Practice Paper – II

1. If a, b, c, $\in$ **R**, $a^2 > b$ and $b < 0$. Then number of complex numbers z satisfying the equation $|z|^2 + 2a|z| + b = 0$ is:
   (a) 0 (b) 1
   (c) 2 (d) infinite
2. The number of real solutions of the equation
   $\sqrt{x^2+6x+25}+\sqrt{2x^2+12x+22}=6$ is:
   (a) 0 (b) 1
   (c) 2 (d) 4
3. Suppose $a \in \mathbf{R}$ and $f(x) = \begin{vmatrix} 1-x & a & a^2 \\ a & a^2-x & a^3 \\ a^2 & a^3 & a^4-x \end{vmatrix}$
   The number of non-zero roots of $f(x) = 0$ is:
   (a) 0 (b) 1
   (c) 2 (d) 3
4. Let $A = (a_{ij})_{3\times 3}$ where $a_{ij} \in \mathbf{C}$. We denote the transpose conjugate of A by A^*, that is, $A^* = (b_{ij})_{3\times3}$ where $b_{ij} = a_{ji}$ $\forall$ i and j. If $A^* = A$ and $A^* = A^{-1}$, then
   (a) $A^2 = I$ (b) $A^2 = -I$
   (c) $A^2 = 0$ (d) none of these.
5. A bag contains 6 identical cards marked with –1; 6 identical cards marked with 0 and 6 identical cards marked with 1, eighteen cards in all. The number of ways of picking up 6 cards one by one from the bag so that their sum is 1, is:
   (a) 126 (b) 141
   (c) 147 (d) 137
6. If the ratio of 2nd term to that of 3rd term in the expansion of $(ax^{-1/2} + xa^{-1/2})^n$ is 2/11, then n is equal to:
   (a) 11 (b) 12
   (c) 23 (d) 22
7. Sum to n terms of the series
   $$\frac{1}{(2)(3)}(2)+\frac{2}{(3)(4)}(2^2)+\frac{3}{(4)(5)}(2^3)+\ldots \text{ is}$$
   (a) $\frac{2^n}{2n+4}$ (b) $\frac{2^n}{n+1}-\frac{1}{2}$
   (c) $\frac{2^{n+1}}{n+2}-1$ (d) $\frac{2^{n+1}}{n+2}-2$
8. Suppose p, q and r are three logical statements given as follows:
   p: Rehman is rich
   q: Rehman is a doctor
   r: Rehman is a dermatologist. Contrapositive of
   $$qVr \to p$$
   is
   (a) Rehman is not rich if Rehman is not a doctor.
   (b) If Rehman is not rich then Rehman is not a dermatologist.
   (c) If Rehman is not rich then Rehman is neither a doctor nor a dermatologist.
   (d) None of these.
9. Suppose A and B are two events such that $P(A \cap B) > 0$.
   Statement-1: If $P(A|B) \geq P(A)$
   then $P(B|A) \geq P(B)$
   Statement-2: If $P(B \cup C|A) = P((B|A) + P(C|A) - P(B \cap C|A)$
10. **Statement-1:** Let $A = \begin{bmatrix} (1+x)^2 & (1-x)^2 & -(2+x^2) \\ 2x+1 & 3x & 1-5x \\ x+1 & 2x & 2-3x \end{bmatrix}$
    and $B = \begin{bmatrix} (1+x)^2 & 2x+1 & (x+1) \\ (1-x)^2 & 3x & 2x \\ 1-2x & 3x-2 & 2x-3 \end{bmatrix}$
    then $A + B'$ is a singular matrix.

Statement-2: In a square matrix use of $R_i \to R_i + k R_j$ for $i \neq j$, $k \in \mathbf{R}$ does not change it from non-singular to singular matrix.

