Practice Paper – I

1. If z, z^2 and z^3 lie on a straight line, then
   (a) Re (z) + Im $(z) = 0$ (b) Im $(z) = 0$
   (c) Re $(z) = 0$ (d) Re (z) = Im (z)

2. Suppose $a \in \mathbf{N}$ $a > 1$. Let $\phi(x)$ be a polynomial with integral coefficients such that

   $(x^2 - x + a)\ \phi\ (x) = x^{13} + x + 90,$

   then a is equal to:
   (a) 2 (b) 3
   (c) 5 (d) 9

3. Suppose a, b, c are three complex number such that $|a| = |b| = |c| = 1$. Let

   $$z = \begin{vmatrix} 1 & 1 & 1 \\ a & b & c \\ \bar{a} & \bar{b} & \bar{c} \end{vmatrix}$$

   then z is equal to
   (a) $(\bar{a} + \bar{b} + \bar{c})\,(bc + ca + ab)$
   (b) $(1 - \bar{a}b)(1 - \bar{b}c)(1 - \bar{c}a)$
   (c) $(a - \bar{b})(b - \bar{c})(c - \bar{a})$
   (d) $a\bar{b} + b\bar{c} + c\bar{a}$

4. Number of 3×3 matrices A with real entries such that $A^2 = -I$ is
   (a) 0 (b) 1
   (c) 27 (d) infinite

5. The sum of all the four digit numbers that can be formed by using the digits 0, 1, 2, and 3 is
   (a) 38660 (b) 38662
   (c) 38664 (d) 38666

6. Coefficient of x^{35} in the expansion of $(1 + x^2)^{40}$ $\left(x^2 + 2 + \frac{1}{x^2}\right)^{-7}$ is
   (a) ${}^{40}C_{15}$ (b) ${}^{35}C_{15}$
   (c) ${}^{30}C_{10}$ (d) ${}^{25}C_{15}$

7. If sum of first 10 terms of an *A.P.* is 22/7 and sum of next 10 terms is π then comman difference of the *A.P.* is
   (a) 0
   (b) a non-zero integer
   (c) a non-zero rational number other than an integer
   (d) an irrational number.

8. If the statement $q \wedge r \to p$ is false, then truth values of p, q, r are respectively:
   (a) F, T, T (b) F, F, F
   (c) T, T, T (d) F, T, F

9. **Statement-1:** If $|ax| < 1$, $|bx| < 1$, then coefficient of x^n in the expansion of

   $$\frac{1}{(ax-1)\,(bx-1)} \text{ is } \frac{1}{a-b}\,(a^{n+1} - b^{n+1})$$

   (a) **Statement-2:** If $|ax| < 1$, coefficient of x^n in the expansion of $(1 - ax)^n$ is a^n

10. *A* machine when correctly set up, it produces 90% of acceptable items. If it is incorrectly set up, it produces only 50% acceptable items. Past experience shows that 80% of the set ups are correctly done. Suppose after a certain set, the machine produces 2 acceptable items.

    Statement-1: The probability that the machine is not correctly set up is 25/349

    Statement-2: The probability that the machine is correctly set up is 324/349

11. **Statement-1:** The cartesian equation of the plane $\mathbf{r} = (1 + \lambda - \mu)\mathbf{i} + (2 - \lambda)\mathbf{j} + (3 - 2\lambda + 2\mu)\mathbf{k}$ is $2x + z = 5$

    Statement-2: The vector equation of a line passing through (2, – 1, 1) and parallel to the line whose equation is $\frac{x-3}{2} = \frac{y+1}{7} = \frac{z-2}{3}$ is

    $$\mathbf{r} = 2\mathbf{i} + 7\mathbf{j} - 3\mathbf{k} + \lambda\,(2\mathbf{i} - \mathbf{j} + \mathbf{k})$$

12. **Statement-1:** A quadratic equation whose roots are $\operatorname{cosec}^2 \theta$ and $\sec^2 \theta$ can be.

$x^2 - 5x + 5 = 0.$

Statement-2: If $\alpha + \beta = \pi/4$, then the maximum value of $2 \sin\alpha \sin\beta$ is $\dfrac{\sqrt{2}-1}{\sqrt{2}}$

13. *ABCD* is a square of unit area *E* and *H* are the middle points of *AB* and *AD* respectively, *F* divides *BC* and *G* divides *DC* in the ratio 2 : 1. Area of the quadrilateral *EFGH* (in sq. units) is.

