

# JEE ADVANCED 2017: PAPER - I

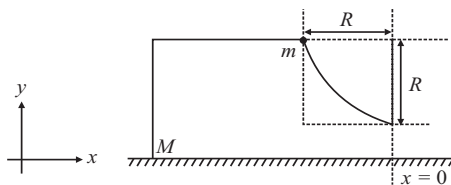
## (PHYSICS)

### QUESTIONS

#### SECTION - I

Multiple Choice Questions will ONE or MORE THAN ONE correct options.

1. A block of mass  $M$  has a circular cut with a frictionless surface as shown. The block rests on the horizontal frictionless surface of a fixed table. Initially the right edge of the block is at  $x = 0$ , in a co-ordinate system fixed to the table. A point mass  $m$  is released from rest at the topmost point of the path as shown and it slides down. When the mass loses contact with the block, its position is  $x$  and the velocity is  $v$ . At that instant, which of the following options is/are correct?



- (a) The velocity of the point mass  $m$  is:

$$v = \sqrt{\frac{2gR}{1 + \frac{m}{M}}}$$

- (b) The velocity of the block  $M$  is:

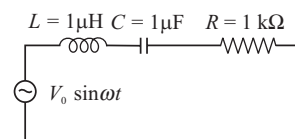
$$v = -\frac{m}{M}\sqrt{2gR}$$

- (c) The position of the point mass is:

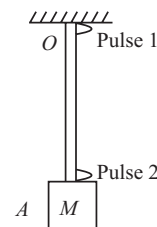
$$x = -\sqrt{2}\frac{mR}{M+m}$$

- (d) The  $x$  component of displacement of the center of mass of the block  $M$  is:  $-\frac{mR}{M+m}$

2. In the circuit shown,  $L = 1\mu\text{H}$ ,  $C = 1\mu\text{F}$  and  $R = 1\text{ k}\Omega$ . They are connected in series with an a.c. source  $V = V_0 \sin \omega t$  as shown. Which of the following options is/are correct?



- (a) At  $\omega \sim 0$  the current flowing through the circuit becomes nearly zero
- (b) At  $\omega \gg 10^6 \text{ rad. s}^{-1}$ , the circuit behaves like a capacitor
- (c) The frequency at which the current will be in phase with the voltage is independent of  $R$
- (d) The current will be in phase with the voltage if  $\omega = 10^4 \text{ rad. s}^{-1}$
3. A block  $M$  hangs vertically at the bottom end of a uniform rope of constant mass per unit length. The top end of the rope is attached to a fixed rigid support at  $O$ . A transverse wave pulse (Pulse I) of wavelength  $\lambda_0$  is produced at point  $O$  on the rope. The pulse takes time  $T_{OA}$  to reach point  $A$ . If the wave pulse of wavelength  $\lambda_0$  is produced at point  $A$  (Pulse 2) without disturbing the position of  $M$  it takes time  $T_{AO}$  to reach point  $O$ . Which of the following options is/are correct?

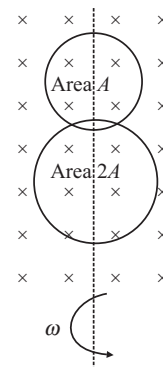


- (a) The velocities of the two pulses (Pulse 1 and Pulse 2) are the same at the midpoint of rope
- (b) The velocity of any pulse along the rope is independent of its frequency and wavelength
- (c) The wavelength of Pulse 1 becomes longer when it reaches point  $A$
- (d) The time  $T_{AO} = T_{OA}$

4. A flat plate is moving normal to its plane through a gas under the action of a constant force  $F$ . The gas is kept at a very low pressure. The speed of the plate  $v$  is much less than the average speed  $u$  of the gas molecules. Which of the following options is/are true?
- The pressure difference between the leading and trailing faces of the plate is proportional to  $uv$
  - The plate will continue to move with constant non-zero acceleration, at all times
  - At a later time external force  $F$  balances the resistive force
  - The resistive force experienced by the plate is proportional to  $v$
5. A human body has a surface area of approximately  $1\text{m}^2$ . The normal body temperature is  $10\text{K}$  above the surrounding room temperature  $T_0$ . Take the room temperature to be  $T_0 = 300\text{K}$ . For  $T_0 = 300\text{K}$ , the value of  $\sigma T_0^4 = 460\text{Wm}^{-2}$  (where  $\sigma$  is the Stefan-Boltzmann constant). Which of the following options is/are correct?
- Reducing the exposed surface area of the body (e.g. by curling up) allows humans to maintain the same body temperature while reducing the energy lost by radiation
  - If the body temperature rises significantly then the peak in the spectrum of electromagnetic radiation emitted by the body would shift to longer wavelengths
  - The amount of energy radiated by the body in 1 second is close to 60 Joules
  - If the surrounding temperature reduces by a small amount  $\Delta T_0 \ll T_0$ , then to maintain the same body temperature the same (living) human being needs to radiate  $\Delta W = 4\sigma T_0^3 \Delta T_0$  more energy per unit time
6. For an isosceles prism of angle  $A$  and refractive index  $\mu$ , it is found that the angle of minimum deviation  $\delta_m = A$ . Which of the following options is/are correct?
- At minimum deviation, the incident angle  $i_1$  and the refracting angle  $r_1$  at the first refracting surface are related by  $r_1 = (i_1/2)$

