# JEE ADVANCED 2017: PAPER - II (PHYSICS)

## SECTION – I

**Multiple Choice Questions having ONLY ONE correction option.**

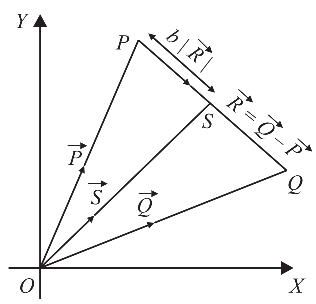
**1.** A photoelectric material having work function $\phi_0$ is illuminated with light of wavelength $\lambda\left(\lambda < \frac{hc}{\phi_0}\right)$

The fastest photoelectron has a de Broglie wavelength $\lambda_d$. A change in wavelength of the incident light by $\Delta\lambda$ results in a change $\Delta\lambda_d$ in $\lambda_d$. Then the ratio $\Delta\lambda_d/\Delta_\lambda$ is proportional to:

(a) $\lambda_d^3/\lambda^2$ (b) $\lambda_d^2/\lambda^2$

(c) $\lambda_d/\lambda$ (d) $\lambda_d^3/\lambda$

**2.** Three vectors $\vec{P}$, $\vec{Q}$ and $\vec{R}$ are shown in the figure. Let $S$ be any point on the vector $\vec{R}$. The distance between the points $P$ and $S$ is $b\,|\vec{R}|$. The general relation among vectors $\vec{P}$, $\vec{Q}$ and $\vec{S}$ is:



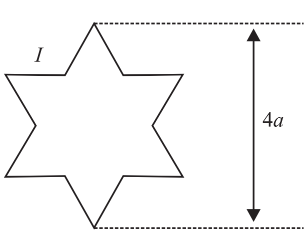

(a) $\vec{S} = (1-b)\vec{P} + b^2\vec{Q}$

(b) $\vec{S} = (1-b^2)\vec{P} + b\vec{Q}$

(c) $\vec{S} = (1-b)\vec{P} + b\vec{Q}$

(d) $\vec{S} = (b-1)\vec{P} + b^2\vec{Q}$

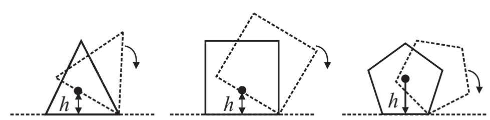
**3.** A symmetric star shaped conducting wire loop is carrying a steady state current $I$ as shown in the figure. The distance between the diametrically opposite vertices of the star is $4a$. The magnitude of the magnetic field at the centre of the loop is:




(a) $\frac{\mu_0 I}{4\pi a}6[\sqrt{3}-1]$ (b) $\frac{\mu_0 I}{4\pi a}3[\sqrt{3}-1]$

(c) $\frac{\mu_0 I}{4\pi a}6[\sqrt{3}+1]$ (d) $\frac{\mu_0 I}{4\pi a}3[2-\sqrt{3}]$

**4.** Consider regular polygons with number of sides $n$ = 3, 4, 5… as shown in the figure. The centre of mass of all the polygons is at height $h$ from the ground. They roll on a horizontal surface about the leading vertex without slipping and sliding as depicted. The maximum increase in height of the locus of the centre of mass for each polygon is $\Delta$. Then $\Delta$ depends on $n$ and $h$ as:




(a) $\Delta = h \sin\left(\text{—}\right)$ (b) $\Delta = h \sin^2\left(\frac{\pi}{n}\right)$

(c) $\Delta = h \tan^2\left(\frac{\pi}{2n}\right)$ (d) $\Delta = h\left[\frac{1}{\cos\left(\frac{\pi}{n}\right)} - 1\right]$

**5.** Consider an expanding sphere of instantaneous radius $R$ whose total mass remains constant. The expansion is such that the instantaneous density $\rho$ remains uniform throughout the volume. The rate of fractional change in density $\left(\frac{1}{\rho}\frac{d\rho}{dt}\right)$ is constant. The velocity $v$ of any point on the surface of the expanding sphere is proportional to:

(a) $R^{2/3}$ (b) $R$

(c) $R^3$ (d) $\frac{1}{R}$

**6.** A person measures the depth of a well by measuring the time interval between dropping a stone and receiving the sound of impact with the bottom of the well. The error in his measurement of time is $\delta T = 0.01$ seconds and he measures the depth of the well to be $L = 20$ meters. Take the acceleration due to gravity $g = 10\ \text{ms}^{-2}$ and the velocity of sound is 300 $ms^{-1}$. Then the fractional error in the measurement $\delta L/L$, is closest to:

(a) 0.2% (b) 5%
(c) 1% (d) 3%

**7.** A rocket is launched normal to the surface of the Earth, away from the Sun, along the line joining the Sun and the Earth. The Sun is $3 \times 10^5$ times heavier than the Earth and is at a distance $2.5 \times 10^4$ times larger than the radius of the Earth. The escape velocity from Earth's gravitational field is $v_e = 11.2$ $kms^{-1}$. The minimum initial velocity ($v_s$) required for the rocket to be able to leave the Sun-Earth system is closest to

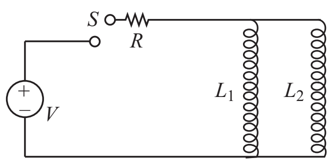
*(Ignore the rotation and revolution of the Earth and the presence of any other planet)*

(a) $v_s = 62$ $kms^{-1}$ (b) $v_s = 42$ $kms^{-1}$
(c) $v_s = 72$ $kms^{-1}$ (d) $v_s = 22$ $kms^{-1}$

## SECTION 2

**(Multiple choice Questions having ONE or MORE THAN ONE Correct Options)**

**8.** A source of constant voltage $V$ is connected to a resistance $R$ and two ideal inductors $L_1$ and $L_2$ through a switch $S$ as shown. There is no mutual inductance between the two inductors. The switch $S$ is initially open. At $t = 0$, the switch is closed and current begins to flow. Which of the following options is/are correct?