(a) $\frac{37}{72}$ (b) $\frac{1}{2}$
(c) $\frac{35}{72}$ (d) $\frac{2}{3}$.

14. The number of points with integral coordinates that are interior to the circle $x^2 + y^2 = 16$ is

(a) 45 (b) 46
(c) 44 (d) 49.

15. From a point *A* (*a*, 0), normals are drawn to the parabola $y^2 = 8x$. If the foot of the normals on the parabola form an equilateral triangle, then the value of *a* is

(a) 24 (b) 28
(c) 20/3 (d) 4/3.

16. e_1 is the eccentricity of the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ when latus rectum is half of its major axis and e_2 is the eccentricity when latus rectum is half of its minor axis, the value of $e_1^2 + e_2^2$ is equal to

(a) 3/4 (b) 1/2
(c) 5/4 (d) 7/4

17. The distance of the point *P*(3, 8, 2) from the line $\frac{x-1}{2} = \frac{y-3}{4} = \frac{z-2}{3}$ measured parallel to the plane $3x + 2y - 2z + 15 = 0$ in units is

(a) 5 (b) 6
(c) 7 (d) 8

18. The value of $\sin \frac{\pi}{14} - \sin \frac{3\pi}{14} + \sin \frac{5\pi}{14}$ is equal to

(a) 3/2 (b) 1/2
(c) – 1/2 (d) – 3/2

19. The value of $\sin (3 \sin^{-1} (2/3) + \cos (2 \cos^{-1} (1/3))$ is equal to

(a) 28/27 (b) 40/27
(c) 1/27 (d) 22/27

20. *A* tower *AP* subtends an angle α at a point *B* on the ground. Another tower *BQ* subtends an angle β at a point *C* on the ground lying on the line *AB* produced to *C*. Tower *AP* substends an angle γ at *C*. If the height of *AP* is twice the height of *BQ*, then $\cot \beta$ is equal to

(a) $2(\cot \alpha - \cot \gamma)$ (b) $2(\cot \gamma - \cot \alpha)$
(c) $2(\cot \alpha + \cot \gamma)$ (d) $2 \cot \gamma \cot \alpha$.

21. Let *f* be a real valued function satisfying $f(x) + 4 f\left(\frac{100}{x}\right) = 5x$ for all $x \in \mathbf{R}$, then $f(5)$ is equal to

(a) 25 (b) 10
(c) 15 (d) $\frac{27}{5}$

22. $\lim_{x \to 0} \left[(1-e^x) \frac{\sin x}{|x|} \right]$ is equal to ([*x*] being greatest integer less than or equal to *x*)

(a) 1 (b) – 1
(c) 0 (d) 1/2

23. If $f(x) = \{x^2\} - (\{x\})^2$, where $\{x\}$ denotes the fractional part of *x* then

(a) *f* is continuous at $x = 3$ but not at $x = -3$
(b) *f* is continuous at $x = -3$ but not at $x = 3$
(c) *f* is continuous at $x = 3, -3$
(d) *f* is discontinuous at $x = 3, -3$.

24. If $x \sin y - \cos y + \cos 2y = 0$, then $\frac{dy}{dx}$ at $x = 0$ is

(a) 1 (b) $\frac{1}{2\sin 2y - \sin y}$
(c) $\frac{1}{2\cos y - 1}$ (d) $\frac{\sin y}{2\sin 2y - \sin y}$

25 On the curve $y = x^2 (x-2)^2$, number of points where the tangent is parallel to *x*-axis is

(a) 1 (b) 2
(c) 3 (d) 4

26. The value of $\int \frac{dx}{x^6 + x^4}$ is equal to

(a) $\tan^{-1} x + \frac{1}{x} - \frac{1}{x^3} + C$ (b) $\tan^{-1} x - \frac{1}{3x^3} + C$
(c) $\tan^{-1} x + \frac{1}{x} - \frac{1}{3x^3} + C$ (d) $\tan^{-1} x + \frac{2}{x} - \frac{1}{x^3} + C$

27. $\int_{-2}^{2} x^{10} \{x^7\} \, dx$ is equal to ($\{x\}$being the fractional part of *x*)

(a) $\frac{2^{17}}{17}$ (b) 2^{10}
(c) 2^{11} (d) $\frac{2^{11}}{11}$

28. For the differential equation $x\frac{dy}{dx} = y(\log y - \log x + 1)$, the general solution is given by

(a) $\log\frac{y}{x} = cx$ (b) $\log\frac{x}{y} = x + y + c$

(c) $\log\frac{x}{y} = cx$ (d) $\log\frac{y}{x} = Cy$

29. **Statement-1:** The area of the traingle whose vertices are the points (1, 2, 3), (– 2, 1, – 4), (3, 4, – 2) is $\sqrt{1218}$.