- For this prism, the refractive index  $\mu$  and the angle of prism  $A$  are related as  $A = \frac{1}{2} \cos^{-1} \left( \frac{\mu}{2} \right)$
- For the angle of incidence  $i_1 = A$ , the ray inside the prism is parallel to the base of the prism
- For this prism, the emergent ray at the second surface will be tangential to the surface when the angle of incidence at the first surface is  $i_1 = \sin^{-1} \left[ \sin A \sqrt{4 \cos^2 \frac{A}{2} - 1} - \cos A \right]$

7. A circular insulated copper wire loop is twisted to form two loops of area  $A$  and  $2A$  as shown in the figure. At the point of crossing the wires remain electrically insulated from each other. The entire loop lies in the plane (of the paper). A uniform magnetic field  $\vec{B}$  points into the plane of the paper. At  $t = 0$ , the loop starts rotating about the common diameter as axis with a constant angular velocity  $\omega$  in the magnetic field. Which of the following options is/are correct?

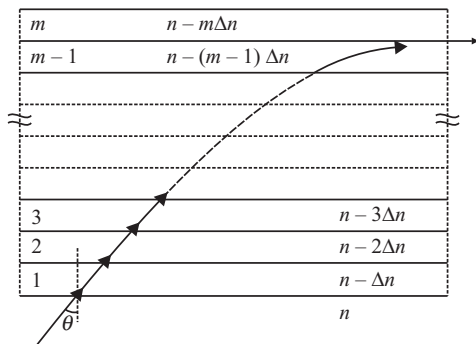


- The amplitude of the maximum net emf induced due to both the loops is equal to the amplitude of maximum emf induced in the smaller loop alone
- The rate of change of the flux is maximum when the plane of the loops is perpendicular to plane of the paper
- The net emf induced due to both the loops is proportional to  $\cos \omega t$
- The emf induced in the loop is proportional to the sum of the areas of the two loops

SECTION 2

Single Digit Integer Answer Questions

8. An electron in a hydrogen atom undergoes a transition from an orbit with quantum number  $n_i$  to another with quantum number  $n_f$ .  $V_i$  and  $V_f$  are respectively the initial and final potential energies of the electron. If  $\frac{V_i}{V_f} = 6.25$ , then the *smallest possible*  $n_f$  is \_\_\_\_\_.
9. A monochromatic light is travelling in a medium of refractive index  $n = 1.6$ . It enters a stack of glass layers from the bottom side at an angle  $\theta = 30^\circ$ . The interfaces of the glass layers are parallel to each other. The refractive indices of different glass layers are monotonically decreasing as  $n_m = n - m\Delta n$ , where  $n_m$  is the refractive index of the  $m^{\text{th}}$  slab and  $\Delta n = 0.1$  (see the figure).



The ray is refracted out parallel to the interface between the  $(m - 1)^{\text{th}}$  and  $m^{\text{th}}$  slabs from the right side of the stack. What is the value of  $m$ ?