11. **Statement-1:** If LM and $L'M'$ are the latus rectums of the parabola $4y^2 = x$ and $y^2 = 8x$ respectively, then area of the quadrilateral $LMM'L'$ is equal to $\frac{1023}{128}$ sq. units

Statement-2: Area of a trapezium is equal to half the sum of the lengths of parallel sides multiplied by the distance between them.

12. **Statement-1:** $\sin 3(\tan^{-1}x + \cot^{-1}x) + \tan\frac{1}{2}(\sin^{-1}x + \cos^{-1}x) = 0$

Statement-2: $\tan^{-1}\frac{2}{5} + \cot^{-1}\frac{7}{3} = \frac{\pi}{4}$.

13. A line passing through the point $A(1, 2)$ makes an angle $\pi/4$ with the positive direction of x-axis length of the line segment cut off between the point A and the line $x + y + 1 = 0$ (in units) is

(a) $\sqrt{2}$ (b) $2\sqrt{2}$
(c) 2 (d) $4\sqrt{2}$

14. Two chords of the circle $x^2 + y^2 - 2x - 2y + c = 0$ passing through the point $(1, 1+c)$ are bisected by the line $y = x$ for

(a) no real value of c (b) all real values of c
(c) only two values of c (d) only one value of c

15. The minimum distance between the curves $y^2 = 4x$ and $x^2 + y^2 - 12x + 31 = 0$ is

(a) $2\sqrt{5}$ (b) $2+\sqrt{5}$
(c) $\sqrt{5}$ (d) $\sqrt{2}+\sqrt{5}$

16. If the latus rectum of a hyperbola through one focus subtends an angle of 60° at the other focus, then the eccentricity of the hyperbola is

(a) $\sqrt{3}$ (b) 2
(c) $2+\sqrt{3}$ (d) none of these

17. If the lines $\frac{x-1}{2} = \frac{y-2}{1} = \frac{z+1}{k}$ and $\frac{x+1}{k} = \frac{y-1}{2} = \frac{z+2}{1}$ are coplanar, then cosine of the angle θ between these lines is given by

(a) 5/6 or –1/6 (b) 5/6 or 2/3
(c) 8/9 or 2/3 (d) 8/9 or 2/5

18. If $\sin(\pi \sin x) = \cos(\pi \cos x)$, then $\sin(x + \pi/4)$ is equal to

(a) $\cos(\pi/4)\cos(\pi/6)$ (b) $\sin(\pi/4)\sin(\pi/6)$
(c) $-\cos(\pi/4)\sin(\pi/6)$ (d) $\sin(\pi/4)\cos(\pi/4)$

19. The number of positive integral pairs (a, b) satisfying the equation $\tan^{-1}a = \tan^{-1}b = \tan^{-1}(-2)$ is

(a) 0 (b) 1
(c) 2 (d) infinite

20. A tower subtends angle $\tan^{-1}(1/\sqrt{7})$ and $\tan^{-1}(1/3)$ at two points A and B on the ground, 100 m. apart. AB subtends a right angle at O, the foot of the tower on the ground. The height of the tower is

(a) 25 m. (b) $10\sqrt{10}$ m
(c) 50 m. (d) 10 m.

21. The domain of the function $y = \sqrt{\sin x} + \sqrt{9 - x^2}$ is

(a) $[-3, \pi]$ (b) $[0, \pi]$
(c) $[0, 3]$ (d) $-3, 3]$

22. $\lim_{n\to\infty}\left(\cos\frac{x}{2}\cos\frac{x}{4}....\cos\frac{x}{2^{n+1}}\right)$ is equal to

(a) $\frac{\cos x}{x}$ (b) $\frac{\sin x}{x}$
(c) $\frac{\sin x}{2x}$ (d) $\frac{2\sin x}{x}$

23. If $f(x) = \frac{x - e^x + \cos 2x}{\sin^2 x}$, $x \neq 0$ and f is defined at 0 so that f is a continuous function then

(a) $f(0) = \frac{5}{2}$ (b) $[f(0)] = -2$
(c) $[f(0)]\{f(0)\} = -\frac{3}{2}$ (d) $\{f(0)\} = -\frac{1}{2}$.

where $[x]$ and $\{x\}$ are greatest integer $\leq x$ and fractional part of x respectively.