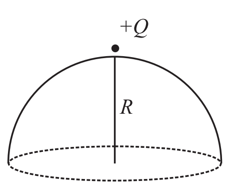

(a) At $t = 0$, the current through the resistance $R$ is $\frac{V}{R}$

(b) After a long time, the current through $L_2$, will be $\frac{V}{R}\frac{L_1}{L_1+L_2}$

(c) After a long time, the current through $L_1$ will be $\frac{V}{R}\frac{L_2}{L_1+L_2}$

(d) The ratio of the currents through $L_1$, and $L_2$ is fixed at all times ($t > 0$)

**9.** A point charge $+Q$ is placed just outside an imaginary hemispherical surface of radius $R$ as shown in the figure. Which of the following statements is/are correct?



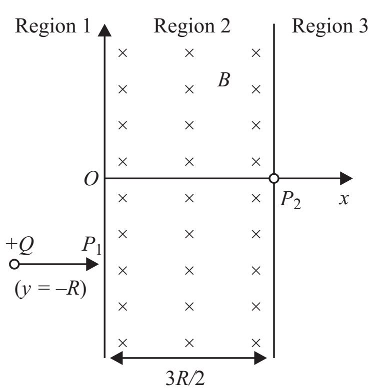

(a) The electric flux passing through the curved surface of the hemisphere is $-\frac{Q}{2\varepsilon_0}\left(1-\frac{1}{\sqrt{2}}\right)$

(b) The component of the electric field normal to the flat surface is constant over the surface

(c) Total flux through the curved and the flat surfaces is $\frac{Q_0}{\varepsilon_0}$

(d) The circumference of the flat surface is an equipotential surface

**10.** A uniform magnetic field $B$ exists in the region between $x = 0$ and $x = \frac{3R}{2}$ (region 2 in the figure) pointing normally into the plane of the paper. A particle with charge $+Q$ and momentum $p$ directed along $x$-axis enters region 2 from region 1 at point $P_1(y = -R)$. Which of the following option(s) is/are correct?



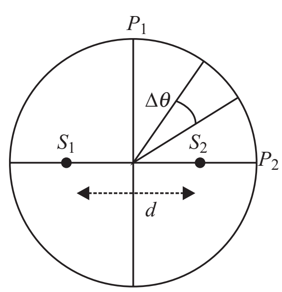

(a) For $B = \frac{8}{13}\frac{p}{QR}$, the particle will enter region 3 through the point $P_2$ on $x$-axis

(b) For $B > \frac{2}{3}\frac{p}{QR}$, the particle will re-enter region I

(c) For a fixed $B$, particles of same charge $Q$ and same velocity $v$, the distance between the point $P_1$ and the point of re-entry into region 1 is inversely proportional to the mass of the particle

(d) When the particle re-enters region 1 through the longest possible path in region 2, the magnitude of the change in its linear momentum between point $P_1$ and the farthest point from $y$-axis is $p/\sqrt{2}$

**11.** Two coherent monochromatic point sources $S_1$ and $S_2$ of wavelength $\lambda$ = 600 nm are placed symmetrically on either side of the centre of the circle as shown. The sources are separated by a distance $d$ = 1.8 mm. This arrangement produces interference fringes visible as alternate bright and dark spots on the circumference of the circle. The angular separation between two consecutive bright spots is $\Delta\theta$. Which of the following options is/are correct?




(a) The total number of fringes produced between $P_1$ and $P_2$ in the first quadrant is close to 3000

(b) A dark spot will be formed at the point $P_2$

(c) At $P_2$ the order of the fringe will be maximum

(d) The angular separation between two consecutive bright spots decreases as we move from $P_1$ and $P_2$ along the first quadrant

**12.** The instantaneous voltages at three terminals marked $X$, $Y$ and $Z$ are given by

$$V_x = V_0 \sin \omega t,$$

$$V_y = V_0 \sin \left(\omega t + \frac{2\pi}{3}\right) \text{ and}$$

$$V_z = V_0 \sin \left(\omega t + \frac{4\pi}{3}\right)$$

An ideal voltmeter is configured to read *rms* value of the potential difference between its terminals. It is connected between points X and Y and then between Y and Z. The reading(s) of the voltmeter will be

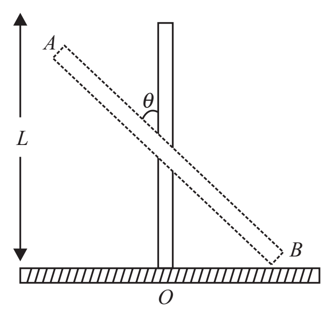
(a) $V_{YZ}^{rms} = V_0\sqrt{\frac{1}{2}}$

(b) $V_{XY}^{rms} = V_0\sqrt{\frac{3}{2}}$

(c) $V_{XY}^{rms} = V_0$

(d) independent of the choice of the two terminals

**13.** A rigid uniform bar $AB$ of length $L$ is slipping from its vertical position on a frictionless floor (as shown in the figure). At some instant of time, the angle made by the bar with the vertical is $\theta$. Which of the following statements about its motion is/are correct?



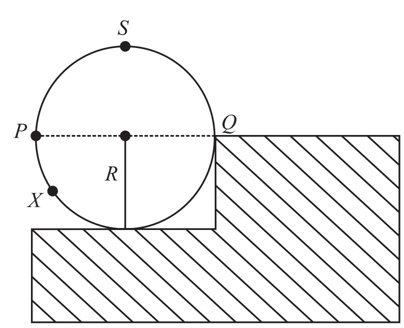

(a) The trajectory of the point $A$ is a parabola

(b) Instantaneous torque about the point in contact with the floor is proportional to sin $\theta$.