10.  $^{131}\text{I}$  is an isotope of Iodine that  $\beta$  decays to an isotope of Xenon with a half-life of 8 days. A small amount of a serum labelled with  $^{131}\text{I}$  is injected into the blood of a person. The activity of the amount of  $^{131}\text{I}$  injected was  $2.4 \times 10^5$  Becquerel (Bq). It is known that the injected serum will get distributed uniformly in the blood stream in less than half an hour. After 11.5 hours, 2.5 ml of blood is drawn from the person's body, and gives an activity of 115 Bq. The total volume of blood in the person's body, in liters is approximately. (you may use  $e^x \approx 1 + x$  for  $|x| \ll 1$  and  $\ln 2 \approx 0.7$ ).
11. A stationary source emits sound of frequency  $f_0 = 492$  Hz. The sound is *reflected* by a large car *approaching* the source with a speed of  $2 \text{ ms}^{-1}$ . The reflected signal is received by the source and superposed with the original. What will be the beat frequency of the resulting signal in Hz? (Given that the speed of sound in air is  $330 \text{ ms}^{-1}$  and the car reflects the sound at the frequency it has received).
12. A drop of liquid of radius  $R = 10^{-2}\text{m}$  having surface tension  $S = \frac{0.1}{4\pi} \text{ Nm}^{-1}$  divides itself into  $K$  identical drops. In this process the total change in the surface energy  $\Delta U = 10^{-3} \text{ J}$ . If  $K = 10^\alpha$  then the value of  $\alpha$  is \_\_\_\_\_.

SECTION 3

Matching Type Questions

Answer Q.13, Q.14 and Q.15 by appropriately matching the information given in the three columns of the following table.

A charged particle (electron or proton) is introduced at the origin ( $x = 0, y = 0, z = 0$ ) with a given initial velocity  $\vec{v}$ . A uniform electric field  $\vec{E}$  and a uniform magnetic field  $\vec{B}$  exist everywhere. The velocity  $\vec{v}$ ,

electric field  $\vec{E}$  and magnetic field  $\vec{B}$  are given in columns 1, 2 and 3, respectively. The quantities  $E_0, B_0$  are positive in magnitude.

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Column 1	Column 2	Column 3
(I) Electron with $\vec{v} = 2\frac{E_0}{B_0}\hat{x}$	(i) $\vec{E} = E_0\hat{z}$	(P) $\vec{B} = -B_0\hat{x}$
(II) Electron with $\vec{v} = \frac{E_0}{B_0}\hat{y}$	(ii) $\vec{E} = -E_0\hat{y}$	(Q) $\vec{B} = B_0\hat{x}$
(III) Proton with $\vec{v} = 0$	(iii) $\vec{E} = -E_0\hat{x}$	(R) $\vec{B} = B_0\hat{y}$
(IV) Proton with $\vec{v} = 2\frac{E_0}{B_0}\hat{x}$	(iv) $\vec{E} = E_0\hat{x}$	(S) $\vec{B} = B_0\hat{z}$

13. In which case will the particle move in a straight line with *constant* velocity?

- (a) (IV) (i) (S)                      (b) (III) (ii) (R)  
 (c) (III) (iii) (P)                    (d) (II) (iii) (S)

14. In which case would the particle move in a straight line along the negative direction of  $y$ -axis (*i.e.*, move along  $-\hat{y}$ )?

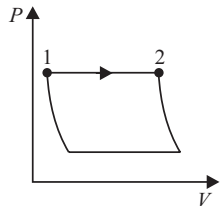
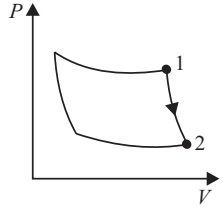
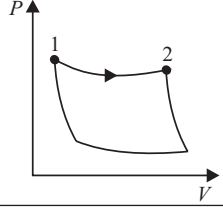
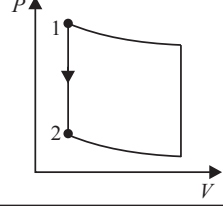
- (a) (III) (ii) (P)                      (b) (III) (ii) (R)  
 (c) (IV) (ii) (S)                      (d) (II) (iii) (Q)

15. In which case will the particle describe a helical path with axis along the positive  $z$  direction?

- (a) (III) (iii) (P)                      (b) (II) (ii) (R)  
 (c) (IV) (ii) (R)                      (d) (IV) (i) (S)

**Answer Q.16, Q.17 and Q.18 by appropriately matching the information given in the three columns of the following table.**

An ideal gas is undergoing a cyclic thermodynamic process in different ways as shown in the corresponding  $P$ - $V$  diagrams in column 3 of the table. Consider only the path from state 1 to state 2.  $W$  denotes the corresponding work done on the system. The equations and plots in the table have standard notations as used in thermodynamic processes. Here  $\gamma$  is the ratio of heat capacities at constant pressure and constant volume. The number of moles in the gas is  $n$ .