Statement-2: Area of a triangle with vertices A, B, C is $\frac{1}{2}|\mathbf{AB} \times \mathbf{AC}|$.

30. If a distribution takes values 0, 1, 2,... 10 with frequencies 1, 10, ${}^{10}C_2$,..., ${}^{10}C_9$, 1.

Statement-1: The mean of the distribution is 5

Statement-2: $\sum_{x=0}^{n} x\,{}^nC_x = n2^{n-1}$.

Answers

1. (b)	**2.** (a)	**3.** (b)	**4.** (a)
5. (c)	**6.** (b)	**7.** (d)	**8.** (a)
9. (a)	**10.** (a)	**11.** (c)	**12.** (b)
13. (c)	**14.** (a)	**15.** (b)	**16.** (c)
17. (c)	**18.** (b)	**19.** (c)	**20.** (b)
21. (a)	**22.** (b)	**23.** (a)	**24.** (d)
25. (c)	**26.** (c)	**27.** (d)	**28.** (a)
29. (d)	**30.** (d)		

Hints and Solutions

1. Suppose z, z^2 and z^3 lie on the straight line

$\bar{a}z + a\bar{z} + b = 0$

where $a \in \mathbf{C}$, $a \neq 0$ and $b \in \mathbf{R}$.

Then

$\bar{a}z + a\bar{z} + b = 0$

$\bar{a}z^2 + a\bar{z}^2 + b = 0$

$\bar{a}z^3 + a\bar{z}^3 + b = 0.$

Eliminating $\bar{a}$, a and b from the above equations, we get

$$\begin{vmatrix} z & \bar{z} & 1 \\ z^2 & \bar{z}^2 & 1 \\ z^3 & \bar{z}^3 & 1 \end{vmatrix} = 0$$

$\Rightarrow z\bar{z}(z - \bar{z})(\bar{z} - 1)(1 - z) = 0.$

If $z \notin \mathbf{R}$, then $z\bar{z} \neq 0$, z, $\bar{z} \neq 1$

$\Rightarrow z = \bar{z} \Rightarrow z \in \mathbf{R}$.

A contradiction. Thus, $z \in \mathbf{R}$ and hence, Im $(z) = 0$.

2. $x = 0$ and 1, we get $a\,\phi(0) = 90$, $a\,\phi(1) = 92$

$\Rightarrow a\,[\phi(1) - \phi(0)] = 2.$

As a and $\phi(1) - \phi(0)$are integers and $a > 1$, we must have $a = 2$.

3. As $a\bar{a} = b\bar{b} = c\bar{c} = 1$, $\bar{a} = 1/a$, $\bar{b} = 1/b$ and $\bar{c} = 1/c$. Thus

$$z = \begin{vmatrix} 1 & 1 & 1 \\ a & b & c \\ 1/a & 1/b & 1/c \end{vmatrix}$$

$$= \frac{1}{abc}\begin{vmatrix} 1 & 1 & 1 \\ a & b & c \\ bc & ca & ab \end{vmatrix}$$

$$= \frac{1}{abc}(a - b)(b - c)(c - a)$$

$$= (1 - \bar{a}b)(1 - \bar{b}c)(1 - \bar{c}a)$$

4. Det $(A^2) = \det(-I)$

$(\det(A))^2 = -1$

There is no solution in $\mathbf{R}$.

5. Note that occurrence of zero at any of the four places does not affect the sum of the digits at that place.

Number of numbers having 1 at the unit's (or ten's or hundred's)place is $2 \times 2 \times 1 = 4$. Number of number's having 1 at thousand's place is $3 \times 2 \times 1 = 6$. Same is true for digits 2 and 3. Thus, sum of the digits at unit's, ten's or hundred's place is $(1 + 2 + 3) \times 4$.

Sum of the digits at the thousand's place is $6(1 + 2 + 3)$.