(c) The midpoint of the bar will fall vertically downward

(d) When the bar makes an angle $\theta$ with the vertical, the displacement of its midpoint from the initial position is proportional to (1 – cos $\theta$)

**14.** A wheel of radius $R$ and mass $M$ is placed at the bottom of a fixed step of height $R$ as shown in the figure. A constant force is continuously applied on the surface of the wheel so that it just climbs the step without slipping. Consider the torque $\tau$ about an axis normal to the plane of the paper passing through the point $Q$. Which of the following options is/are correct?



(a) If the force is applied tangentially at point $S$ then $\tau \neq 0$ but the wheel never climbs the step

(b) If the force is applied normal to the circumference at point $P$ then $\tau$ is zero

(c) If the force is applied normal to the circumference at point $X$ then $\tau$ is constant

(d) If the force is applied at point $P$ tangentially then $\tau$ decreases continuously as the wheel climbs

## SECTION 3

**(Paragraph Based Questions having ONLY ONE Correct Option)**

**Paragraph for Questions 15-16**

Consider a simple $RC$ circuit as shown in Figure 1.

**Process 1 :** In the circuit the switch $S$ is closed at $t = 0$ and the capacitor is fully charged to voltage $V_0$, (i.e., charging continues for time $T \gg RC$). In the process some dissipation ($E_D$) occurs across the resistance $R$. The amount of energy finally stored in the fully charged capacitor is $E_C$.

**Process 2:** In a different process the voltage is first set to $\frac{V_0}{3}$ and maintained for a charging time $T \gg RC$. Then the voltage is raised to $\frac{2V_0}{3}$ without discharging the capacitor and again maintained for a time $T >> RC$. The process is repeated one more time by raising the voltage to $V_0$ and the capacitor is charged to the same final voltage $V_0$ as in Process 1. These two processes are depicted in Figure 2.

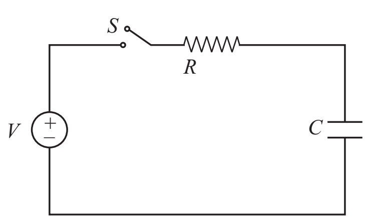



**Fig. 1**

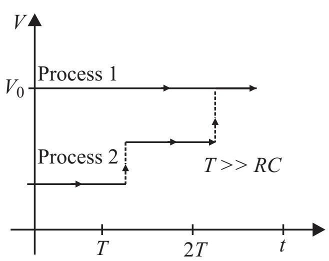



**Fig. 2**

**15.** In Process 2, total energy $E_D$ dissipated across the resistance is:

(a) $E_D = 3\left(\frac{1}{2}CV_0^2\right)$ (b) $E_D = \frac{1}{2}CV_0^2$

(c) $E_D = 3CV_0^2$ (d) $E_D = \frac{1}{3}\left(\frac{1}{2}CV_0^2\right)$

**16.** In Process 1, the energy stored in the capacitor $E_C$ and heat dissipated $E_D$ across resistance are related by:

(a) $E_C = \frac{1}{2}E_D$ (b) $E_C = 2E_D$

(c) $E_C = E_D$ (d) $E_C = E_D \ln 2$

**Paragraph for Questions 17-18**

One twirls a circular ring (of mass $M$ and radius $R$) near the tip of one's finger as shown in Figure 1. In the process the finger never loses contact with the inner rim of the ring. The finger traces out the surface of a cone, shown by the dotted line. The radius of the path traced out by the point where the ring and the finger is in contact is $r$. The finger rotates with an angular velocity $\omega_0$. The rotating ring *rolls without slipping* on the outside of a smaller circle described by the point where the ring and the finger is in contact (Figure 2). The coefficient of friction between the ring and the finger is $\mu$ and the acceleration due to gravity is $g$.

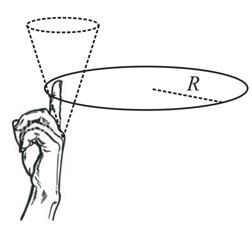



**Fig. 1**

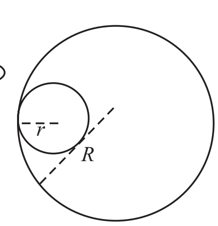



**Fig. 2**

**17.** The minimum value of $\omega_0$ below which the ring will drop down is :

(a) $\sqrt{\frac{2g}{\mu(R-r)}}$ (b) $\sqrt{\frac{g}{\mu(R-r)}}$

(c) $\sqrt{\frac{3g}{2\mu(R-r)}}$ (d) $\sqrt{\frac{g}{2\mu(R-r)}}$

**18.** The total kinetic energy of the ring is:

(a) $\frac{3}{2}M\omega_0^2(R - r)^2$

(b) $\frac{1}{2}M\omega_0^2(R - r)^2$

(c) $M\omega_0^2 (R - r)^2$

(d) $M\omega_0^2 R^2$

## Answers

### Section I

| | |
|---|---|
| **1.** (a) | **2.** (c) |
| **3.** (a) | **4.** (d) |
| **5.** (b) | **6.** (c) |

### Section II

| | |
|---|---|
| **7.** (b) | **8.** (b), (c), (d) |
| **9.** (a), (d) | **10.** (a), (b) |
| **11.** (a), (c) | **12.** (b), (d) |
| **13.** (b), (c), (d) | **14.** (b), (d) |