Column 1	Column 2	Column 3
(I) $W_{1 \rightarrow 2} = \frac{1}{\gamma - 1}(P_2V_2 - p_1V_1)$	(i) Isothermal	(P) 
(II) $W_{1 \rightarrow 2} = -PV_2 + PV_1$	(ii) Isochoric	(Q) 
(III) $W_{1 \rightarrow 2} = 0$	(iii) Isobaric	(R) 
(IV) $W_{1 \rightarrow 2} = -nRT \ln \left( \frac{V_2}{V_1} \right)$	(iv) Adiabatic	(S) 

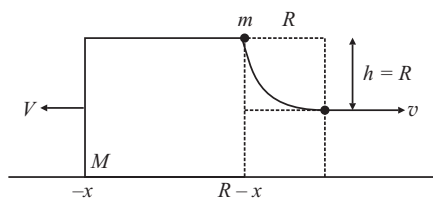
16. Which of the following options is the only correct representation of a process in which  $\Delta U = \Delta Q - P\Delta V$  ?  
 (a) (II) (iv) (R)                      (b) (III) (iii) (P)  
 (c) (II) (iii) (P)                      (d) (II) (iii) (S)
17. Which one of the following options correctly represents a thermodynamic process that is used as a correction in the determination of the speed of sound in an ideal gas?  
 (a) (I) (iv) (Q)                      (b) (III) (iv) (R)  
 (c) (I) (ii) (Q)                      (d) (IV) (ii) (R)
18. Which one of the following options is the correct combination?  
 (a) (IV) (ii) (S)                      (b) (III) (ii) (S)  
 (c) (II) (iv) (R)                      (d) (II) (iv) (P)

**ANSWERS KEY**

- |                  |                  |
|------------------|------------------|
| 1. (a), (d)      | 2. (a), (c)      |
| 3. (a), (b), (d) | 4. (a), (c), (d) |
| 5. (a), (c), (d) | 6. (a), (c), (d) |
| 7. (a), (b)      | 8. 5             |
| 9. 8             | 10. 5            |
| 11. 6            | 12. 6            |
| 13. (d)          | 14. (b)          |
| 15. (d)          | 16. (c)          |
| 17. (a)          | 18. (b)          |

**SOLUTIONS**

1. Let  $V$  be the velocity of the block when mass  $m$  loses contact with the block (see figure).



Since no net external force acts on the system, the momentum is conserved, i.e.,

$$MV = mv \Rightarrow V = \frac{mv}{M}$$

From conservation of energy,

Loss in gravitational P.E. of  $m =$  gain in K.E. of  $M +$  gain in K.E. of  $m$

$$\Rightarrow mgh = \frac{1}{2}MV^2 + \frac{1}{2}mv^2$$

$$\Rightarrow mgR = \frac{1}{2}M\left(\frac{mv}{M}\right)^2 + \frac{1}{2}mv^2$$

$$\Rightarrow 2gR = \left(1 + \frac{m}{M}\right)v^2$$

$$\Rightarrow v = \sqrt{\frac{2gR}{\left(1 + \frac{m}{M}\right)}}$$

Hence, option (a) is correct.

$$V = \frac{m}{M}v = \frac{m}{M} \sqrt{\frac{2gR}{\left(1 + \frac{m}{M}\right)}}$$

along negative  $x$ -direction. So option (b) is incorrect.

Let  $\Delta x$  be the displacement of the centre of mass of  $m$  and  $\Delta X$  be the displacement of the centre of mass of the block of mass  $M$ . Since no external force acts on the system, its centre of mass has no net displacement, i.e.,

$$m\Delta x + M\Delta X = 0$$

$$\Rightarrow m(R - x) + M(-x) = 0$$

$$\Rightarrow x = \frac{mR}{m + M}$$

Hence, option (d) is correct.

Now,  $\Delta x = R - x$

$$= R - \frac{mR}{m + M} = \frac{MR}{M + m}$$

$\therefore$  Final position of  $m = 0 - x$

$$= 0 - \frac{MR}{M + m} = -\frac{mR}{M + m}$$

Hence, option (c) is incorrect.

2. Capacitive reactance  $X_C = \frac{1}{\omega C}$  and inductive

reactance  $X_L = \omega L$ .

At  $\omega = 0$ ,  $X_C \rightarrow \infty$  and  $X_L = 0$ . Hence the current flowing through the circuit is nearly zero. So option (a) is correct. When  $\omega \gg 10^6 \text{ rad s}^{-1}$ ,  $X_C \rightarrow 0$  and  $X_L \gg 1$ . Thus the circuit does not behave like a capacitor and option (b) is incorrect.