$\therefore$ required sum of the numbers is
$4(1 + 2 + 3)(1 + 10 + 100) + 6(1 + 2 + 3)(1000)$

$= 38664$

6. Let $E = (1 + x^2)^{40}\left(x^2 + 2 + \frac{1}{x^2}\right)^{-5}$

$$= (1 + x^2)^{40}\left(x + \frac{1}{x}\right)^{-5}$$

$$= (1 + x^2)^{40}\left(\frac{x^2 + 1}{x}\right)^{-5}$$

$$= (1 + x^2)^{40}\left(\frac{x}{x^2+1}\right)^5$$

$$= x^5 (1 + x^2)^{35}$$

$\therefore$ Coefficient of x^{35} in E

$$= \text{coefficient of } x^{30}(1 + x^2)^{35}$$
$$= \text{coefficient of } t^{15} (1 + t)^{35}$$
$$= {}^{35}C_{15}$$

7. Let a be the first term and d be the common difference of the A.P., then

$$\frac{10}{2}[2a+9d] = \frac{22}{7}$$

$$\frac{10}{2}[2a+19d] = \pi$$

substracting and simplifying we obtain

$$d = \frac{1}{50}\left(\pi - \frac{22}{7}\right),$$

which is an irrational number.

8. As $q \wedge r \to p$ is false, $q \wedge r$ must be T and p must be F.

But $q \wedge r$ is T if and only if $q = T$, $r = T$.

$\therefore$ truth values of p, q, r are respectively : F, T, T.

9. For $|ax| < 1$,

$(1 - ax)^{-1} = 1 + ax + a^2x^2 + ...$

$\therefore$ coefficient of x^n in the expansion of $(1 - ax)^{-1}$ is a^n.

Also, $\dfrac{1}{(ax-1)(bx-1)} = \dfrac{1}{(1-ax)(1-bx)}$

$$= \frac{1}{(a-b)x}\left[\frac{1}{1-ax} - \frac{1}{1-bx}\right]$$

Coefficient of x^n in the expansion of $\dfrac{1}{(ax-1)(bx-1)}$ is

$\dfrac{1}{a-b}$ [coefficient of x^{n+1} in $(1 - ax)^n$ – coefficient of x^{n+1} in $(1 - bx)^n$]

$$= \frac{a^{n+1} - b^{n+1}}{a-b}$$

10. Let E_1 denote the event that the machine is correctly set up and E_2 denote the event that the machine is not correctly set up and A denote the event that machine produces two acceptable items.

We are given

$$P(E_1) = 0.8, P(E_2) = 0.2$$
$$P(A|E_1) = (0.9)(0.9) = 0.81 \text{ and}$$
$$P(A|E_2) = (0.5)(0.5) = 0.25$$

Now, by the Bayes' rule

$$P(E_1|A) = \frac{P(E_2)P(A \mid E_2)}{P(E_1)P(A \mid E_1) + P(E_2)P(A \mid E_2)}$$

$$= \frac{(0.8)(0.81)}{(0.8)(0.81)+(0.2)(0.25)}$$

$$= \frac{648}{698} = \frac{324}{349}$$

Also, $P(E_2|A) = 1 - P(E_2'|A)$

$$= 1 - P(E_1|A) = \frac{25}{349}$$

11. In statement-1. $\mathbf{r} = \mathbf{i} + 2\mathbf{j} + 3\mathbf{k} + \lambda(\mathbf{i} - \mathbf{j} - 2\mathbf{k}) + \mu(-\mathbf{i} + 2\mathbf{k})$

$\Rightarrow$ The plane passes through

$\mathbf{a} = \mathbf{i} + 2\mathbf{j} + 3\mathbf{k}$ and $\mathbf{k}$ is parallel to $\mathbf{b}$ and $\mathbf{c}$

where $\mathbf{b} = \mathbf{i} - 2\mathbf{j} - \mathbf{k}$, $\mathbf{c} = -\mathbf{i} + 2\mathbf{k}$.

So the equation of the plane is

$(\mathbf{r} - \mathbf{a}).(\mathbf{b} \times \mathbf{c}) = 0$, $\mathbf{b} \times \mathbf{c} = -2\mathbf{i} - \mathbf{k}$

$\Rightarrow -2(x - 1) - (z - 3) = c \Rightarrow 2x + z - 5 = 0$

so statement-1 is true.