### Section III

| | |
|---|---|
| **15.** (d) | **16.** (c) |
| **17.** (b) | **18.** None. |

## Hints and Solutions

**1.** If $p$ is the momentum of the fastest photoelectron, its K.E. is

$$K_{max} = \frac{p^2}{2m}; \; m = \text{mass of electron}$$

$$= \frac{1}{2m}\left(\frac{h}{\lambda_d}\right)^2 \qquad (\because \; \lambda_d = \frac{h}{p})$$

or $$K_{max} = \frac{h^2}{2m\lambda_d^2}$$

From Einstein's photo-electric equation,

$$\frac{hc}{\lambda} = K_{max} + \phi_0$$

$$\Rightarrow \quad \frac{hc}{\lambda} = \frac{h^2}{2m\lambda_d^2} + \phi_0$$

$$\Rightarrow \quad -\frac{hc}{\lambda^2}\Delta\lambda = -\frac{2h^2}{2m\lambda_d^3}\Delta\lambda_d$$

$$\Rightarrow \quad \frac{\Delta\lambda_d}{\Delta\lambda} = \left(\frac{c}{2hm}\right)\left(\frac{\lambda^3{}_d}{\lambda^2}\right)$$

$\therefore \frac{\Delta\lambda_d}{\Delta\lambda} \propto \frac{\lambda_d^3}{\lambda^2}$, which is option (a).

**2.** Applying triangle law to $\Delta OPS$, $\vec{S}$ is the resultant of vectors $\vec{P}$ and $b\,|\vec{R}|$, i.e.

$$\vec{P} + b\,|\vec{R}| = \vec{S}$$

$$\Rightarrow \quad b\,|\vec{R}| = \vec{S} - \vec{P}$$

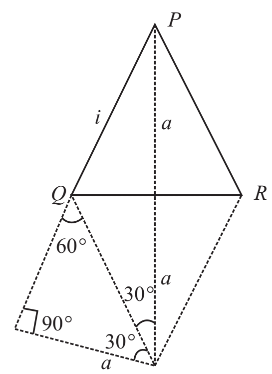
It is given in the figure that $\vec{R} = \vec{Q} - \vec{P}$

or $\quad \vec{P} = \vec{Q} - \vec{R}$

or $\quad \vec{P} = \vec{Q} - \frac{1}{b}(\vec{S} - \vec{P})$

$\Rightarrow \quad \vec{S} = (1 - b)\vec{P} + b\vec{Q}$, which is option (c).

**3.** Refer to the following figure.




The magnitude of the magnetic field due to one segment $PQ$ of the star at the centre of the loop is

$$B = -\frac{\mu_0 i}{4\pi a}\int_{60°}^{30°} \sin\theta \, d\theta$$

$$= \frac{\mu_0 i}{4\pi a}\left|\cos\theta\right|_{60°}^{30°}$$

$$= \frac{\mu_0 i}{4\pi a}(\cos 30° - \cos 60°)$$

$$= \frac{\mu_0 i}{8\pi a}(\sqrt{3} - 1) \text{ directed into the page.}$$

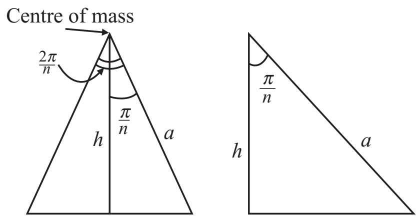
The magnitude of the magnetic field at the centre of the loop = $12B$ as all segments of stars produce magnetic field directed into the page. Hence

$$B_{loop} = 12B = \frac{12\mu_0 i}{8\pi a}(\sqrt{3} - 1)$$

$$= 6\left(\frac{\mu_0 i}{4\pi a}\right)(\sqrt{3} - 1)$$

So the correct option is (a).

**4.** Refer to the following figure. For $n$-sided regular polygon, the angle subtended at the centre of mass $= \frac{2\pi}{n}$




$$\frac{h}{a} = \cos\left(\frac{\pi}{n}\right) \Rightarrow a = \frac{h}{\cos\left(\frac{\pi}{n}\right)}$$

$$\Delta = a - h = \frac{h}{\cos\left(\frac{\pi}{n}\right)} - h = h\left[\frac{1}{\cos\left(\frac{\pi}{n}\right)} - 1\right]$$

So the correct option is (d).

**5.** The total mass of the sphere is constant. Thus

$$\frac{4\pi R^3}{3}\rho = \text{constant}$$

or $$R^3\rho = \text{constant}$$

Differentiating with respect to time,

$$3R^2\rho\frac{dR}{dt} + R^3\frac{d\rho}{dt} = 0$$

or $$3R^2\rho v + R^3\frac{d\rho}{dt} = 0$$

$$\Rightarrow \quad v = -\frac{R}{3}\frac{1}{\rho}\frac{d\rho}{dt}$$

It is given that $\frac{1}{\rho}\frac{d\rho}{dt}$ = constant. Hence $v \propto R$.

So the correct option is (b).

**6.** Time taken for the stone to reach the bottom of the well is given by

$$-L = -\frac{1}{2}gt^2$$

$$\Rightarrow \quad t = \sqrt{\frac{2L}{g}}$$

Time taken for sound to travel from the bottom of the well to the observer is

$t' = \frac{L}{v}$; $v$ = speed of sound

Total time taken is

$$T = t + t' = \sqrt{\frac{2L}{g}} + \frac{L}{v}$$

Differentiating partially with respect to $L$

$$\frac{\Delta T}{\Delta L} = \sqrt{\frac{2}{g}} \times \left(-\frac{1}{2\sqrt{L}}\right) + \frac{1}{v}$$