The current will be in phase with voltage at resonant frequency i.e. when  $X_L = X_C$  or  $\omega L = \frac{1}{\omega C} \Rightarrow \omega = \frac{1}{\sqrt{LC}}$  which is independent of  $R$ . So

option (c) is correct. The resonant frequency is

$$\omega = \frac{1}{\sqrt{LC}} = \frac{1}{\sqrt{(1 \times 10^{-6}) \times (1 \times 10^{-6})}} = 10^6 \text{ rad s}^{-1}$$

So option (d) is incorrect.

3. Mass of rope is  $m = \mu L$  where  $\mu$  is the mass per unit length of the rope and  $L$  is its length. The velocity of a pulse is given by

$$v = \sqrt{\frac{T}{\mu}}$$

where  $T$  is the tension. Since the rope has a finite mass, the tension will not be the same at points on the rope. At  $A$  the tension is  $T_A = Mg$  and at  $O$ , the tension is  $T_O = (M + m)g$ . At the mid-point, the tension is  $\left(M + \frac{m}{2}\right)g$  which is the same for

both pulses. Hence pulses 1 and 2 have the same velocity at the mid-point, so option (a) is correct.

Since  $\mu$  is constant and  $T$  has nothing to do with wavelength or frequency, the velocity of any pulse along the rope is independent of wavelength and frequency.

So option (b) is correct.

Since  $v = v\lambda$

$$\lambda = \frac{v}{\nu} = \frac{1}{\nu} \sqrt{\frac{T}{\mu}}$$

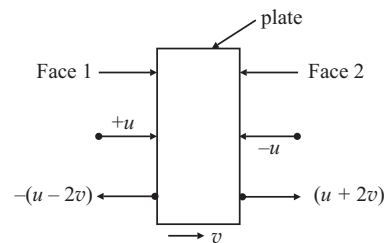
The frequency  $\nu$  of the pulse cannot change as it depends only on the frequency of the source producing the pulse and since  $\mu$  is also constant,

$$\frac{\lambda_1}{\lambda_2} \propto \sqrt{T}$$

Since the tension decreases as we go from  $O$  to  $A$ , the velocity of pulse 1 decreases as it travels from  $O$  to  $A$ . So the wavelength of pulse 1 becomes shorter when it reaches point  $A$ . Hence option (c) is incorrect.

Since the average speed of each pulse is the same, the time taken for pulse 1 to go from  $O$  to  $A$  = time taken for pulse 2 to go from  $A$  to  $O$ . So option (d) is correct. Thus the correct options are (a), (b) and (d).

4. Let  $m$  be the mass of each molecule. Let velocity directed to the right be taken as positive and velocity to the left be taken as negative.



The rate of change of momentum of a molecule due to collision with face 1 is

$$\Delta p_1 = \text{Momentum after collision} - \text{momentum before collision}$$

$$= -m(u - 2v) - m(u + 2v)$$

$$= -mu + 2mv - mu - 2mv$$

$$= -2m(u + v) \text{ directed to the left}$$

The rate of change of momentum of a molecule due to collision with face 2 is

$$\Delta p_2 = m(u + 2v) - (-mu)$$

$$= 2m(u - v) \text{ directed to the right}$$

Let  $\tau_1$  and  $\tau_2$  be the time of collision with face 1 and 2 respectively. Then the forces on face 1 and

on face 2 will be proportional to  $\frac{\Delta p_1}{\tau_1}$  and  $\frac{\Delta p_2}{\tau_2}$

Let  $R_1$  and  $R_2$  be the number of collisions per unit time with faces 1 and 2 respectively, the  $R_1 \propto \frac{1}{\tau_1}$

and  $R_2 \propto \frac{1}{\tau_2}$ . If  $F_1$  and  $F_2$  are the magnitudes of

forces on faces 1 and 2, then

$$F_1 \propto R_1 |\Delta p_1|$$

$$\text{or } F_1 \propto R_1 \Delta p_1 \text{ and } F_2 \propto R_2 \Delta p_2$$

Now  $R_1 \propto (u + v)$  and  $R_2 \propto (u - v)$ . Hence

$$F_1 \propto 2m(u + v)^2 \text{ and}$$

$$F_2 \propto 2m(u - v)^2$$

$\therefore$  Net Force acting on the plate is

$$F \propto F_1 - F_2$$

$$\propto 2m(u + v)^2 - 2m(u - v)^2$$

or  $F \propto 8muv$

Thus  $F \propto uv$ . So option (a) is correct.