24. Let f be defined by

$$f(x) = \begin{Bmatrix} x^n \sin\frac{1}{x} & , & x \neq 0 \\ O & , & x = 0 \end{Bmatrix}$$

The least value of $n \in \mathbf{N}$ for which f' is continuous is

(a) 1 (b) 4
(c) 2 (d) 3

25. Let f be defined by $f(x) = |x|^m\,|x-1|^n$, $m, n > 1$ and $x \in \mathbf{R}$. The maximum value of f on [0, 1] is given by

(a) $\frac{m^m n^n}{(m+n)^{m+n}}$ (b) 1
(c) $\frac{m^n n^m}{(m+n)^{m+n}}$ (d) $\frac{(mn)^{mn}}{(m+n)^{m+n}}$

26. $\int \frac{\operatorname{cosec}^2 x - n}{\cos^n x} dx$ is equal to

(a) $-\frac{\cot x}{\cos^n x} + C$ (b) $\frac{\tan x}{\cos^n x} + C$

(c) $-\tan x \sec^n x + C$ (d) $\cot x \sec^n x + C$

27. The scalar triple product [**a** + **b**, **b** + **c**, **c** + **a**] is equal to (**a**, **b**, **c** are non-coplanar vectors)

(a) 0 (b) [**a b c**]

(c) 2[**a b c**] (d) – [**a b c**]

28. The mean and range of $x + y$, $x - y$, $2x + y$, $2y + x$ is 20 and 30 respectively ($0 < y \le x$). The median will be

(a) 40 (b) 25

(c) 30 (d) 20

29. **Statement 1:**

$$\int_0^{\pi} x \sin^5 x \cos^4 x \, dx = \frac{\pi}{2} \int_0^{\pi} \sin^5 x \cos^4 x \, dx$$

Statement 2: $\int_a^b x\, f(x) dx = \frac{a+b}{2} \int_a^b f(x) dx$

30. **Statement 1:** The differential equation of the family of circles touching the x-axis at origin is a homogeneous first order differential equation.

Statement 2: The first order homogeneous equation can be solved by putting $y = vx$

Answers

1. (d)	**2.** (b)	**3.** (b)	**4.** (a)
5. (a)	**6.** (b)	**7.** (c)	**8.** (c)
9. (b)	**10.** (a)	**11.** (a)	**12.** (b)
13. (b)	**14.** (b)	**15.** (c)	**16.** (a)
17. (b)	**18.** (b)	**19.** (c)	**20.** (a)
21. (c)	**22.** (b)	**23.** (c)	**24.** (d)
25. (a)	**26.** (a)	**27.** (c)	**28.** (b)
29. (c)	**30.** (b)		

Hints and Solutions

1. $|z| = -a \pm \sqrt{a^2 - b}$

As $b < 0$, $\sqrt{a^2 - b} > |a| \ge a$

$\Rightarrow -a + \sqrt{a^2 - b} > 0.$

Also, $|z| = -a + \sqrt{a^2 - b}$ is satisfied by infinite values of z.

2. Write the given equation as

$$\sqrt{(x+3)^2 + 16} + \sqrt{2(x+3)^2 + 4} = 6. \quad (1)$$

For each $x \in \mathbf{R}$, we get

LHS of (1) $\ge \sqrt{16} + \sqrt{4} = 4 + 2 = 6.$

and the equality holds if $x + 3 = 0$

i.e. if $x = -3$.