Since errors do not cancel each other,

$$\frac{\Delta T}{\Delta L} = \sqrt{\frac{2}{g}} \times \frac{1}{2\sqrt{L}} + \frac{1}{v}$$

$$= \sqrt{\frac{2}{10}} \times \frac{1}{2\times\sqrt{20}} + \frac{1}{300}$$

$$= \frac{1}{20} + \frac{1}{300} = \frac{16}{300}$$

$$\Rightarrow \quad \frac{0.01}{\Delta L} = \frac{16}{300}$$

$$\Rightarrow \quad \Delta L = \frac{0.01\times 300}{16} = \frac{3}{16}$$

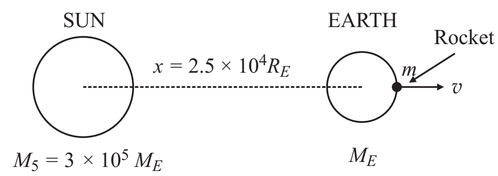
$$\therefore \quad \frac{\Delta L}{L} = \frac{3}{16}\times\frac{1}{20}$$

$$\text{Percentage error} = \frac{\Delta L}{L}\times 100 = \frac{3}{16}\times\frac{1}{20}\times 100$$

$$= \frac{15}{16} \simeq 1\%$$

So the correct option is (c).

**7.** Refer to the following figure.




The escape velocity for earth is

$$v_e = \sqrt{\frac{2GM_E}{R_E}} = 11.2 \text{ kms}^{-1} \text{ (given)}$$

The gravitational potential energy of the SUN–EARTH system is ($m$ = mass of the rocket)

$$U = -\frac{GM_E m}{R_E} - \frac{GM_S m}{x}$$

$$= -\frac{GM_E m}{R_E} - \frac{G\times 3\times 10^5 M_E m}{2.5\times 10^4 R_E}$$

$$= -\frac{GM_E m}{R_E}(1 + 12)$$

or $$U = -\frac{13GM_E m}{R_E}$$

The minimum initial velocity $v$ required for to escape from the sun-earth system is given by

$$\frac{1}{2}mv^2 = \frac{13GM_E m}{R_E}$$

$$\Rightarrow \quad v = \sqrt{13} \times \sqrt{\frac{2GM_E}{R_E}}$$

$$= \sqrt{13}\, v_e$$

$$= \sqrt{13} \times 11.2$$

$$= 40.4 \text{ kms}^{-1}$$

The closest option is (b).

**8.** At time $t = 0$ when the switch $S$ is open, the source of voltage is not connected to the circuit. Hence no current flows in the circuit at time $t = 0$. So option (a) is incorrect.

Immediately after the switch is closed, a current begins to flow. Since the inductors offer reactance, it takes some time to grow to its steady state value $= i_0$. Since the inductors $L_1$ and $L_2$ are connected in parallel, the induced voltage $V_L$ is the same across each inductor. If $i_1$ and $i_2$ are the currents through $L_1$ and $L_2$ at any time $t$ during the growth of current, then

$$V_L = L_1 \frac{di_1}{dt} = L_2 \frac{di_2}{dt}$$

or $\quad L_1 di_1 = L_2\, di_2$

$$\Rightarrow \quad L_1 \int di_1 = L_2 \int di_2$$

$\Rightarrow L_1 i_1 = L_2 i_2 \Rightarrow \dfrac{i_1}{i_2} = \dfrac{L_2}{L_1}$, which is independent of time. So option (d) is correct.

Now, $i_0 = i_1 + i_2$ or $i_2 = i_0 - i_1$

or $\quad \dfrac{i_2}{i_1} = \dfrac{i_0}{i_1} - 1$

or $\quad \dfrac{L_1}{L_2} = \dfrac{i_0}{i_1} - 1 \qquad \left(\because \dfrac{i_1}{i_2} = \dfrac{L_2}{L_1}\right)$

$$\Rightarrow \quad i_1 = \frac{i_0 L_2}{L_1 + L_2}$$

Since, in the steady state, the inductors offer no reactance, $i_0 = \dfrac{V}{R}$. Hence

$$i_1 = \frac{V}{R} \frac{L_2}{(L_1 + L_2)}$$

Now, $\quad i_2 = i_0 - i_1$

$$= \frac{V}{R} - \frac{V}{R} \frac{L_2}{(L_1 + L_2)}$$

$$= \frac{V}{R} \frac{L_1}{(L_1 + L_2)}$$

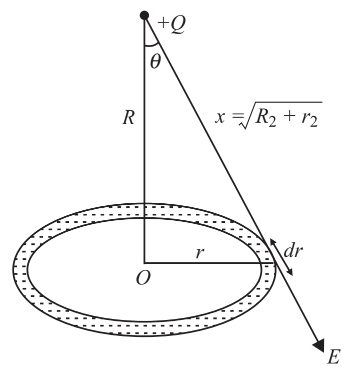
So option (b) and (c) are both correct. Thus the correct options are (b), (c) and (d).

**9.** The hemisphere consists of a curved surface of radius $R$ and a flat (bottom) surface which is a disc of radius $R$. Since the charge $+Q$ lies outside the hemisphere, the net electric flux ($\phi_e$) through the curved surface and the flux ($\phi_f$) through the flat surface is zero, i.e..

$$\phi_c + \phi_f = 0 \Rightarrow \phi_c = -\phi_f$$

So option (c) is incorrect.

To calculate flux through the flat surface (disc), we divide the disc into very small elements of length $dr$. Consider one such element at a distance $r$ from the centre O of the disc as shown in the figure.