Now  $F \propto v$ . Hence the plate will accelerate and will eventually acquire a terminal velocity because

the gas will tend to reduce the velocity until its acceleration becomes zero. Hence choice (c) is correct.

From Stokes' law, it follows that for small velocities the resistive force experienced by the plates is proportional to its velocity  $v$ . So option (d) is correct. Thus the correct options are (a), (c) and (d).

5. Assuming that human body behaves as a block body, the heat energy radiated per second from its surface is

$$Q = \sigma A (T^4 - T_0^4),$$

$\sigma$  = Stefan's constant

$A$  = exposed surface area of the body

For small temperature difference

$$\Delta T = T - T_0 \Rightarrow T = T_0 + \Delta T$$

$$\begin{aligned} \therefore Q &= \sigma A [(T_0 + \Delta T)^4 - T_0^4] \\ &= \sigma A \left[ T_0^4 \left( 1 + \frac{\Delta T}{T_0} \right)^4 - T_0^4 \right] \\ &= \sigma A \left[ T_0^4 \left( 1 + \frac{4\Delta T}{T_0} \right)^4 - T_0^4 \right] \end{aligned}$$

or  $Q = 4\sigma A T_0^3 \Delta T$

It follows that  $Q$  decreases with decrease in  $A$ . So option (a) is correct.

From Wien's displacement law, it follows that the peak of the graph of  $Q$  versus  $\lambda$  shifts to shorter wavelength if the temperature  $T$  rises. So option (b) is incorrect.

$$\begin{aligned} Q &= 4\sigma A T_0^3 \Delta T = 4\sigma A T_0^3 (T - T_0) \\ &= (\sigma T_0^4) \times 4A \times \left( \frac{T}{T_0} - 1 \right) \\ &= 460 \times 4 \times 1 \times \left( \frac{310}{300} - 1 \right) \\ &= 61.3 \text{ Js}^{-1} \text{ (or W)} \end{aligned}$$

So option (c) is correct.

If  $T_0$  decreases to  $T_0 - \Delta T_0$ , the heat energy radiated per second is given by

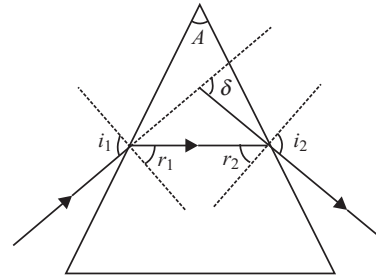
$$\begin{aligned} Q' &= \sigma A [T^4 - (T_0 - \Delta T_0)^4] \\ \text{or } &= \sigma A \left[ T^4 - T_0^4 \left( 1 - \frac{\Delta T_0}{T_0} \right)^4 \right] \end{aligned}$$

$$= \sigma A \left[ T^4 - T_0^4 \left( 1 - \frac{4\Delta T_0}{T_0} \right) \right]$$

$$\begin{aligned} \text{or } Q' &= \sigma A [T^4 - T_0^4 + 4T_0^3 \Delta T_0] \\ &= Q + 4\sigma A T_0^3 \Delta T_0 \end{aligned}$$

Thus to maintain the same temperature  $T$ ,  $Q'$  must be greater than  $Q$ . So option (d) is correct. Hence the correct options are (a), (c) and (d).

6. Refer to the following figure.



$$\delta = i_1 + i_2 - A \text{ and } r_1 + r_2 = A$$

For minimum deviation,  $i_1 = i_2$  and  $r_1 = r_2$ . Hence

$$\delta_m = 2i_1 - A \text{ and } 2r_1 = A$$

Given  $\delta_m = A$ . Hence

$$A = 2i_1 - A \rightarrow i_1 = A = 2r_1 \Rightarrow r_1 = \frac{i_1}{2}$$

So, option (a) is correct.

From Snell's law,

$$\sin i_1 = \mu \sin r_1$$

$$\Rightarrow \sin A = \mu \sin \left( \frac{A}{2} \right)$$

$$\Rightarrow 2 \sin \left( \frac{A}{2} \right) \cos \left( \frac{A}{2} \right) = \mu \sin \left( \frac{A}{2} \right)$$

$$\Rightarrow 2 \cos \left( \frac{A}{2} \right) = \mu$$

$$\Rightarrow \cos \left( \frac{A}{2} \right) = \frac{\mu}{2}$$

$$\Rightarrow A = 2 \cos^{-1} \left( \frac{\mu}{2} \right)$$

So option (b) is incorrect.