In statement-2, the line passes through the vector $2\mathbf{i} - \mathbf{j} + \mathbf{k}$ and is parallel to $2\mathbf{i} + 7\mathbf{j} - 3\mathbf{k}$ so its equation is $\mathbf{r} = 2\mathbf{i} - \mathbf{j} + \mathbf{k} + \lambda(2\mathbf{i} + 7\mathbf{j} - 3\mathbf{k})$ So statement-2 is false.

12. In statement-1, $\operatorname{cosec}^2\theta + \sec^2\theta = \dfrac{4}{\sin^2 2\theta} \geq 4$.

and $\operatorname{cosec}^2\theta \sec^2\theta = \dfrac{4}{\sin^2 2\theta} \geq 4$.

So statement-1 is true. In statement-2

$2 \sin \alpha \sin \beta = 2 \sin \alpha \sin (\pi/4 - \alpha)$

$$= \frac{2}{\sqrt{2}}\sin\alpha(\cos\alpha - \sin\alpha)$$

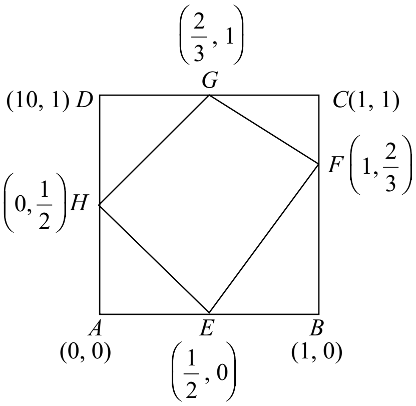
$$= \frac{1}{\sqrt{2}}\sin 2\alpha - \frac{1}{\sqrt{2}}(1-\cos 2\alpha)$$

$$= \frac{1}{\sqrt{2}}(\sin 2\alpha + \cos 2\alpha) - \frac{1}{\sqrt{2}}$$

$$= \sin(\pi/4 + 2\alpha) \quad \frac{1}{\sqrt{2}} \leq 1 - \frac{1}{\sqrt{2}} = \frac{\sqrt{2}-1}{\sqrt{2}}$$

so statement-2 is also true but not a correct explanation for statement-1.

13. $A(0, 0)$, $B(1, 0)$, $C(1, 1)$, $D(0, 1)$ be the vertices of the square $ABCD$, then



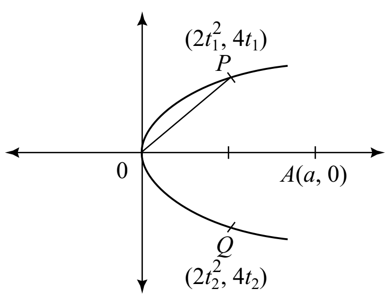
$E\left(\frac{1}{2},0\right), F\left(1,\frac{2}{3}\right), G\left(\frac{2}{3},1\right), H\left(0,\frac{1}{2}\right)$. and GF ∥ HE so $EFGH$ is a trapezium with $GF \parallel HE$, $=$ $|GF| = \frac{\sqrt{2}}{3} |HE| = \frac{\sqrt{2}}{2}$, Equation of HE is $x + y = \frac{1}{2}$ and of GF is $x + y = \frac{5}{3}$.

Distance between GF and HE = $\left|\frac{\frac{5}{3}-\frac{1}{2}}{\sqrt{1+1}}\right| = \frac{7}{6\sqrt{2}}$.

So the required Area $= \frac{1}{2}\left(\frac{\sqrt{2}}{3}+\frac{\sqrt{2}}{2}\right)\times\frac{7}{6\sqrt{2}} = \frac{35}{72}$

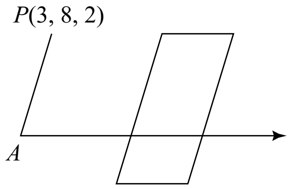
14. $x = 0, \pm 1, \pm 2, \pm 3$, $y = 0, \pm 1, \pm 2$
$x = 0, \pm 1, \pm 2$ $\quad$ $y = 0, \pm 1, \pm 2, \pm 3$
Required points are
(0, 0), (0 ± 1), (± 1, 0), (0, ± 2), (± 2, 0), (0 ± 3), (± 3, 0), (± 1, ± 1), (± 1, ± 2), (± 1, ± 3), (± 2, ± 1), (± 2, ± 2), (± 2, ± 3), (± 3, ± 1), (± 3, ± 2) which are 45.