3. Write $f(x) = \Delta_1 - x\Delta_2$

where

$$\Delta_1 = \begin{vmatrix} 1 & a & a^2 \\ a & a^2 - x & a^3 \\ a^2 & a^3 & a^4 - x \end{vmatrix}$$

Applying $C_2 \to C_2 - aC_1$

and $C_3 \to C_3 - a^2C_1$, we get

$$\Delta_1 = \begin{vmatrix} 1 & a & 0 \\ a & -x & 0 \\ a^2 & 0 & -x \end{vmatrix} = x^2$$

and $$\Delta_2 = \begin{vmatrix} 1 & a & a^2 \\ 0 & a^2 - x & a^3 \\ 0 & a^3 & a^4 - x \end{vmatrix}$$

$$= (a^2 - x)(a^4 - x) - a^6$$

$$= -x(a^2 + a^4) + x^2$$

Thus $f(x) = x^2(1 + a^2 + a^4) - x^3$

This shows that $f(x) = 0$,

for only one non-zero value of x.

4. $A = A^* = A^{-1} \Rightarrow A^2 = I$

5. Suppose a cards marked with –1, b cards marked with 0 and c cards marked with 1 are taken out. Then

$(-1)a + (0)b + (1)c = 1$

and $a + b + c = 6$.

$\Rightarrow \quad c = a + 1,\ 2a + b = 5.$

Possible solutions are $(a, b, c) = (0, 5, 1), (1, 3, 2)$ and $(2, 1, 3)$.

Thus, the number of drawing cards is

$$\frac{6!}{5!1!} + \frac{6!}{1!3!2!} + \frac{6!}{2!1!3!} = 126$$

6. $T_2 = {}^nC_1 (ax^{-1/2})^{n-1} (xa^{-1/2})$

and $T_3 = {}^nC_2 (ax^{-1/2})^{n-2} (xa^{-1/2})^2$

Now,

$$\frac{2}{11} = \frac{T_2}{T_3} = \frac{2}{n-1} \cdot \frac{ax^{-1/2}}{xa^{-1/2}}$$

$$= \frac{2}{n-1}\left(\frac{a}{x}\right)^{3/2}$$

We must have $n = 12$ and $x = a$.

7. rth term of the series is

$$t_r = \frac{r(2^r)}{(r+1)(r+2)}$$

$$= \left[\frac{2}{r+2} - \frac{1}{r+1}\right](2^r)$$

$$= \frac{2^{r+1}}{r+2} - \frac{2^r}{r+1}$$

$$\Rightarrow \quad \sum_{r=1}^{n} t_r = \sum_{r=1}^{n}\left(\frac{2^{r+1}}{r+2} - \frac{2^r}{r+1}\right)$$

$$= \frac{2^{n+1}}{n+2} - \frac{2}{2} = \frac{2^{n+1}}{n+2} - 1$$

8. Contrapositive of $qVr \to p$ is

$$\sim p \to \sim(q \vee r)$$

or $\quad \sim p \to (\sim q) \wedge (\sim r)$

It means if Rehann is not rich then he is neither a doctor nor a dermatologist.

9. As $P(\cdot|A)$ is a prabability function, statement-2 is true.

Also, $P(A)$, $P(B) \geq 0, P(A \cap B) > 0$, and

$$P(A|B) = \frac{P(A \cap B)}{P(B)} \geq P(A)$$

$$\Rightarrow \quad P(A \cap B) \geq P(A)\, P(B)$$

$$\Rightarrow \quad P(B|A) = \frac{P(B \cap A)}{P(A)} \geq \frac{P(A)P(B)}{P(A)} = P(B)$$

Thus, both statements are true but statement-2 is not a correct explanation for statement-1.