At minimum deviation, the ray inside the prism is parallel to the base. So option (c) is correct.

Applying Snell's to the second refraction.

$$\mu \sin r_2 = \sin i_2$$

For tangential emergence,  $i_2 = 90^\circ$ . Hence

$$\mu \sin r_2 = 1$$

$$\Rightarrow r_2 = \sqrt{1 - \frac{1}{\mu^2}}$$

Now,  $r_1 = A - r_2$

$$\sin r_1 = \sin(A - r_2) = \sin A \cos r_2 - \cos A \sin r_2$$

$$= (\sin A) \frac{\sqrt{\mu^2 - 1}}{\mu} - (\cos A) \frac{1}{\mu}$$

From Snell's law applied at the first refraction,

$$\sin i_1 = \mu \sin r_1$$

$$\Rightarrow \sin r_1 = \sin(A) (\mu^2 - 1)^{1/2} - \cos A$$

$$\text{or } i_1 = \sin^{-1} \left[ \sin A \sqrt{4 \cos^2 \left( \frac{A}{2} \right) - 1} - \cos A \right]$$

So option (d) is correct. Thus the correct options are (a), (c) and (d).

7. At any time  $t$ , the area vector of each loop makes an angle  $\theta = \omega t$  with  $\vec{B}$ . The instantaneous magnetic flux in loop 1 is  $\phi_1 = BA \cos \omega t$  and in loop 2 is  $\phi_2 = B(2A) \cos \omega t$ . The magnitudes of induced emfs in loops 1 and 2 are

$$|\mathcal{E}_1| = \left| \frac{d\phi_1}{dt} \right| = BA\omega \sin \omega t$$

and

$$|\mathcal{E}_2| = \left| \frac{d\phi_2}{dt} \right| = 2 BA\omega \sin \omega t$$

The net instantaneous flux is (because the two emfs oppose each other)

$$\mathcal{E}_{\text{net}} = 2BA\omega \sin \omega t - BA\omega \sin \omega t$$

$$= BA\omega \sin \omega t$$

The amplitude of the  $\mathcal{E}_{\text{net}} = BA\omega$  which is equal to that due to loop 1 alone. So option (a) is correct.

When  $\theta = \frac{\pi}{2}$ ,  $\mathcal{E}_{\text{net}}$  is maximum. So option (b) is correct.  $\mathcal{E}_{\text{net}}$  is proportional to  $\sin \omega t$ . So option (c) is incorrect. Option (d) is incorrect because the orientation of the loop is such that  $\mathcal{E}_1$  and  $\mathcal{E}_2$  oppose each other. Hence  $\mathcal{E}_{\text{net}}$  is proportional to the difference in areas. So the correct options are (a) and (b).

8.  $V \propto \frac{1}{n^2}$ . So

$$\frac{V_i}{V_f} = \left( \frac{n_f}{n_i} \right)^2$$

$$\text{or } 6.25 = \left( \frac{n_f}{n_i} \right)^2 \Rightarrow \frac{n_f}{n_i} = 2.5 = \frac{5}{2}$$

So the smallest integral value of  $n_f = 5$

9. The refractive index of layers decreases by a constant amount  $\Delta n$  as we go from lower to upper layers. For total reflection at the interface between the  $(m - 1)$ th and  $m$ th layers, we have ( $\because n \sin \theta = \text{constant}$ )