Electric Field due to $+Q$ at the element is

$$E = \frac{Q}{4\pi\varepsilon_0 x^2} = \frac{Q}{4\pi\varepsilon_0 (R^2 + r^2)}$$

Area of element is $dA = 2\pi r\, dr$. The electric flux through the flat surface is

$$\phi_f = \int_0^R \vec{E} \cdot \vec{dA} = \int_0^R E\, dA \cos\theta = \int_0^R E\, dA \times \frac{R}{x}$$

$$= \int_0^R \frac{Q}{4\pi\varepsilon_0 (R^2 + r^2)} \times 2\pi r dr \times \frac{R}{(R^2 + r^2)^{1/2}}$$

$$= \frac{2\pi QR}{4\pi\varepsilon_0} \int_0^R \frac{r dr}{(R^2 + r^2)^{3/2}}$$

$$= \frac{QR}{2\varepsilon_0} \left[ \frac{1}{2} \frac{(R^2 + r^2)^{-1/2}}{-\frac{1}{2}} \right]_0^R$$

$$= \frac{QR}{2\varepsilon_0} \left[ \frac{1}{R} - \frac{1}{\sqrt{R^2 + R^2}} \right]$$

$$= \frac{Q}{2\varepsilon_0} \left( 1 - \frac{1}{\sqrt{2}} \right)$$

Now $\phi_c = -\phi_f = -\dfrac{Q}{2\varepsilon_0}\left(1-\dfrac{1}{\sqrt{2}}\right)$

So option (a) is correct.

Electric potential at any point on the circumference of the flat surface is

$$V = \frac{1}{4\pi\varepsilon_0}\frac{Q}{\sqrt{R^2+R^2}} = \frac{Q}{4\pi\varepsilon_0(\sqrt{2}R)}$$

which is constant. Hence the circumference of the flat surface is equipotential. So option (d) is correct.

The component of the electric field normal to the flat surface is

$$\frac{1}{4\pi\varepsilon_0}\frac{Q}{(R/\cos\theta)^2} = \frac{Q\cos^2\theta}{4\pi\varepsilon_0 R^2}, \text{ which}$$

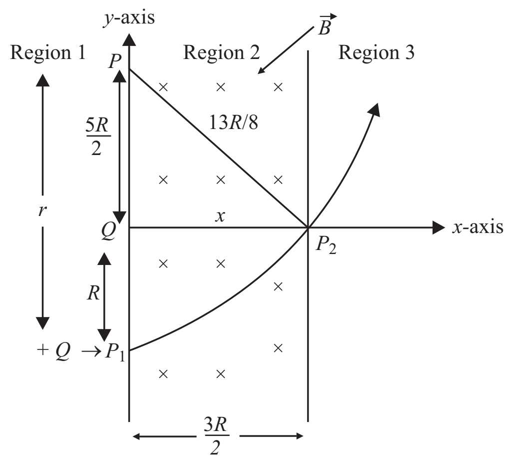
varies with $\theta$ and is constant. So option (b) is incorrect. Thus the correct options are (a) and (d).

10. The particle will describe a circle of radius $r$ given by $r = \dfrac{mv}{QB} = \dfrac{p}{QB}$ in the anticlockwise sense.

**Option (a)**: If $B = \dfrac{8}{13}\dfrac{p}{QR}$,

$$r = \frac{p}{Q}\times\frac{13QR}{8p} = \frac{13}{8}R$$

Refer to the following figure. It follows from the figure that




$$x = \sqrt{\left(\frac{13R}{8}\right)^2 - \left(\frac{5R}{2}\right)^2} = \frac{3R}{2} \text{ which is equal to } OP_2.$$

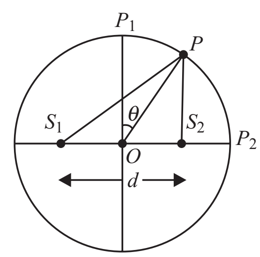
Hence the particle will enter region 3 at point $P_2$. So option (a) is correct.

**Option (b)** The particle will re-enter region 1 if it describes a semicircle in region 2 of radius $r < x$ i.e. if $r < \dfrac{3R}{2}$, i.e. if $\dfrac{p}{QB} < \dfrac{3R}{2}$ or $B > \dfrac{2p}{3QR}$. So option (b) is also correct.

**Option (c):** The distance between $P_1$ and $P$ (the point of re-entry in region 1 = $2r = \dfrac{2mv}{QB}$ which is directly proportional to $m$ (for given $v$ and $B$). So option (c) is incorrect.

**Option (d):** The particle leaves region 2 and re-enters region 1 moving horizontal along the negative $x$ direction with momentum $-p$, it entered region 2 with momentum $+p$. So the change in momentum $= -p - (+p) = -2p$. So option (d) is incorrect. Thus the correct options are (a) and (b).

**11.** Refer to the following figure.




At point $P_1$, the path difference is $\Delta x = S_1P_1 - S_2P_1 = 0$. Hence there is a bright fringe at $P_1$. So option (b) is incorrect.

At point $P_2$, $\Delta x = S_1P_2 - S_2P_2 = S_1S_2 = d$. There will be $n$th bright spot at $P_2$ if

$$d = n\lambda$$

or $$n = \frac{d}{\lambda} = \frac{1.8\times10^{-3}}{600\times10^{-9}} = 3000$$

So there will be 3000th bright spot at $P_2$. So option (c) is correct. Hence there will be $n - 1 = 3000 - 1 = 2999$ bright spots between $P_1$ and $P_2$. So option (a) is correct.

Let $P$ be the point on the circle for which angle between $OP$ and $OP_1$ is, say $\theta$. Assuming that the circle has a large radius, the path difference at point $P$ is

$$\Delta x = d\sin\theta$$

The rate of change of path difference with angle $\theta$ is

$$\frac{d}{d\theta}(\Delta x) = d\cos\theta,$$

which decreases will increase in $\theta$. So the angular separation between consecutive bright spots increases as we go from $P_1$ to $P_2$. Hence option

(d) is incorrect. Thus the correct option are (a) and (c).