10. For truth of statement-2, see theory of Chapters 4 and 5.

We have
$$A + B' = \begin{bmatrix} (1+x)^2 & (1-x)^2 & -(1+x)^2 \\ (2x+1) & 3x & -(2x+1) \\ (x+1) & 2x & -(x+1) \end{bmatrix}$$

Using $R_1 \to R_1 + R_3$, we get

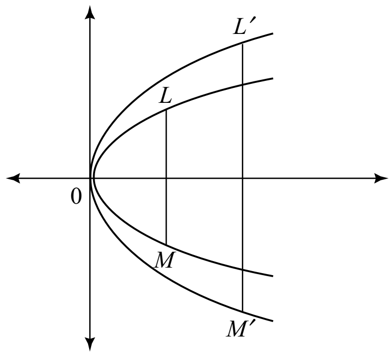
$$\det(A + B') = \begin{vmatrix} 0 & (1-x)^2 & -(x+1)^2 \\ 0 & 3x & -(2x+1) \\ 0 & 2x & -(x+1) \end{vmatrix} = 0.$$

Thus, $A + B'$ is not invertible.

11. Equation of LM, the latus rectum of $y^2 = \frac{1}{4}x$ is $x = \frac{1}{16}$ and its length is $\frac{1}{4}$. Equation of $L'M'$, the latus rectum of $y^2 = 8x$ is $x = 2$ and its length is 8.

So $LMM'L'$ is trapezium. Statement-2 is true and using the result, the area of the required quadrilateral is

$$\frac{1}{2}\left(\frac{1}{4}+8\right)\left(2-\frac{1}{16}\right) = \frac{1023}{128}.$$



12. In statement-1, $\sin 3(\pi/2) + \tan(1/2)(\pi/2)$

$$= \sin(3\pi/2) + \tan(\pi/4) = -1 + 1 = 0$$

So statement-1 is True.

In statement-2, $\tan^{-1}\frac{2}{5} + \cot^{-1}\frac{7}{3} = \tan^{-1}\frac{2}{5} + \tan^{-1}\frac{3}{7}$

$$= \tan^{-1}\frac{\frac{2}{5}+\frac{3}{7}}{1-\frac{2}{5}\times\frac{3}{7}} = \tan^{-1}1 = \pi/4.$$

So statement-2 is also true but is not a correct explanation for statement-1.

13. Equation of the line through $A(1, 2)$ and making an angle $\pi/4$ with positive x-axis is

$$\frac{x-1}{\cos\pi/4} = \frac{y-2}{\sin\pi/2} = r \text{ (say)}$$

Coordinates of any point P on this line at a distance r from A are $\left(1+\frac{r}{\sqrt{2}}, 2+\frac{r}{\sqrt{2}}\right)$

If P lies on $x + y + 1 = 0$, Then

$$1+\frac{r}{\sqrt{2}}+2+\frac{r}{\sqrt{2}}+1 = 0 \quad \Rightarrow r = -2\sqrt{2}$$

So the required length $= 2\sqrt{2}$

14. Let (α, α) be any point on the line $y = x$.

Equation of the chord of the circle with (α, α) as the mid-point is

$$x\alpha + y\alpha - (x + \alpha) - (y + \alpha) + c$$

$= \alpha^2 + \alpha^2 - 2\alpha - 2\alpha + c$

If it passes through $(1, 1+c)$, then

$1 \cdot (\alpha - 1) + (\alpha - 1)(1 + c) = 2\alpha^2 - 2\alpha$

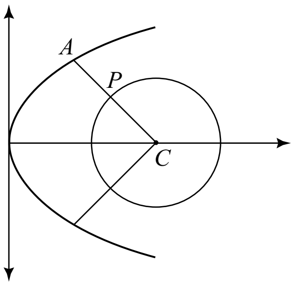
$\Rightarrow 2\alpha^2 - (4 + c)\alpha + 2 + c = 0$

which gives two real distinct values of α.

if $(4 + c)^2 > 8(2 + c)$ or if $c^2 > 0$

which is true for all real values of c.

