after the collision respectively are

$$(v)_{\text{bullet}} = \frac{100}{1} = 100 \text{ ms}^{-1}$$

$$(v)_{\text{ball}} = \frac{20}{1} = 20 \text{ ms}^{-1}$$

From conservation of momentum,
Total initial momentum = total final momentum

$\Rightarrow \quad (m)_{\text{bullet}} \times V = (m)_{\text{bullet}} \times (v)_{\text{bullet}} + (m)_{\text{ball}} \times (v)_{\text{ball}}$

$\Rightarrow \quad 0.01\ V = 0.01 \times 100 + 0.2 \times 20$

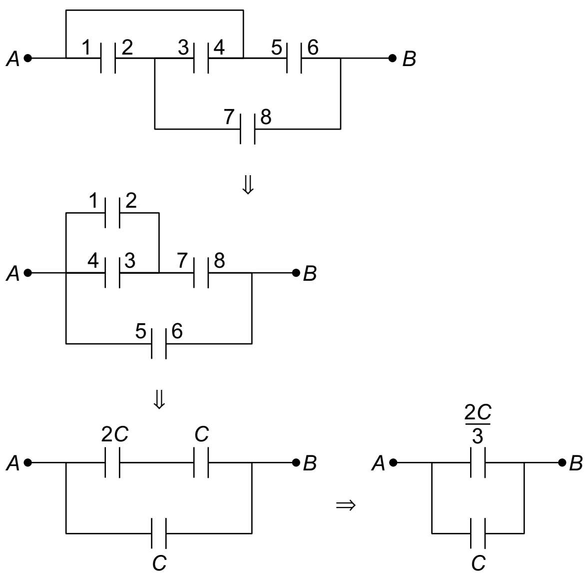
$\Rightarrow \quad V = 500\ \text{ms}^{-1}$

6. The force exerted on charge $+Q$ by the electric field is $\vec{E}$ is

$$\vec{F} = Q\vec{E}$$

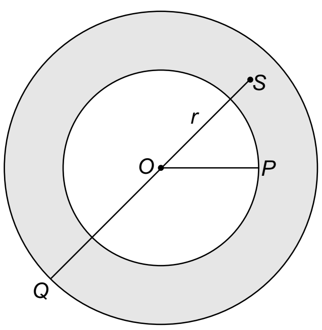
in the direction of $\vec{E}$. Since $\vec{F}$ is constant, a constant force is added to the applied force. Hence only the mean position will change and the frequency of oscillation will remain the same.

7. The circuit can be redrawn as follows.



$$C_{\text{eq}} = \frac{2C}{3} + C = \frac{5C}{3}$$

8.



$OP = \frac{R}{2}$, $OQ = R$, $OS = r$,

Inside the cavity (i.e. for r lying between zero and $\frac{R}{2}$); $B = 0$

Outside the cylinder. (i.e. for $r > R$),

$$B = \frac{\mu_0 I}{2\pi r}$$

In the shaded region (i.e. for $\frac{R}{2} < r < R$). From Ampere's circuital law,

$$B \times 2\pi r = \mu_0 I$$

where $J = \frac{I}{A}$ is the current density and A is the area of the shaded region. Now

$$A = \pi r^2 - \pi\left(\frac{R}{2}\right)^2$$

$$\therefore \quad B \times 2\pi r = \mu_0 J\left[\pi r^2 - \frac{\pi R^2}{4}\right]$$

$$\Rightarrow \quad B = \frac{\mu_0 J}{2}\left[\frac{r^2 - R^2/4}{r}\right]$$

$$= \frac{\mu_0 J}{2}\left[r - \frac{R^2}{4r}\right]$$

Hence the correct graph is (d).

9. End correction $e = 0.3d = 0.3 \times 4 = 1.2$ cm

Wavelength $\lambda = \frac{v}{\nu} = \frac{\cdot 336}{512} = 0.656$ m $= 65.6$ cm

Now $L + e = \frac{\lambda}{4}$

$$\Rightarrow \quad L = \frac{\lambda}{4} - e = \frac{65.6}{4} - 1.2$$

$$= 16.4 - 1.2 = 15.2 \text{ cm}$$

10. Since $\lambda_R > \lambda_G > \lambda_B$ and $\beta = \frac{\lambda D}{d}$, $\beta_R > \beta_G > \beta_B$.

11. Magnetic Moment $\vec{M}$ = current × area of the loop

$= I\vec{A}$

$$= I \times \left[a^2 + \pi\left(\frac{a}{2}\right)^2 \times 2\right]\hat{k}$$

$$= Ia^2\left[1 + \frac{\pi}{2}\right]\hat{k}$$

The direction of area vector $\vec{A}$ is along $\hat{k}$.

12. When the pebble hits the block, the distance travelled by the pebble (S_p) = distance travelled by the block (S_b).

$$S_p = 4.9 + 0.2 \cos \omega t$$