15. One of the vertices of the traingle is the origin O. Let the other two vertices be $P(2t_1^2, 4t_1)$ and $Q(2t_2^2, 4t_2)$
Normal at P is $y = -t_1x + 4t_1 + 2t_1^3$ which passes through $(a, 0)$ if $t_1 = 0$ or $2t_1^2 = a - 4$., similarly $2t_2^2 = a - 4$
$\Rightarrow t_2 = -t_1$. $\Rightarrow PQ \perp OA$
$\underline{|POA} = \pi/6 \Rightarrow t_1 = 2\sqrt{3} \Rightarrow a = 28$



16. Let $a > b$, Length of the latus rectum $= 2b^2/a$
when $\frac{2b^2}{a} = a \Rightarrow 2b^2 = a^2$ and $e_1^2 = \frac{a^2-b^2}{a^2} = \frac{1}{2}$
when $\frac{2b^2}{a} = b \Rightarrow a = 2b$ and
$e_2^2 = \frac{a^2-b^2}{a^2} = \frac{3}{4}$. So $e_1^2 + e_2^2 = \frac{5}{4}$.

17. Let the coordinates of A be $(2r + 1, 4r + 3, 3r + 2)$ AP in parallel to the plane $3x + 2y - 2z + 15 = 0$

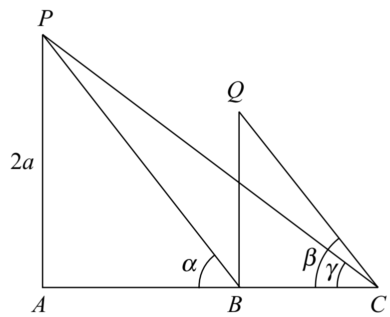


$\Rightarrow 3(2r + 1 - 3) + 2(4r + 3 - 8) - 2(3r + 2 - 2) = 0$
$\Rightarrow r = 2 \Rightarrow$ coordinates of A are (5, 11, 8) and AP = 7 units.

18. $\sin\frac{\pi}{14} - \sin\frac{3\pi}{14} + \sin\frac{5\pi}{14}$.
$= -\left(\cos\frac{4\pi}{7} + \cos\frac{2\pi}{7} + \cos\frac{6\pi}{7}\right) = \frac{1}{2}$(Example 29, chapter 22)

19. Required value is equal to
$3(2/3) - 4(2/3)^3 + 2(1/9)^{-1} = 1/27$

20. Let $BQ = a$, $AP = 2\alpha$ $AB = 2a \cot\alpha$, $BC = a \cot\beta$ $AC = 2a \cot\gamma$.
$AC = AB + BC$
$\Rightarrow 2 \cot\gamma = 2 \cot\alpha + \cot\beta$.



21. Replace x by $\frac{100}{x}$ in the given equation
$$f(x) + 4f\left(\frac{100}{x}\right) = 5x \qquad \text{(i)},$$
we have $$f\left(\frac{100}{x}\right) + 4f(x) = \frac{500}{x} \qquad \text{(ii)}$$

Multiply the equation (ii) and subtracting (i), we have

$$16f(x) - f(x) = \frac{2000}{x} - 5x$$

$$\Rightarrow 15f(x) = \frac{2000}{x} - 5x$$

$$\Rightarrow \quad f(x) = \frac{1}{3}\left[\frac{400}{x} - x\right]$$

so $\quad f(5) = \frac{1}{3}[80 - 5] = 25.$

22. $1 - e^x < 0$, for $x > 0$ and $0 < \frac{\sin x}{|x|} = \frac{\sin x}{x} < 1$

so for $x > 0$, $(1 - e^x)\frac{\sin x}{|x|} < 0$. For small $x > 0$, $(1 - e^x)\frac{\sin x}{x} > -1$. Hence $\left[(1-e^x)\frac{\sin x}{x}\right] = -1$, $x > 0$

Hence $\lim\limits_{x\to 0+}\left[(1-ex)\frac{\sin x}{|x|}\right] = -1.$

For $x < 0$, $1 - e^x > 0$ and $\frac{\sin x}{|x|} = -\frac{\sin x}{x} < 0$ so $-1 < (1 - e^x)\frac{\sin x}{|x|} < 0$, hence $\lim\limits_{x\to 0-}\left[(1-e^x)\frac{\sin x}{|x|}\right] = -1.$