15. Equation of normal to the parabola at $A(t^2, 2t)$ is $y = -tx + 2t + t^3$ which is a normal to the circle, if it passes through the centre $C(6, 0)$ of the give circle $\Rightarrow 0 = t^3 - 4t$ $\Rightarrow t = \pm 2$



Minimum distance between the curves is along a common normal. Coordinates of A are (4, 4)

Required distance is $AP = AC - PC$

$$= \sqrt{(4-6)^2 + (4-0)^2} - \sqrt{6^2 - 31}$$

$$= 2\sqrt{5} - \sqrt{5} = \sqrt{5}$$

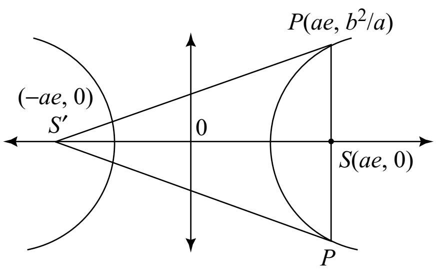
16. $\underline{|PS'S} = 30°$

$$\Rightarrow \quad \tan 30° = \frac{b^2/a}{2ae}$$

$$\Rightarrow \quad \frac{2}{\sqrt{3}} = \frac{a^2(1-e^2)}{a^2 e}$$

$$\Rightarrow \quad \sqrt{3}e^2 - 2e - \sqrt{3} = 0$$

$$\Rightarrow \quad e = \sqrt{3}$$



17. The lines are coplanar if

$$\begin{vmatrix} 1+1 & 2-1 & -1+2 \\ 2 & 1 & k \\ k & 2 & 1 \end{vmatrix} = 0 \Rightarrow k = 1 \text{ or } 4.$$

$$\cos\theta = \frac{2(k) + 1(2) + k(1)}{2^2 + 1^2 + k^2} = \frac{3k+2}{k^2+5}$$

$$= \frac{5}{6} \text{ or } \frac{2}{3}$$

18. $\sin(\pi \sin x) = \sin(\pi/2 - \cos x)$

$\Rightarrow \pi(\sin x + \cos x) = \pi/2$

$$\Rightarrow \frac{1}{2}(\sin x + \cos x) = \frac{1}{2\sqrt{2}}$$

$$\Rightarrow \sin(x + \pi/4) = \frac{1}{2\sqrt{2}} = \cos\frac{\pi}{4}\cos\frac{\pi}{3} = \sin\frac{\pi}{4}\sin\frac{\pi}{6}$$

19. We have $\dfrac{a+b}{1-ab} = -2$

$\Rightarrow a = \dfrac{b+2}{2b-1}$ so $(a, b) = (1, 3)$ or $(3, 1)$

[Note for $b \geq 4$, $b + 2 < 2b - 1$]

20. $\tan\alpha = \dfrac{1}{\sqrt{7}}, \tan\beta = \dfrac{1}{3}$

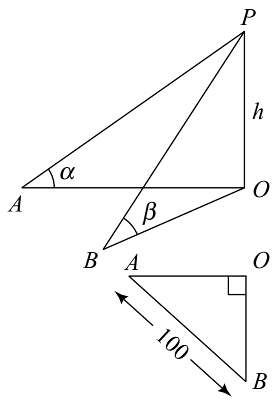
$OA = h\cot\alpha, OB = h\cot\beta$

$(AB)^2 = (OA)^2 + (OB)^2$

$\Rightarrow (100)^2 = h^2(\cot^2\alpha + \cot^2\beta)$

$$\Rightarrow \quad h^2 = \frac{100 \times 100}{7+9}$$

$$\Rightarrow \quad h = 25\,\text{m}.$$



21. y is defined if $\sin x \geq 0$ and $9 - x^2 \geq 0$

i.e. $x \in \bigcup_{n \in \mathbf{I}} [2n\pi, (2n+1)\pi]$ and $-3 \leq x \leq 3$

The intersection of two sets is [0, 3]