**12.** When the voltmeter is connected between $X$ and $Y$

$$V_{XY} = V_X - V_Y$$
$$= V_0 \sin \omega t - V_0 \sin\left(\omega t + \frac{2\pi}{3}\right)$$
$$= V_0 \left[\sin \omega t - \sin\left(\omega t + \frac{2\pi}{3}\right)\right]$$
$$= 2V_0 \left[\cos\left(\omega t + \frac{\pi}{3}\right)\sin\left(-\frac{\pi}{3}\right)\right]$$
$$= -\frac{\sqrt{3}\times 2}{2} V_0 \cos\left(\omega t + \frac{\pi}{3}\right)$$
$$= +\sqrt{3}\, V_0 \sin\left(\omega t - \frac{\pi}{3}\right)$$

Maximum value of $V_{XY} = \sqrt{3}V_0$

$\therefore$ rms value of $V_{XY} = \frac{\sqrt{3}V_0}{\sqrt{2}}$. So option (b) is correct and option (c) is incorrect.

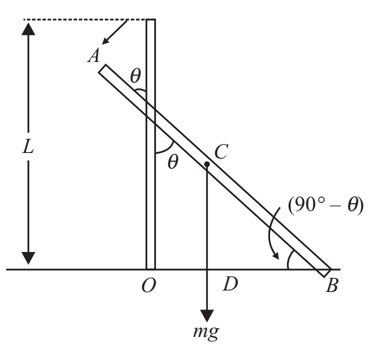
When the voltmeter is connected between $Y$ and $Z$.

$$V_{YZ} = V_Y - V_Z$$
$$= V_0 \sin\left(\omega t + \frac{2\pi}{3}\right) - V_0 \sin\left(\omega t + \frac{4\pi}{3}\right)$$
$$= V_0 \left[\sin\left(\omega t + \frac{2\pi}{3}\right) - \sin\left(\omega t + \frac{4\pi}{3}\right)\right]$$
$$= \sqrt{3}\, V_0 \cos \omega t$$

Maximum value of $V_{YZ} = \sqrt{3}\, V_0$

$\therefore$ *rms* value of $V_{YZ} = \frac{\sqrt{3}V_0}{\sqrt{2}}$, which is the same as the rms value of $V_{xy}$. So option (a) is incorrect and option (d) is correct. Hence the correct options are (b) and (d).

**13.** Refer to the following figure.




**Option (a):** The rod rotates about an axis passing through $B$ and perpendicular to the plane of the page. Hence point $A$ describes a circular trajectory. So option (a) is incorrect.

**Option (b):** At any instant of time, the magnitude of the torque about $B = mg \times BD = mg\, B < \sin\theta$ $= \left(\frac{mgL}{2}\right) \sin\theta$

This is so because the entire mass $m$ of the rod can be assumed to be acting at its centre of mass $C$. Since the rod is uniform, $BC = \frac{AB}{2} = \frac{L}{2}$.

So option (b) is correct.

**Option (c):** Since there is no external horizontal force and the initial velocity of the rod is zero, the centre of mass will move vertically downwards. So option (c) is correct.

**Option (d):** Initial $y$-coordinate of the centre of mass $= \frac{L}{2}$. When the rod subtends an angle $\theta$ with the vertical, the $y$-coordinate $= \frac{L}{2}\cos\theta$. Shift in the mid-point $= \frac{L}{2} - \frac{L}{2}\cos\theta = \frac{L}{2}(1 - \cos\theta)$.

So option (d) is correct. Thus the correct options are (b), (c) and (d).

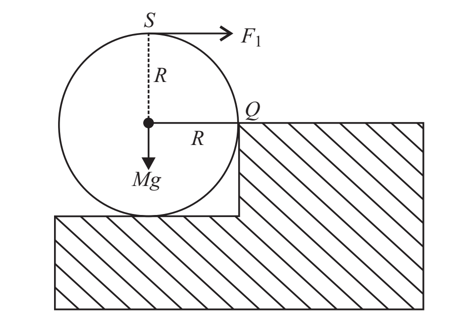



**Fig. 1**

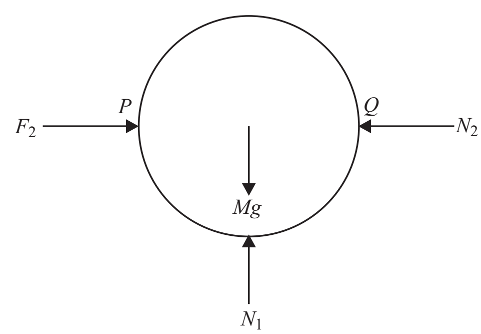



**Fig. 2**

**Option (a):** If the force $F_1$ is applied at S as shown in Fig. 1, torque of $F_1$ about $Q = F_1R$ (anticlockwise). Torque of $Mg$ about $Q = MgR$ (clockwise). The net torque depends on the magnitudes of $F_1$ and $Mg$ is given by

$$\tau = F_1R - MgR = (F_1 - Mg)\ R$$

If $F_1 > Mg$, $\tau \neq 0$ and the wheel will climb the step. So option (a) is incorrect.

**Option (b):** If the force $F_2$ is applied normal to point $P$ as shown in Fig. 2, then $F_2$ balances with normal reaction $N_2$ of the wall at $Q$ and force $Mg$ balances with the normal reaction $N_1$ of the base of the step as shown.

Hence no net force acts on the wheel and $\tau = 0$. So option (b) is correct.

**Option (c):** If force $F_3$ is applied at $X$ as shown in Fig. 3, the torque due to $F_3$ about $Q = F_3QT$ (clockwise). The torque due to $Mg$ about $Q = MgR$ (anticlockwise). The net torque on the wheel is

$$\tau = F_3QT - MgR$$

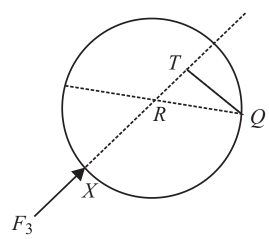



**Fig. 3**

If $F_3QT > MgR$, a net clockwise torque acts on the wheel, as a result the wheel begins to climb up the step. As it climbs $QT$ increases. Hence $\tau$ does not remain constant. So option (c) is incorrect.