$$n \sin \theta = (n - m \Delta n) \sin 90^\circ$$

$$\Rightarrow 1.6 \times \sin 30^\circ = 1.6 - m \Delta n$$

$$\Rightarrow 0.8 = 1.6 - m \times 0.1 \Rightarrow m = \frac{0.8}{0.1} = 8$$

10. Activity  $R = R_0 e^{-\lambda t}$  where the decay constant  $\lambda$  is given by

$$\lambda = \frac{\ln(2)}{T}; T = \text{half life}$$

$$= \frac{\ln(2)}{8} \text{ per day} = \frac{0.7}{8} \text{ per day}$$

$$t = 12 \text{ hours} = \frac{1}{2} \text{ day. So}$$

$$R = R_0 e^{-\frac{0.7}{8} \times \frac{1}{2}} = R_0 e^{-\frac{0.7}{16}}$$

Using  $e^x = 1 + x$ , we get

$$R = R_0 \left( 1 - \frac{0.7}{16} \right) = R_0 \times \frac{15.3}{16}$$

$$= \frac{2.4 \times 10^5 \times 15.3}{16}$$

$$= 2.3 \times 10^5 \text{ Bq}$$

Now,  $V$  litre is the volume of the blood, then

$$115 = \frac{R \times 2.5 \times 10^{-3}}{V}$$

$$= \frac{2.3 \times 10^5 \times 2.5 \times 10^{-3}}{V}$$

$$\Rightarrow V = 5 \text{ litre}$$

11. Apparent frequency is

$$\mu' = \left( \frac{330 + 2}{330 - 2} \right) \times 492 = 498 \text{ Hz}$$

$$\therefore \text{Beat frequency} = 498 - 492 = 6 \text{ Hz}$$

12. Let  $r$  be the radius of each small drop. Since the total volume remains constant,

$$\frac{4\pi}{3} R^3 = K \times \frac{4\pi}{3} r^3 \Rightarrow R^3 = Kr^3$$



Initial P.E. is

$$U_i = S \times 4\pi R^2$$

Final P.E. is

$$U_f = KS \times 4\pi r^2$$

$$\Delta U = U_f - U_i$$

$$\Rightarrow 10^{-3} = KS \times 4\pi r^2 - 5 \times 4\pi R^2$$

$$= 4\pi S(Kr^2 - R^2)$$

$$= 4\pi SR^2 \left( \frac{Kr^2}{R^2} - 1 \right)$$

$$= 4\pi SR^2 (k^{1/3} - 1) \quad (\because R^3 = Kr^3)$$

or  $10^{-3} = 4\pi SR^2 \left( 10^{\frac{\alpha}{3}} - 1 \right) \quad (\because K = 10^\alpha)$

Substituting the given values of  $S$  and  $R$  and solving we get

$$\alpha \approx 6$$

13. The charged particle will move in a straight line with a constant velocity if no net force acts on it, i.e. if magnetic force = electrical force or

$$qvB = qE$$

$$\Rightarrow v = \frac{E}{B}$$

The electric and magnetic field must be perpendicular to each other.

If the particle is an electron, then  $\vec{v} = \frac{E_0}{B_0} \hat{y}$  if

$$\vec{E} = -E_0 \hat{x} \text{ and } \vec{B} = B_0 \hat{z}$$

In this case  $\vec{F}_B = q(\vec{v} \times \vec{B})$

$$= -e \left( \frac{E_0}{B_0} \hat{y} \times B_0 \hat{z} \right) = -e E_0 \hat{x}$$

and  $\vec{F}_E = q\vec{E} = -e \times (-E_0 \hat{x}) = eE_0 \hat{x}$

So the correct choices are (II), (ii) and (S) which is option (d).

14. If  $v = 0$ , the proton will move along the negative  $y$  direction if  $\vec{E}$  is along negative  $y$  direction because if  $\vec{v} = 0$ , magnetic force is zero or parallel or antiparallel to  $\vec{E}$ . So the correct choices are (III), (ii) and (R) which is option (b).

15. The proton will describe a helical path with velocity  $\vec{v} = 2 \left( \frac{E_0}{B_0} \right) \hat{x}$  if  $\vec{E} = E_0 \hat{z}$  and  $\vec{B} = B_0 \hat{z}$

because then  $\vec{F}_E$  will be along the  $+z$  axis and  $\vec{F}_B$  will be along  $-y$  axis. so the correct choices are (IV), (i) and (S) which is option (d).

16. Since  $W$  represents work done on the gas, it is negative. For an isobaric process ( $P = \text{constant}$ ),

$$W = - \int P dV = -P \int_{V_1}^{V_2} dV = P (V_1 - V_2),$$

which is (II).

For an isobaric process, the  $P$ - $V$  graph is parallel to the  $V$ -axis which is option (P). No other combination is possible. So the correct choices are (II), (iii) and (P), which is option (c).

17. Laplace gave the correct formula for the speed of sound in a gas. He corrected Newton's formula by assuming that the propagation of sound in a gas is an adiabatic process for which

$$W = \frac{1}{\gamma - 1} (P_2 V_2 - P_1 V_1) \text{ which is choice (I). Also}$$

choice (iv) is correct. In option (Q), process  $1 \rightarrow 2$  is adiabatic. So the correct option is (a).

18. Option (a) is incorrect because process  $1 \rightarrow 2$  is isochoric for which  $W = 0$ . Choice (c) is incorrect because  $W = P (V_1 - V_2)$  is wrong for an adiabatic process. For the same reason choice (d) is also incorrect. Hence the only correct option is (b).