23. $\lim\limits_{x\to 3+} f(x) = 0 - 0 = 0$ as $x \to 3+ \Rightarrow x^2 \to 9+$

$\lim\limits_{x\to 3-} f(x) = 1 - 1 = 0$ since $x \to 3- \Rightarrow x^2 \to 9-$

so $\{x^2\} \to 1$.

f is continuous at $x = 3$.

$$\lim_{x\to -3-} f(x) = 0 - 1 = -1$$

$$\lim_{x\to -3+} f(x) = 1 - 0 = 1$$

Hence f is discontinuous at $x = -3$

24. Differentiating, we get

$$x\cos y\frac{dy}{dx} + \sin y + \sin y\frac{dy}{dx} - 2\sin 2y\frac{dy}{dx} = 0$$

$$(x\cos y + \sin y - 2\sin 2y)\frac{dy}{dx} = \sin y$$

$$\left.\frac{dy}{dx}\right|_{x=0} = \frac{\sin y}{2\sin 2y - \sin y}.$$

25. $\frac{dy}{dx} = 2x\,(x-2)^2 + 2x^2(x-2) = 2x(x-2)[x - 2 + x]$
$= 4x\,(x-2)\,(x-1)$

$$\frac{dy}{dx} = 0 \Rightarrow x = 0, 1, 2$$

For $x = 0$, $y = 0$; for $x = 1$, $y = 1$ and $x = 2$, $y = 0$
Hence the required points are (0, 0), (1, 1), (2, 0).

26. $\frac{1}{x^6 + x^4} = \frac{1}{x^4\,(x^2+1)} = \frac{1}{x^2}\left[\frac{1}{x^2} - \frac{1}{x^2+1}\right]$
$= \frac{1}{x^4} - \frac{1}{x^2} + \frac{1}{x^2+1}$

$$\int\frac{dx}{x^6 + x^4} = -\frac{1}{3x^3} + \frac{1}{x} + \tan^{-1}x + C$$

27. $\int_{-2}^{2} x^{10}\{x^7\}dx = \int_{-2}^{0} x^{10}\{x^7\}dx + \int_{0}^{2} x^{10}\{x^7\}dx = I_1 + I_2$

$$I_1 = \int_{-2}^{0} x^{10}\{x^7\}dx = -\int_{2}^{0} t^{10}\{-t^7\}dt = \int_{0}^{2} t^{10}\{-t^7\}dt$$

Thus $I_1 + I_2 = \int_0^2 x^{10}(\{x\} + \{-x\})\,dx$

$$= \int_0^1 x^{10}(\{x\} + \{-x\})\,dx + \int_0^2 x^{10}(\{x\} + \{-x\})\,dx$$

$$= \int_0^1 x^{10}\,dx + \int_1^2 x^{10}\,dx\ (\{x\} + \{-x\} = 1, x \notin I)$$

$$= \int_0^2 x^{10}\,dx = \frac{2^{11}}{11}.$$

28. The given differential can be written as

$$\frac{dy}{dx} = \frac{y}{x}\left[\log\frac{y}{x} + 1\right]$$

Put $y = Vx$, so $V + x\frac{dV}{dx} = V(\log V + 1)$

$$\Rightarrow \quad x\frac{dV}{dx} = V\log V$$

$\Rightarrow \quad \frac{dV}{V\log V} = \frac{dx}{x}$. Integrating we have

$$\log(\log V) = \log x + \text{Const.}$$

$\Rightarrow \quad \log V = Cx$ i.e. $\log\frac{y}{x} = Cx.$

29. **AB** = − 3**i** − **j** −7**k**, **AC** = 2**i** + 2**j** − 5**k**
AB × **AC** = 19**i** − 29**j** − 4**k**

Area of the triangle $= \frac{1}{2}|\mathbf{AB}\times\mathbf{AC}|$

$$= \frac{1}{2}\sqrt{19^2 + 29^2 + 4^2}$$

$$= \frac{1}{2}\sqrt{1218}\,.$$

30. $N = \Sigma f = 1 + {}^{10}C_1 + \ldots + {}^{10}C_9 + {}^{10}C_{10} = (1+1)^{10} = 2^{10}$

$$\Sigma f x = \sum_{x=0}^{10} x\,{}^{n}C_x = 10.2^9$$

(According to statement 2 which is true)

$$\bar{x} = \frac{10.2^9}{2^{10}} = 5$$