22. $\cos\dfrac{x}{2}\cos\dfrac{x}{4}\ldots\cos\dfrac{x}{2^{n+1}}$

$$= \frac{1}{2\sin\frac{x}{2^{n+1}}}\cos\frac{x}{2}\cos\frac{x}{4}\ldots\cos\frac{x}{2^n}\left(2\sin\frac{x}{2^{n+1}}\cos\frac{x}{2^{n+1}}\right)$$

$$= \frac{1}{2\sin\frac{x}{2^{n+1}}}\cos\frac{x}{2}\cos\frac{x}{4}\ldots\cos\frac{x}{2^n}\sin\frac{x}{2^n}$$

$$= \frac{\sin x}{2^{n+1}\sin\frac{x}{2^{n+1}}} = \frac{\sin x}{x}\frac{x/2^{n+1}}{\sin\left[x/2^{n+1}\right]}$$

Hence $\lim_{n\to\infty}\left(\cos\frac{x}{2}\cos\frac{x}{4}\ldots\cos\frac{x}{2^{n+1}}\right) = \frac{\sin x}{x}.$

23. $f(0) = \lim_{x\to 0} f(x) = \lim_{x\to 0}\frac{x - e^x + \cos 2x}{\sin^2 x}$

$$= \lim_{x\to 0}\frac{1 - e^x - 2\sin 2x}{2\sin x\cos x}\left(\frac{0}{0}\text{ form}\right)$$

$$= -\lim_{x\to 0}\frac{e^x - 1}{\sin 2x} - 2$$

$$= \frac{-1}{2}\lim_{x\to 0}\frac{e^x - 1}{x}\cdot\frac{2x}{\sin 2x} - 2 = -\frac{5}{2}$$

$[f(0)] = -3$, $\{f(0)\} = \frac{1}{2}$. Thus

$[f(0)]\ \{f(0)\} = -\frac{3}{2}$.

24. Since f is continuous, so $n \ge 1$, $f'(0) = \lim_{h\to 0}\frac{h^n \sin 1/h}{h}$

$= \lim_{h\to 0} h^{n-1}\sin^{1/h}$ which exists if $n > 1$. In this case

$$f'(x) = \begin{cases} n\,x^{n-1}\sin\frac{1}{x} \quad -x^{n-2}\cos\frac{1}{x} & , x \ne 0 \\ 0 & , x = 0 \end{cases}$$

Since f' is continuous so $\lim_{h\to 0} f'(x)$ exists

Thus $\lim_{h\to 0}\left(n\,h^{n-1}\sin\frac{1}{h} - h^{n-2}\cos\frac{1}{h}\right)$ exists

but this is possible if $n > 2$ Hence $n = 3$.

25. $$f(x) = \begin{cases} (-1)^{m+n}x^m(x-1)^n & , \text{ if } x < 0 \\ (-1)^n x^m (x-1)^n & , 0 \le x < 1 \\ x^m (x-1)^n & , x \ge 1 \end{cases}$$

$$f'(x) = 0 \text{ if } \frac{d}{dx}(x^m(x-1)^n) = 0$$

$\Rightarrow mx^{m-1}(x-1)^n + x^m n(x-1)^{n-1} = 0$

$\Rightarrow x^{m-1}(x-1)^{n-1}[m(x-1) + nx] = 0$

$\Rightarrow x = 0, 1, \frac{m}{m+n}$

$f(0) = 0 = f(1)$ and $f\left(\frac{m}{m+n}\right) = (-1)^n\frac{m^n n^n(-1)^n}{(m+n)^{m+n}}$

$$= \frac{m^m n^n}{(m+n)^n}.$$

26. $\int\frac{\operatorname{cosec}^2 x - n}{\cos^n x}dx = \int\cos^{-n}x\operatorname{cosec}^2 x\,dx - n\int\cos^{-n}x\,dx$

$$= -\cos^{-n}x\cot x + (n)\int\cos^{-(n+1)}x\sin x\cot x\,dx - n\int\cos^{-n}x\,dx + C$$

$$= -\frac{\cot x}{\cos^n x} + n\int\cos^{-n}x\,dx - n\int\cos^{-n}x\,dx + C$$

$$= -\frac{\cot x}{\cos^n x} + C$$

27. $[\mathbf{a} + \mathbf{b}, \mathbf{b} + \mathbf{c}, \mathbf{c} + \mathbf{a}] = ((\mathbf{a} + \mathbf{b}) \times (\mathbf{b} + \mathbf{c}))\cdot(\mathbf{c} + \mathbf{a})$