**Option (d):** If the force $F_4$ is applied tangentially at $P$, as shown in Fig. 4, the normal reaction at $Q$ will be absent. Hence the frictional force is zero. As a result the wheel will start slipping as it rises up the step. It is given that there is no slipping, the wheel will rise up causing point $Q$ to come down as shown in the figure.

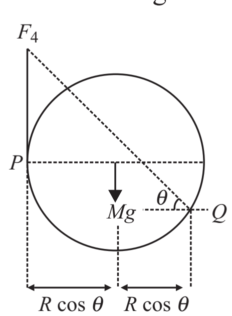



**Fig. 4**

Torque due to $F_4$ about $Q = F_4 \times 2\ R \cos\theta$ (clockwise)

Torque due to $Mg$ about $Q = Mg \times R \cos\theta$ (anticlockwise)

Net clockwise torque

$= F_4 \times 2R \cos\theta - Mg \times R \cos\theta$ (clockwise)

$= (2\ F_4R - MgR) \cos\theta$.

Initially $\theta = 0$. As the wheel rises $\theta$ increase from zero to $\dfrac{\pi}{2}$. Hence $\cos\theta$ decreases from 1 to zero.

So option (d) is correct. Thus the correct options are (b) and (d).

**15.** At time $T > RC$, the capacitor is fully charged to the voltage of the battery (here $RC$ is the time constant of the $RC$ circuit).

When the voltage is $V_1 = \dfrac{V_0}{3}$, the charge supplied by the battery is $Q_1 = CV_1 = \dfrac{CV_0}{3}$

When the voltage is increased to $V_2 = \dfrac{2V_0}{3}$, the additional charge supplied by the battery is

$$Q_2 = C\left(\frac{2V_0}{3}\right) - C\frac{2V_0}{3} = \frac{CV_0}{3}$$

When the voltage is increased to $V_3 = V_0$, the additional charge supplied by the battery is $Q_3 = CV_0 - \dfrac{2CV_0}{3} = \dfrac{CV_0}{3}$

Total energy stored in the capacitor is

$$E = \frac{1}{2}Q_1V_1 + \frac{1}{2}Q_2\ V_2 + \frac{1}{2}Q_3V_3$$

$$= \frac{1}{2}\left(\frac{CV_0}{3}\right) \times \frac{V_0}{3} + \frac{1}{2}\left(\frac{CV_0}{3}\right) \times \frac{2V_0}{3} + \frac{1}{2}\left(\frac{CV_0}{3}\right) \times V_0$$

$$= \frac{1}{18}CV_0^2 + \frac{1}{9}CV_0^2 + \frac{1}{6}CV_0^2$$

$$= \frac{1}{3}CV_0^2$$

Since the final voltage is $V_0$, final charge on the capacitor is $Q = CV_0$ and the final energy stored in the capacitor is

$$E' = \frac{1}{2}CV_0^2$$

$\therefore$ Energy dissipated through the resistor is

$$E_D = E' - E$$
$$= \frac{1}{2}CV_0^2 - \frac{1}{3}CV_0^2 = \frac{1}{6}CV_0^2$$

So the correct option is (d).

16. In this case, the charge supplied by the battery is $Q_0 = CV_0$

$\therefore$ Energy supplied by battery $= Q_0V_0 = CV_0^2$

Energy stored in the capacitor is

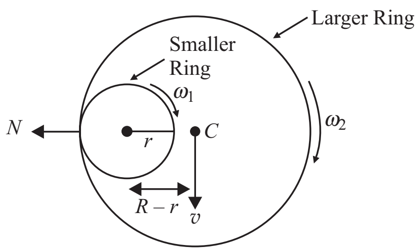
$$E_C = \frac{1}{2}Q_0V_0^2 = \frac{1}{2}CV_0^2$$

Energy dissipated through the resistor is

$$E_D = CV_0^2 - \frac{1}{2}CV_0^2 = \frac{1}{2}CV_0^2$$

$\therefore$ $E_C = E_D$. So in this case the correct option is (c).

**17.** Refer to the following figure.




Let the angular speeds of the smaller and larger rings be $\omega_1$ and $\omega_2$ respectively. Let $v$ be the linear speed of the centre $C$ of the larger ring. The centre of the larger ring moves in a circle of radius $(R - r)$. So

$$v = (R - r)\omega_2 \qquad ...(1)$$

Since there is no slipping at the point of contact,

$$r\omega_1 = R\omega_2 - v \qquad ...(2)$$

Using (1) in (2), we have

$$r\omega_1 = R\omega_2 - (R - r)\ \omega_2 = r\omega_2$$
$$\Rightarrow \quad \omega_1 = \omega_2 = \omega_0$$

Normal reaction at the point of contact is given by

$$N = Ma_c = M\omega_0^2\ (R - r)$$

For translational equilibrium in the vertical direction,

$$Mg = \mu N = \mu M\omega_0^2\ (R - r)$$

$\Rightarrow \omega_0 = \sqrt{\dfrac{g}{\mu(R-r)}}$, which is option (b).

**18.** The total kinetic energy $= \frac{1}{2}Mv_{cm}^2 + \frac{1}{2}I_{cm}\,\omega_0^2$

$$= \frac{1}{2}M\omega_0^2(R - r)^2 + \frac{1}{2}(MR^2)\omega_0^2$$
$$= \frac{1}{2}M\omega_0^2\left[(R-r)^2 + R^2\right]$$

So no option is correct.