$= (\mathbf{a}\times\mathbf{b} + \mathbf{b}\times\mathbf{b} + \mathbf{a}\times\mathbf{c} + \mathbf{b}\times\mathbf{c})\cdot(\mathbf{c} + \mathbf{a})$

$= (\mathbf{a}\times\mathbf{b} + \mathbf{a}\times\mathbf{c} + \mathbf{b}\times\mathbf{c})\cdot(\mathbf{c} + \mathbf{a})$

$= (\mathbf{a}\times\mathbf{b})\cdot\mathbf{c} + (\mathbf{a}\times\mathbf{c})\cdot\mathbf{c} + (\mathbf{b}\times\mathbf{c})\cdot\mathbf{c}$
$\quad + (\mathbf{a}\times\mathbf{b})\cdot\mathbf{a} + (\mathbf{a}\times\mathbf{c})\cdot\mathbf{a} + (\mathbf{b}\times\mathbf{c})\mathbf{a}$

$= [\mathbf{a}, \mathbf{b}, \mathbf{c}] + 0 + 0 + 0 + 0 + \mathbf{a}\cdot(\mathbf{b}\times\mathbf{c})$

$= 2[\mathbf{a}\ \mathbf{b}\ \mathbf{c}]$

28. Since $x-y < x+y$, $x + y < 2y+x$ and $2y+x \le 2x+y$

The mean $= \frac{x+y+x-y+2x+y+2y+x}{4} = \frac{5x+3y}{4}$

So $5x + 3y = 80$

Also range $= 2x + y - (x - y) = x + 2y$

Thus $x + 2y = 30$ Solving, these equations, we have $x = 10$, $y = 10$. Hence the numbers are 0, 20, 30, 30.

Therefore the median is $\frac{20+30}{2} = 25$.

29. Since $\int_0^a f(x)dx = \int_0^a f(a-x)dx$, So

$$\int_0^\pi x\sin^5 x\cos^4 x\,dx = \int_0^\pi(\pi - x)\sin^5(\pi - x)\cos^4(\pi - x)dx$$

$$= \int_0^\pi(\pi - x)\sin^5 x\cos^4 x\,dx$$

$$= \pi\int_0^\pi\sin^5 x\cos^4 x\,dx - \int_0^\pi x\sin^5 x\cos^4 x\,dx$$

$$\int_0^\pi x\sin^5 x\cos^4 x\,dx = \frac{\pi}{2}\int_0^\pi\sin^5 x\cos^4 x\,dx$$

Take $f(x) = x$, $\int_a^b x\,f(x)dx = \frac{b^3 - a^3}{3}$

$$\frac{a+b}{2}\int_a^b f(x)dx = \frac{a+b}{2}\frac{b^2 - a^2}{2}$$

So, Statement-2 is not True.

30. An equation of a circle touching x-axis at origin is $x^2 + (y - a)^2 = a^2$ (i)

a being a parameter.

Differentiating, we get

$$2x + 2(y - a)\frac{dy}{dx} = 0$$

$$\Rightarrow \quad a = y + x\frac{dx}{dy}$$

Putting this in (i), we get

$$x^2 + \left(x\frac{dx}{dy}\right)^2 = \left(y + x\frac{dx}{dy}\right)^2 = y^2 + 2xy\frac{dx}{dy} + \left(x\frac{dx}{dy}\right)^2$$

$$\frac{dy}{dx} = \frac{2xy}{x^2 - y^2}$$

which is homogeneous and of first order. Statement-2 is correct but not correct explanation of Statement-1.